5. The ungrounded systems below has a phase to ground fault on phase "a". Assume that the line to ground (and line to neutral voltages) were balanced three phase set before the fault occurred. Do the following:

(a) Find the symmetrical components of the phase a line-to-neutral voltages when a ground fault is applied \((V_{an0}, V_{an1}, V_{an})\).

(b) Repeat part (a) using line to ground voltages instead of the line to neutral voltages and find \((V_{ag0}, V_{ag1}, V_{ag2})\).

6. Do the following

(a) A set of current transformers reads the following currents (in Amperes). If the current transformers each have a turns ratio of 5:500 (usually referred to as a current transformation ratio or CTR of 500:5) calculate the primary currents in amps.

(b) Calculate the symmetrical components of the secondary currents \((I_{a0}, I_{a1}, I_{a2})\).

(c) Calculate the current measured by the fourth ammeter \((I_r)\) and compare it to the zero sequence current calculated in part (a). How do they compare?

(d) Using the primary current calculated in part (a), repeat part (b) if the CTs are connected in delta.
\[ V_{\text{m,012}} = [Z_{\text{ABC}}]^{-1} V_{\text{m,012}} = [Z_{\text{ABC}}]^{-1} \]

Pre-multiply both sides by \([A_{012}]^{-1}\)
**ECE 523: Lecture 5**

**Impedance in Sequence Domain**

\[ a := e^{j\cdot120\text{deg}} \]

<table>
<thead>
<tr>
<th>Phase A symmetrical components transform</th>
<th>Phase B symmetrical components transform</th>
<th>Phase C symmetrical components transform</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & a & a^2 \\
1 & 1 & 1 \\
1 & a^2 & a \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & a^2 & a \\
1 & a & a^2 \\
1 & 1 & 1 \\
\end{pmatrix}
\] |

Case 1:

\[ Z_{aa} := (14.9 + j\cdot58.4)\text{ohm} \]
\[ Z_{bb} := (14.9 + j\cdot58.4)\text{ohm} \]
\[ Z_{cc} := (14.9 + j\cdot58.4)\text{ohm} \]

\[ Z_{ABC} := \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{pmatrix} \]

\[ Z_{O12} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012} \]

\[
Z_{012} = \begin{pmatrix} 22.9 + 113i & 0 & 0 \\ 0 & 10.9 + 31.1i & 0 \\ 0 & 0 & 10.9 + 31.1i \end{pmatrix} \Omega
\]

\[ Z_s := \frac{1}{3} (Z_{aa} + Z_{bb} + Z_{cc}) \]

\[ Z_m := \frac{1}{3} (Z_{ab} + Z_{bc} + Z_{ac}) \]

\[ Z_s - Z_m = (10.9 + 31.1i) \Omega \]
\[ Z_s + 2Z_m = (22.9 + 113i) \Omega \]

\[ Z_{012}_{1,1} = 0 \Omega \]
\[ Z_{012}_{0,0} = 0 \Omega \]
\[ Z_1 := Z_{012,1,1} \quad Z_1 = (10.9 + 31.1i) \Omega \]
\[ Z_2 := Z_{012,2,2} \quad Z_2 = (10.9 + 31.1i) \Omega \]
\[ Z_0 := Z_{012,0,0} \quad Z_0 = (22.9 + 113i) \Omega \]

\[ Z_{012B} := B_{012}^{-1} \cdot Z_{ABC} \cdot B_{012} \]
\[ Z_{012C} := C_{012}^{-1} \cdot Z_{ABC} \cdot C_{012} \]

\[ Z_{012B} - Z_{012} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Omega \]

\[ Z_{012C} - Z_{012} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Omega \]

Case 2: (now add imbalance to self terms—act as if it is a similar tower structure with mutual coupling to other conductors creating unbalance, although I'm being somewhat unrealistic for comparison purposes)

\[ Z_{aa} := (14.9 + j \cdot 53.4) \text{ohm} \]
\[ Z_{ab} := (4 + j \cdot 27.3) \text{ohm} \]
\[ Z_{bb} := (14.9 + j \cdot 68.4) \text{ohm} \]
\[ Z_{ac} := (4 + j \cdot 27.3) \text{ohm} \]
\[ Z_{cc} := (14.9 + j \cdot 53.4) \text{ohm} \]
\[ Z_{bc} := (4 + j \cdot 27.3) \text{ohm} \]

\[ Z_{ABC} := \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{pmatrix} \]

\[ Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012} \]

\[ Z_{012} = \begin{pmatrix} 22.9 + 113i & 4.33 - 2.5i & -4.33 - 2.5i \\ -4.33 - 2.5i & 10.9 + 31.1i & 4.33 - 2.5i \\ 4.33 - 2.5i & -4.33 - 2.5i & 10.9 + 31.1i \end{pmatrix} \Omega \]

