ECE 523
Symmetrical Components

Session 6

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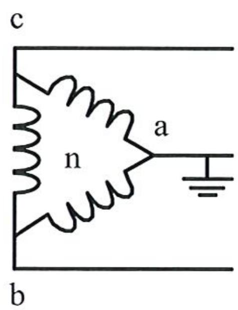
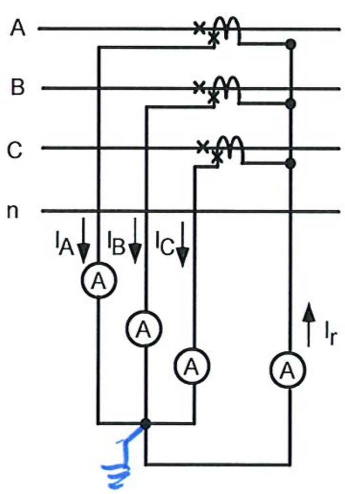
5. The ungrounded system below has a phase to ground fault on phase "a". Assume that the line to ground (and line to neutral voltages) were balanced three phase set before the fault occured. Do the following:
   (a) Find the symmetrical components of the phase a line-to-neutral voltages when a ground fault is applied (V_{an0}, V_{an1}, V_{an2}).
   (b) Repeat part (a) using line to ground voltages instead of the line to neutral voltages and find (V_{ag0}, V_{ag1}, V_{ag2}).



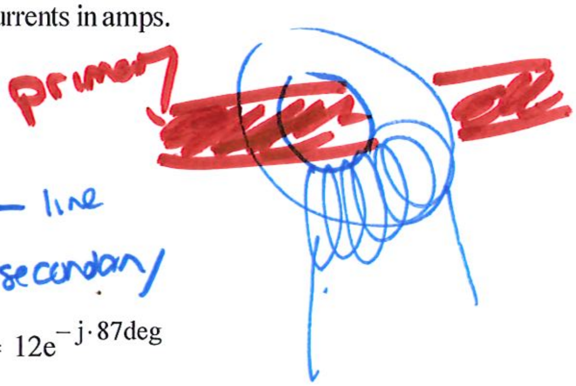
6. Do the following:

(a) A set of current transformers reads the following currents (in Amperes). If the current transformers each have a turns ratio of 5:500 (usually referred to as a current transformation ratio or CTR of 500:5) calculate the primary currents in amps.



Note that the symbol:

is equivalent to:



$I_A := 12e^{-j\cdot 87deg}$

$I_B := 4\cdot e^{-j\cdot 120deg}$

$I_C := 4\cdot e^{j\cdot 120deg}$

(b) Calculate the symmtrical components of the secondary currents (I_{a0}, I_{a1}, I_{a2}).

(c) Calculate the current measured by the fourth ammeter (I_r) and compare it to the zero sequence current calculated in part (b). How do they compare?

(d) Using the primary current calculated in part (a), repeat part (b) if the CTs are connected in delta (relay currents should lag the primary line currents by 30 degrees).

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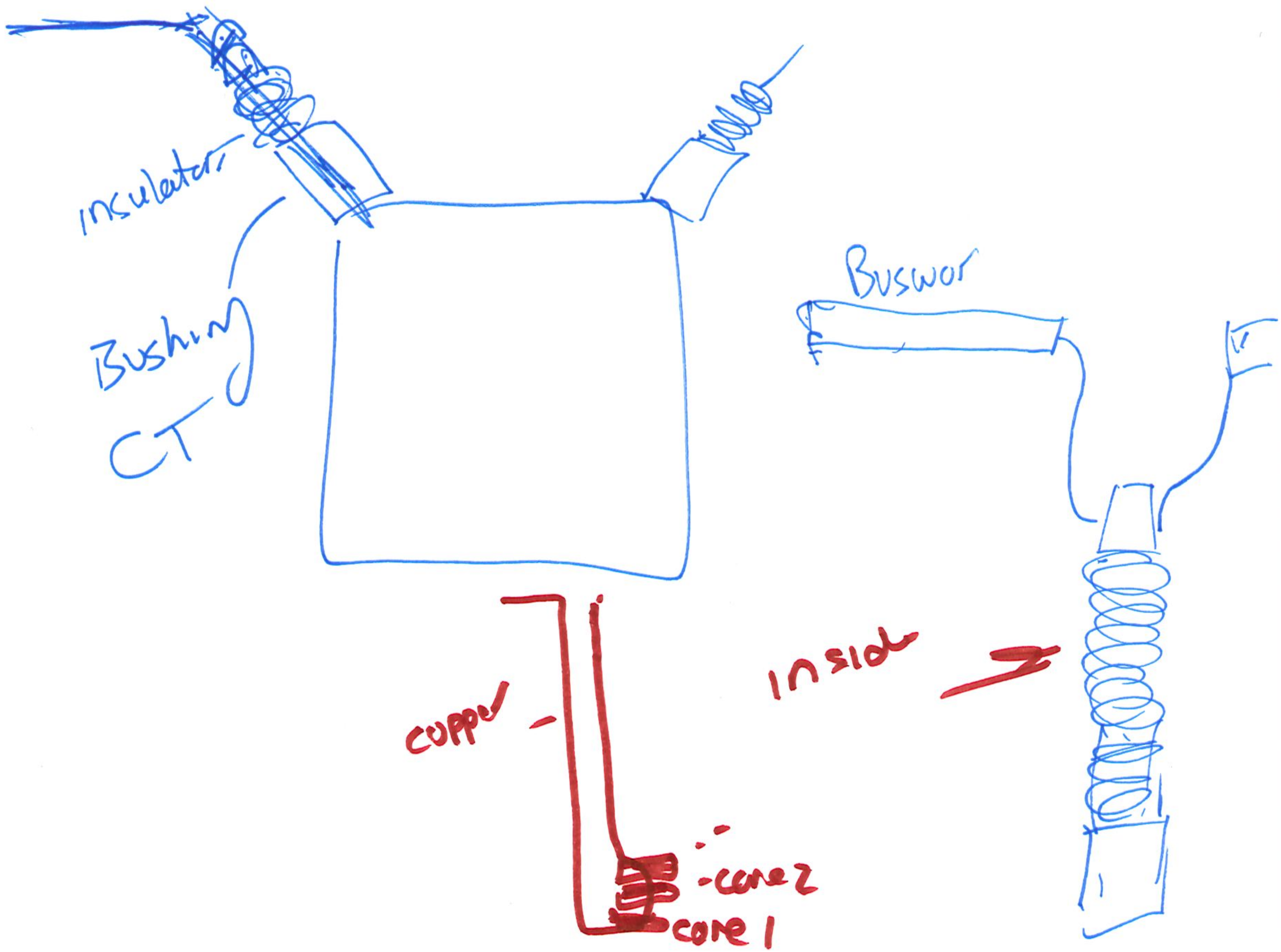