\[ Z_s := \frac{1}{3} (Z_{aa} + Z_{bb} + Z_{cc}) \quad Z_s = (14.9 + 58.4i) \Omega \]

\[ Z_m := \frac{1}{3} (Z_{ab} + Z_{bc} + Z_{ac}) \quad Z_m = (4 + 27.3i) \Omega \]
\[ Z_s - Z_m = (10.9 + 31.1i) \Omega \quad Z_s - Z_m - Z_{012}^{1,1} = 0 \Omega \]
\[ Z_s + 2Z_m = (22.9 + 113i) \Omega \quad Z_s + 2Z_m - Z_{012}^{2,0} = 0 \Omega \]

Compare to original case:
\[ Z_0 - (Z_s + 2Z_m) = 0 \Omega \quad Z_1 - (Z_s - Z_m) = 0 \Omega \]
\[ Z_{012B} := B_{012}^{-1} \cdot Z_{ABC} \cdot B_{012} \quad Z_{012C} := C_{012}^{-1} \cdot Z_{ABC} \cdot C_{012} \]

\[
Z_{012B} - Z_{012} = \begin{pmatrix}
0 & -4.33 + 7.5i & 4.33 + 7.5i \\
4.33 + 7.5i & 0 & -4.33 + 7.5i \\
-4.33 + 7.5i & 4.33 + 7.5i & 0
\end{pmatrix} \Omega
\]

\[
Z_{012C} - Z_{012} = \begin{pmatrix}
0 & -8.66 & 8.66 \\
8.66 & 0 & -8.66 \\
-8.66 & 8.66 & 0
\end{pmatrix} \Omega
\]

Note that the diagonal terms do not change, just the coupling terms.

Case 3: (A more realistic set of unbalances) smaller unbalance

\[ Z_{aa} := (14.9 + j \cdot 57.0) \Omega \quad Z_{ab} := (4 + j \cdot 27.3) \Omega \]
\[ Z_{bb} := (14.9 + j \cdot 58.2) \Omega \quad Z_{ac} := (4 + j \cdot 27.3) \Omega \]
\[ Z_{cc} := (14.9 + j \cdot 60.0) \Omega \quad Z_{bc} := (4 + j \cdot 27.3) \Omega \]

\[
Z_{ABC} := \begin{pmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ab} & Z_{bb} & Z_{bc} \\
Z_{ac} & Z_{bc} & Z_{cc}
\end{pmatrix}
\]

\[ Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012} \quad \text{Phase A Components} \]

\[
Z_{012} = \begin{pmatrix}
22.9 + 113i & -0.52 - 0.7i & 0.52 - 0.7i \\
0.52 - 0.7i & 10.9 + 31.1i & -0.52 - 0.7i \\
-0.52 - 0.7i & 0.52 - 0.7i & 10.9 + 31.1i
\end{pmatrix} \Omega
\]

more diagonal dominant
\[
\frac{1}{3} \text{ left} \quad A \quad B \quad C \\
\frac{1}{3} \text{ left} \quad B \quad C \quad A \\
\frac{1}{3} \quad C \quad A \quad B
\]

\[\rightarrow \text{ need to be transposed}\]
\[ Z_s := \frac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) \quad Z_s = (14.9 + 58.4i) \Omega \]

\[ Z_m := \frac{1}{3}(Z_{ab} + Z_{bc} + Z_{ca}) \quad Z_m = (4 + 27.3i) \Omega \]

\[ Z_s - Z_m = (10.9 + 31.1i) \Omega \quad Z_s - Z_m - Z_{012} = 0 \Omega \]

\[ Z_s + 2Z_m = (22.9 + 113i) \Omega \quad Z_s + 2Z_m - Z_{012} = 0 \Omega \]

Compare to original case:

\[ Z_0 - (Z_s + 2Z_m) = 0 \Omega \quad Z_1 - (Z_s - Z_m) = 0 \Omega \]

Phase B and Phase C components

\[ Z_{012B} := B_{012} - Z_{ABC}B_{012} \quad Z_{012C} := C_{012} - Z_{ABC}C_{012} \]

\[ Z_{012B} - Z_{012} = \begin{pmatrix} 0 & 1.39 + 0.6i & -1.39 + 0.6i \\ -1.39 + 0.6i & 0 & 1.39 + 0.6i \\ 1.39 + 0.6i & -1.39 + 0.6i & 0 \end{pmatrix} \Omega \]

\[ Z_{012C} - Z_{012} = \begin{pmatrix} 0 & 0.17 + 1.5i & -0.17 + 1.5i \\ -0.17 + 1.5i & 0 & 0.17 + 1.5i \\ 0.17 + 1.5i & -0.17 + 1.5i & 0 \end{pmatrix} \Omega \]