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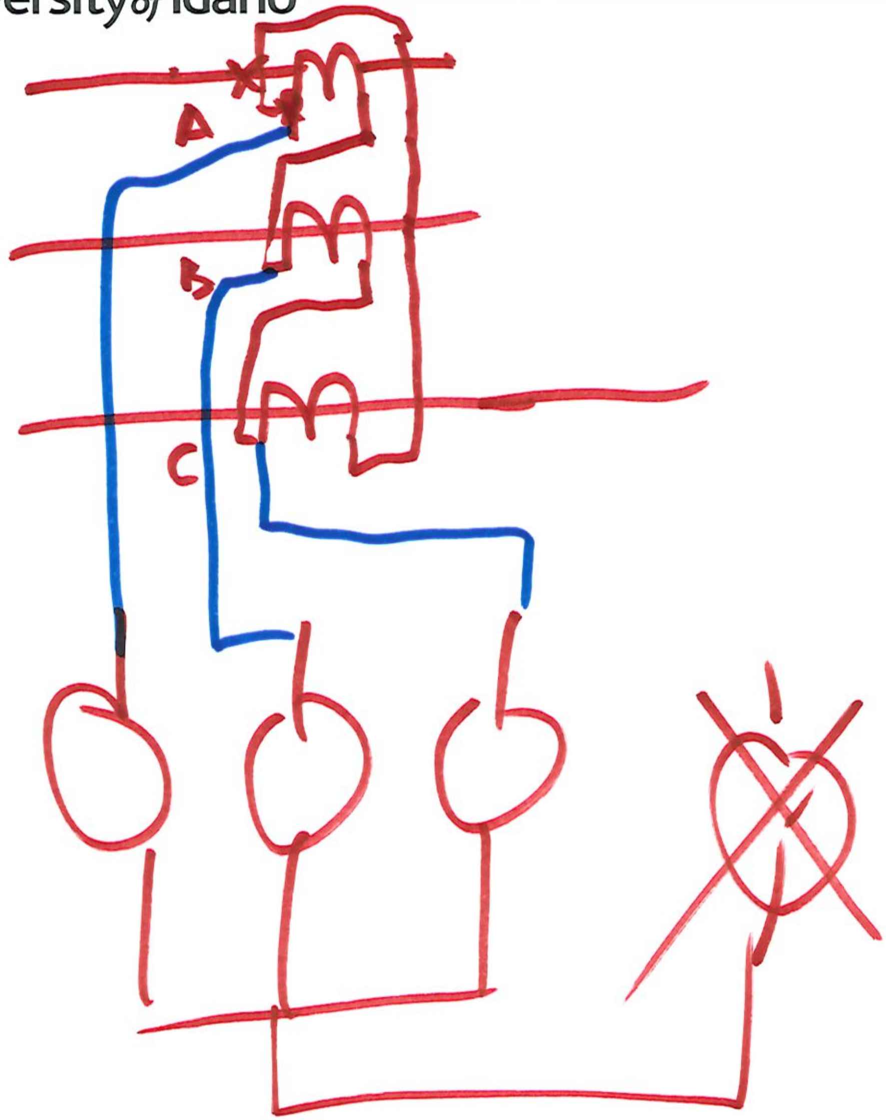

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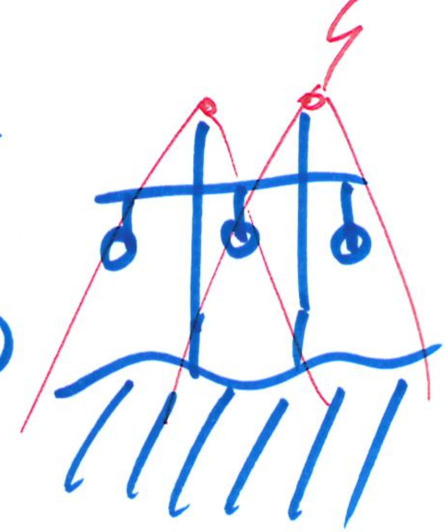
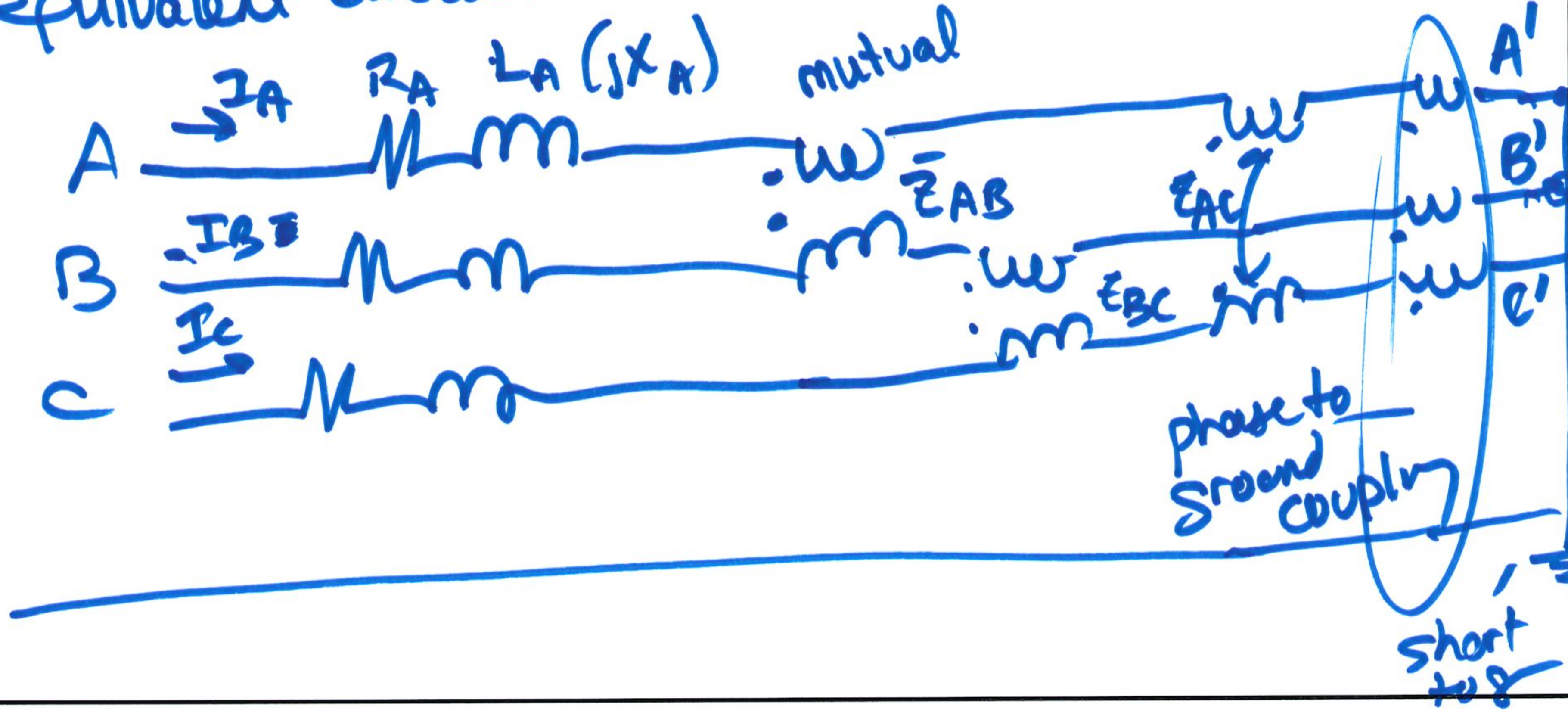
-Impedances in sequence domain

Transmission line



equivalent circuit



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$R_A + jX_A$

$$\begin{bmatrix} V_{A-A'} \\ V_{B-B'} \\ V_{C-C'} \end{bmatrix} = \underbrace{\begin{bmatrix} \bar{Z}_{AA} & \bar{Z}_{AB} & \bar{Z}_{AC} \\ \bar{Z}_{BA} & \bar{Z}_{BB} & \bar{Z}_{BC} \\ \bar{Z}_{CA} & \bar{Z}_{CB} & \bar{Z}_{CC} \end{bmatrix}}_{[Z_{ABC}]} \begin{bmatrix} \bar{I}_A \\ \bar{I}_B \\ \bar{I}_C \end{bmatrix}$$