Again, only the off diagonal terms change

Case 4: (Now add imbalances to mutual terms only)  

\[ Z_{aa} := (14.9 + j\cdot 58.4) \text{ohm} \quad Z_{ab} := (4 + j\cdot 27.4) \text{ohm} \]

\[ Z_{bb} := (14.9 + j\cdot 58.4) \text{ohm} \quad Z_{bc} := (4 + j\cdot 28.0) \text{ohm} \]

\[ Z_{cc} := (14.9 + j\cdot 58.4) \text{ohm} \quad Z_{za} := (4 + j\cdot 26.5) \text{ohm} \]

\[ Z_{ABC} := \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{pmatrix} \]
\[V_{012} = (Z_{012}) \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}\]

Normal
\[
\begin{bmatrix}
0 \\
1000 \\
0
\end{bmatrix}
\]

\[I_0 = 0, \quad I_2 = 0\]

\[V_0 = 0 + (-0.17 + j0.4)1000 + 0\]

\[V_1 = (10.9 + j31.1) \cdot 1000 + 0\]

\[V_2 = 0 + 0 + (-0.35 - j0.8)1000 + 0\]
$Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012}$

$Z_{012} = \begin{pmatrix}
22.9 + 113i & -0.17 + 0.4i & 0.17 + 0.4i \\
0.17 + 0.4i & 10.9 + 31.1i & 0.35 - 0.8i \\
-0.17 + 0.4i & -0.35 - 0.8i & 10.9 + 31.1i
\end{pmatrix} \Omega$

$Z_{s2} := \frac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) \quad Z_{s2} = (14.9 + 58.4i) \Omega \quad Z_s = (14.9 + 58.4i) \Omega$

$Z_m := \frac{1}{3}(Z_{ab} + Z_{bc} + Z_{ac}) \quad Z_m = (4 + 27.3i) \Omega$

$Z_s - Z_m = (10.9 + 31.1i) \Omega \quad Z_s - Z_m - Z_{012,1,1} = 0 \Omega$

$Z_s + 2Z_m = (22.9 + 113i) \Omega \quad Z_s + 2Z_m - Z_{012,0,0} = 0 \Omega$

Compare to original case:

$Z_0 - (Z_s + 2Z_m) = 0 \Omega \quad Z_1 - (Z_s - Z_m) = 0 \Omega$

Phase B and C based components

$Z_{012B} := B_{012}^{-1} \cdot Z_{ABC} \cdot B_{012} \quad Z_{012C} := C_{012}^{-1} \cdot Z_{ABC} \cdot C_{012}$

$Z_{012B} - Z_{012} = \begin{pmatrix}
0 & -0.09 - 0.75i & 0.09 - 0.75i \\
0.09 - 0.75i & 0 & 0.17 + 1.5i \\
-0.09 - 0.75i & -0.17 + 1.5i & 0
\end{pmatrix} \Omega$

$Z_{012C} - Z_{012} = \begin{pmatrix}
0 & 0.61 - 0.45i & -0.61 - 0.45i \\
-0.61 - 0.45i & 0 & -1.21 + 0.9i \\
0.61 - 0.45i & 1.21 + 0.9i & 0
\end{pmatrix} \Omega$

Case 5 (both self and mutual unbalanced):

$Z_{aa} := (14.9 + j \cdot 57.0)\text{ohm} \quad Z_{ab} := (4 + j \cdot 27.4)\text{ohm}$

$Z_{bb} := (14.9 + j \cdot 58.2)\text{ohm} \quad Z_{ac} := (4 + j \cdot 28.0)\text{ohm}$

$Z_{cc} := (14.9 + j \cdot 60.0)\text{ohm} \quad Z_{bc} := (4 + j \cdot 26.5)\text{ohm}$
\[
\begin{align*}
I_a := 100A \cdot e^{j0\text{deg}} &= I_0 \\
I_b := 2000A \cdot e^{j30\text{deg}} &= I_T \\
I_c := 1900A \cdot e^{j210\text{deg}} &= I_0 \\
I_{012} := A_{012}^{-1} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} &\rightarrow \begin{pmatrix} 64.4 \\ 1109.23 \\ 1142.56 \end{pmatrix} \begin{pmatrix} I_{01} \\ I_{12} \end{pmatrix} \\
\text{arg}(I_{012}) &= \begin{pmatrix} 15 \\ 119.37 \end{pmatrix} \cdot \text{deg} \\
I_{PNZ} := H_{PNZ}^{-1} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} &\rightarrow \begin{pmatrix} 1109.23 \\ 1142.56 \\ 64.4 \end{pmatrix} \begin{pmatrix} I_{01} \\ I_{12} \end{pmatrix} \\
\text{arg}(I_{PNZ}) &= \begin{pmatrix} 119.37 \\ -59.39 \end{pmatrix} \cdot \text{deg}
\end{align*}
\]

\[P \equiv \text{Positive}\]
\[N \equiv \text{Negative}\]
\[Z \equiv \text{Zero}\]
Symmetrical Components Transformations
European versus North American Notation

\[ a := e^{j \cdot 120^\circ} \]