$\bar{Z}_{AB} = \bar{Z}_{BA}$ etc

$V_{AG} - V_{A'G} = V_{A-A'}$

Now - add 3Ø short to ground at A', B', C' $V_{A'G} = V_{B'G} = \bar{V}_{C'G} = 0$

$$\begin{bmatrix} \bar{V}_{AG} \\ \bar{V}_{BG} \\ \bar{V}_{CG} \end{bmatrix} = \left[\underline{A_{012}} \right] \begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} \qquad \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \left[A_{012} \right] \begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_{AG} \\ \bar{V}_{BG} \\ \bar{V}_{CG} \end{bmatrix} = \left[\bar{Z}_{ABC} \right] \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \underbrace{\left[A_{012} \right] \begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \left[Z_{ABC} \right] \underbrace{\left[A_{012} \right] \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}}_{\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}}}_{\Downarrow}$$

$$\begin{bmatrix} \bar{V}_{AG} \\ \bar{V}_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} & & \\ & Z_{ABC} & \\ & & \end{bmatrix} \begin{bmatrix} \bar{I}_A \\ \bar{I}_B \\ \bar{I}_o \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \begin{bmatrix} & & \\ & Z_{012} & \\ & & \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$\bar{V}_0 \equiv \bar{V}_{A0}$ $\qquad$ $\bar{I}_0 = \bar{I}_{A0}$

$\bar{V}_1 \equiv \bar{V}_{A1}$ $\qquad$ $\bar{I}_1 = \bar{I}_{A1}$

$\bar{V}_2 = \bar{V}_{A2}$ $\qquad$ $\bar{I}_2 = \bar{I}_{A2}$

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Premultiply both sides by $[A_{012}]^{-1}$

$$\underbrace{[A_{012}]^{-1}[A_{012}]}\begin{bmatrix}\bar{V}_0\\ \bar{V}_1\\ V_2\end{bmatrix} = [A_{012}]^{-1}[Z_{ABC}][A_{012}]\begin{bmatrix}I_0\\ \bar{I}_1\\ \bar{I}_2\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = [I] \qquad [I]\begin{bmatrix}V_0\\ V_1\\ V_2\end{bmatrix} = \begin{bmatrix}\bar{V}_0\\ \bar{V}_1\\ \bar{V}_2\end{bmatrix}$$

$$\begin{bmatrix}\bar{V}_0\\ \bar{V}_1\\ \bar{V}_2\end{bmatrix} = \underbrace{[A_{012}]^{-1}[Z_{ABC}][A_{012}]}_{[Z_{012}]}\begin{bmatrix}I_0\\ I_1\\ I_2\end{bmatrix}$$

$$[Z_{012}] = [A_{012}]^{-1}[Z_{ABC}][A_{012}]$$

If $[Z_{ABC}] = \begin{bmatrix} Z & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} = [Z_{012}]$

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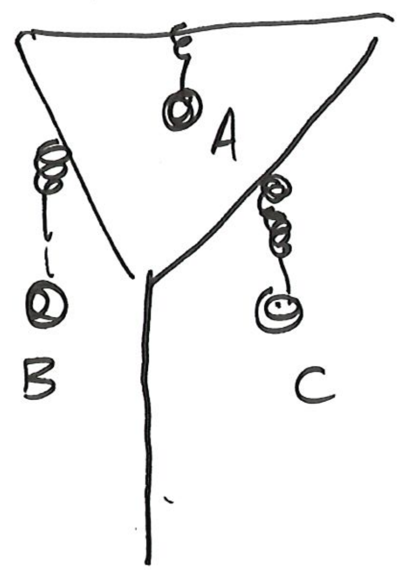
ECE 523: Lecture 5

Impedance in Sequence Domain

$a := e^{j \cdot 120deg}$

Phase A symmetrical components transform	Phase B symmetrical components transform	Phase C symmetrical components transform
$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$	$B_{012} := \begin{pmatrix} 1 & a & a^2 \\ 1 & 1 & 1 \\ 1 & a^2 & a \end{pmatrix}$	$C_{012} := \begin{pmatrix} 1 & a^2 & a \\ 1 & a & a^2 \\ 1 & 1 & 1 \end{pmatrix}$

Case 1:



$Zaa := (14.9 + j \cdot 58.4)ohm$ $\qquad$ $Zab := (4 + j \cdot 27.3)ohm$

$Zbb := (14.9 + j \cdot 58.4)ohm$ $\qquad$ $Zac := (4 + j \cdot 27.3)ohm$

$Zcc := (14.9 + j \cdot 58.4)ohm$ $\qquad$ $Zbc := (4 + j \cdot 27.3)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} \overset{Z_0}{22.9 + 113i} & 0 & 0 \\ 0 & \overset{Z_1}{10.9 + 31.1i} & 0 \\ 0 & 0 & \overset{Z_2}{10.9 + 31.1i} \end{pmatrix} \Omega$$

$$Z_s := \frac{1}{3}(Zaa + Zbb + Zcc)$$

$$Z_m := \frac{1}{3} \cdot (Zab + Zbc + Zac)$$

$Z_2 = Z_1 =$ $\quad$ $Z_s - Z_m = (10.9 + 31.1i)\,\Omega$ $\qquad$ $Z_s - Z_m - Z_{012_{1,1}} = 0\,\Omega$

$Z_0 =$ $\quad$ $Z_s + 2Z_m = (22.9 + 113i)\,\Omega$ $\qquad$ $Z_s + 2Z_m - Z_{012_{0,0}} = 0\,\Omega$

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$$Z_1 := Z_{012_{1,1}} \qquad Z_1 = (10.9 + 31.1i)\,\Omega$$

$$Z_2 := Z_{012_{2,2}} \qquad Z_2 = (10.9 + 31.1i)\,\Omega$$

$$Z_0 := Z_{012_{0,0}} \qquad Z_0 = (22.9 + 113i)\,\Omega$$

$$Z_{012B} := {B_{012}}^{-1} \cdot Z_{ABC} \cdot B_{012} \qquad Z_{012C} := {C_{012}}^{-1} \cdot Z_{ABC} \cdot C_{012}$$

$$\underset{A}{Z_{012B} - Z_{012}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\Omega \qquad \underset{A}{Z_{012C} - Z_{012}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\Omega$$

Case 2: (now add imbalance to self terms--act as if it is a similar tower structure with mutual coupling to other conductors creating unbalance, although I'm being somewhat unrealistic for comparison purposes)

$$Zaa := (14.9 + j \cdot 53.4)\text{ohm} \qquad Zab := (4 + j \cdot 27.3)\text{ohm}$$

$$Zbb := (14.9 + j \cdot 68.4)\text{ohm} \qquad Zac := (4 + j \cdot 27.3)\text{ohm}$$

$$Zcc := (14.9 + j \cdot 53.4)\text{ohm} \qquad Zbc := (4 + j \cdot 27.3)\text{ohm}$$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := {A_{012}}^{-1} \cdot Z_{ABC} \cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} \overset{Z_0}{22.9 + 113i} & 4.33 - 2.5i & -4.33 - 2.5i \\ -4.33 - 2.5i & \overset{Z_1}{10.9 + 31.1i} & 4.33 - 2.5i \\ 4.33 - 2.5i & -4.33 - 2.5i & \underset{Z_2}{10.9 + 31.1i} \end{pmatrix}\Omega$$

Z_0, Z_1, Z_2 each an order of magnitude larger than off diagonal

$$Z_s := \frac{1}{3}(Zaa + Zbb + Zcc) \qquad Z_s = (14.9 + 58.4i)\,\Omega$$

$$Z_m := \frac{1}{3} \cdot (Zab + Zbc + Zac) \qquad Z_m = (4 + 27.3i)\,\Omega$$

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$$Z_s - Z_m = (10.9 + 31.1i)\,\Omega \qquad Z_s - Z_m - Z012_{1,1} = 0\,\Omega$$

$$Z_s + 2Z_m = (22.9 + 113i)\,\Omega \qquad Z_s + 2Z_m - Z012_{0,0} = 0\,\Omega$$