Phase A symmetrical components transform (A)

\[ A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \]

\[ h := e^{j \cdot 120^\circ} \]

Phase R symmetrical components transform (H)

\[ H_{PNZ} := \begin{pmatrix} 1 & 1 & 1 \\ h^2 & h & 1 \\ h & h^2 & 1 \end{pmatrix} \]

\[ Z_{aa} := (14.9 + j \cdot 58.4) \text{ohm} \quad Z_{ab} := (4 + j \cdot 27.3) \text{ohm} \]
\[ Z_{bb} := (14.9 + j \cdot 58.4) \text{ohm} \quad Z_{ac} := (4 + j \cdot 27.3) \text{ohm} \]
\[ Z_{cc} := (14.9 + j \cdot 58.4) \text{ohm} \quad Z_{bc} := (4 + j \cdot 27.3) \text{ohm} \]

\[ Z_{ABC} := \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{pmatrix} \]

\[ Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012} \]

\[ Z_{012} = \begin{pmatrix} 22.9 + 113i & 0 & 0 \\ 0 & 10.9 + 31.1i & 0 \\ 0 & 0 & 10.9 + 31.1i \end{pmatrix} \]

\[ Z_{PNZ} := H_{PNZ}^{-1} \cdot Z_{ABC} \cdot H_{PNZ} \]

\[ Z_{PNZ} \]

\[ \begin{pmatrix} 10.9 + 31.1i & 0 & 0 \\ 0 & 10.9 + 31.1i & 0 \\ 0 & 0 & 22.9 + 113i \end{pmatrix} \]

\[ \Omega \]
**ECE 523: Symmetrical Components Examples**

If a load is unbalanced, its neutral, m, will not be at the same potential as the source neutral, n. Derive the relationship between the neutral shift \( V_{mn} \) and the zero sequence voltage, \( V_{amo} \) for the system shown below. Hint, consider the line to ground voltages, line to neutral voltages and neutral to ground voltages.

\[
V_m - n = V_m_{\text{ground}} - V_n_{\text{ground}}
\]

\[
V_a - n = V_a_{\text{ground}} - V_n_{\text{ground}}
\]

and \( V_a - m = V_a_{\text{ground}} - V_m_{\text{ground}} \)

so \( V_m - n = V_m_{\text{ground}} - V_n_{\text{ground}} = (V_a_{\text{ground}} - V_n_{\text{ground}}) - (V_a_{\text{ground}} - V_m_{\text{ground}}) \)

Then:

\( V_m - n = V_a - n - V_a - m \)

similarly \( V_m - n = V_b - n - V_b - m \)

\( V_m - n = V_c - n - V_c - m \)

\} \text{Take sum of these 3}
- Adding these three expressions, results in:
  \[ 3V_{m_n} = (V_{an} + V_{bn} + V_{cn}) - (V_{am} + V_{bm} + V_{cm}) \]

- Applying the Symmetrical Components transformation:
  \[ 3V_{an0} = (V_{an} + V_{bn} + V_{cn}) = 0 \]
  Since the source is still balanced

Similarly
  \[ 3V_{am0} = (V_{am} + V_{bm} + V_{cm}) \]
  This does not sum to 0, since the load is unbalanced

Therefore
  \[ 3V_{m_n} = 3V_{an0} - 3V_{am0} = -3V_{am0} \]
  \[ V_{m_n} = -V_{am0} \]

As a check, the circuit was simulated with ATPDraw, and then ATP Analyzer was used to determine the symmetrical components using instantaneous quantities.

- First we see \( V_{ag}, V_{bg}, V_{cg}, V_{ng} \) and \( V_{mg} \) (brown line)
Next plot instantaneous values of Vmn and V0 (note that V0 is simply \(1/3*(Vam + Vbm + Vcm)\). Note the 180 degree phase difference.

ATPDraw Schematic

- The capacitors were added to provide a ground reference. The capacitive reactances are quite large and don't impact the results otherwise.
- Control modelling language TACS used to calculate V0