Compare to original case:

unchanged

$$Z_0 - (Z_s + 2Z_m) = 0\,\Omega \qquad Z_1 - (Z_s - Z_m) = 0\,\Omega$$

$$Z_{012B} := B_{012}^{-1} \cdot Z_{ABC} \cdot B_{012} \qquad Z_{012C} := C_{012}^{-1} \cdot Z_{ABC} \cdot C_{012}$$

$$Z_{012B} - Z_{012} = \begin{pmatrix} 0 & -4.33+7.5i & 4.33+7.5i \\ 4.33+7.5i & 0 & -4.33+7.5i \\ -4.33+7.5i & 4.33+7.5i & 0 \end{pmatrix}\Omega$$

$$Z_{012C} - Z_{012} = \begin{pmatrix} 0 & -8.66 & 8.66 \\ 8.66 & 0 & -8.66 \\ -8.66 & 8.66 & 0 \end{pmatrix}\Omega$$

Note that the diagonal terms do not change, just the coupling terms.

Case 3: (A more realistic set of unbalances)

$$Zaa := (14.9 + j\cdot 57.0)\text{ohm} \qquad Zab := (4 + j\cdot 27.3)\text{ohm}$$

$$Zbb := (14.9 + j\cdot 58.2)\text{ohm} \qquad Zac := (4 + j\cdot 27.3)\text{ohm}$$

$$Zcc := (14.9 + j\cdot 60.0)\text{ohm} \qquad Zbc := (4 + j\cdot 27.3)\text{ohm}$$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012}$$

Phase A Components

$$Z_{012} = \begin{pmatrix} 22.9+113i & -0.52-0.7i & 0.52-0.7i \\ 0.52-0.7i & 10.9+31.1i & -0.52-0.7i \\ -0.52-0.7i & 0.52-0.7i & 10.9+31.1i \end{pmatrix}\Omega$$

much smaller off diagonal

$$Z_s := \frac{1}{3}(Zaa + Zbb + Zcc) \qquad Z_s = (14.9 + 58.4i)\,\Omega$$

$$Z_m := \frac{1}{3}\cdot(Zab + Zbc + Zac) \qquad Z_m = (4 + 27.3i)\,\Omega$$

$$Z_s - Z_m = (10.9 + 31.1i)\,\Omega \qquad Z_s - Z_m - Z_{012_{1,1}} = 0\,\Omega$$

$$Z_s + 2Z_m = (22.9 + 113i)\,\Omega \qquad Z_s + 2Z_m - Z_{012_{0,0}} = 0\,\Omega$$

Compare to original case:

$$Z_0 - \left(Z_s + 2Z_m\right) = 0\,\Omega \qquad Z_1 - \left(Z_s - Z_m\right) = 0\,\Omega$$

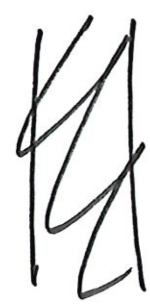
Phase B and Phase C components

$$Z_{012B} := B_{012}{}^{-1}\cdot Z_{ABC}\cdot B_{012} \qquad Z_{012C} := C_{012}{}^{-1}\cdot Z_{ABC}\cdot C_{012}$$

$$Z_{012B} - Z_{012} = \begin{pmatrix} 0 & 1.39+0.6i & -1.39+0.6i \\ -1.39+0.6i & 0 & 1.39+0.6i \\ 1.39+0.6i & -1.39+0.6i & 0 \end{pmatrix}\Omega$$

$$Z_{012C} - Z_{012} = \begin{pmatrix} 0 & 0.17+1.5i & -0.17+1.5i \\ -0.17+1.5i & 0 & 0.17+1.5i \\ 0.17+1.5i & -0.17+1.5i & 0 \end{pmatrix}\Omega$$

Again, only the off diagonal terms change



Case 4: (Now add imbalances to mutual terms only)

$$Zaa := (14.9 + j\cdot 58.4)\text{ohm} \qquad Zab := (4 + j\cdot 27.4)\text{ohm}$$

$$Zbb := (14.9 + j\cdot 58.4)\text{ohm} \qquad Zac := (4 + j\cdot 28.0)\text{ohm}$$

$$Zcc := (14.9 + j\cdot 58.4)\text{ohm} \qquad Zbc := (4 + j\cdot 26.5)\text{ohm}$$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.9 + 113i & -0.17 + 0.4i & 0.17 + 0.4i \\ 0.17 + 0.4i & 10.9 + 31.1i & 0.35 - 0.8i \\ -0.17 + 0.4i & -0.35 - 0.8i & 10.9 + 31.1i \end{pmatrix} \Omega$$

$$Z_{s2} := \frac{1}{3}(Zaa + Zbb + Zcc) \qquad Z_{s2} = (14.9 + 58.4i)\,\Omega \qquad Z_s = (14.9 + 58.4i)\,\Omega$$

$$Z_m := \frac{1}{3} \cdot (Zab + Zbc + Zac) \qquad Z_m = (4 + 27.3i)\,\Omega$$

$$Z_s - Z_m = (10.9 + 31.1i)\,\Omega \qquad Z_s - Z_m - Z_{012_{1,1}} = 0\,\Omega$$

$$Z_s + 2Z_m = (22.9 + 113i)\,\Omega \qquad Z_s + 2Z_m - Z_{012_{0,0}} = 0\,\Omega$$

Compare to original case:

$$Z_0 - \left(Z_s + 2Z_m\right) = 0\,\Omega \qquad Z_1 - \left(Z_s - Z_m\right) = 0\,\Omega$$

Phase B and C based components

$$Z_{012B} := B_{012}^{-1} \cdot Z_{ABC} \cdot B_{012} \qquad Z_{012C} := C_{012}^{-1} \cdot Z_{ABC} \cdot C_{012}$$

$$Z_{012B} - Z_{012} = \begin{pmatrix} 0 & -0.09 - 0.75i & 0.09 - 0.75i \\ 0.09 - 0.75i & 0 & 0.17 + 1.5i \\ -0.09 - 0.75i & -0.17 + 1.5i & 0 \end{pmatrix} \Omega$$

$$Z_{012C} - Z_{012} = \begin{pmatrix} 0 & 0.61 - 0.45i & -0.61 - 0.45i \\ -0.61 - 0.45i & 0 & -1.21 + 0.9i \\ 0.61 - 0.45i & 1.21 + 0.9i & 0 \end{pmatrix} \Omega$$

Case 5 (both self and mutual unbalanced):

$$Zaa := (14.9 + j \cdot 57.0)\text{ohm} \qquad Zab := (4 + j \cdot 27.4)\text{ohm}$$

$$Zbb := (14.9 + j \cdot 58.2)\text{ohm} \qquad Zac := (4 + j \cdot 28.0)\text{ohm}$$

$$Zcc := (14.9 + j \cdot 60.0)\text{ohm} \qquad Zbc := (4 + j \cdot 26.5)\text{ohm}$$

complex power

$$\bar{S}_{3\phi} = \bar{S}_{ABC} \Rightarrow \bar{S}_{012} ?$$

$$\bar{S}_{3\phi} = \cancel{\bar{V}_{LG\,ABC}\left(\bar{I}_{ABC}\right)^{*}}$$

$$\cancel{= \bar{S}_{3\phi}}$$

$$\bar{S}_{3\phi} = 3\bar{V}_{GN}\bar{I}_{\phi}^{*} \quad \text{- Balanced } 3\phi$$

$$= \bar{V}_{AG}\bar{I}_A^{*} + \bar{V}_{BG}\bar{I}_B^{*} + \bar{V}_{CG}\bar{I}_C^{*}$$

$$\bar{S}_{ABC} = \begin{bmatrix} \bar{V}_{AG} & \bar{V}_{BG} & \bar{V}_{CG} \end{bmatrix} \begin{bmatrix} \bar{I}_A^* \\ \bar{I}_B^* \\ \bar{I}_C^* \end{bmatrix}$$

$$= \left[\bar{V}_{ABC}\right]^T \left[\bar{I}_{ABC}\right]^*$$