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ECE 523
Symmetrical Components

Session 7

L7 1/20 ~~L6 15/16~~

complex powe

$$\bar{S}_{3\phi} = \bar{S}_{ABC} \Rightarrow \bar{S}_{012} ?$$

$$\bar{S}_{3\phi} = \cancel{\bar{V}_{LG\,ABC}\left(\bar{I}_{ABC}\right)^*}$$

$$\bar{S}_{3\phi} = 3\bar{V}_{\phi N}\bar{I}_{\phi}^{*} \quad \text{— Balanced } 3\phi$$

$$= \bar{V}_{AG}\bar{I}_A^* + \bar{V}_{BG}\bar{I}_B^* + \bar{V}_{CG}\bar{I}_C^*$$

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$$\bar{S}_{ABC} = \begin{bmatrix} \bar{V}_{AG} & \bar{V}_{BG} & \bar{V}_{CG} \end{bmatrix} \begin{bmatrix} \bar{I}_A^* \\ \bar{I}_B^* \\ \bar{I}_C^* \end{bmatrix}$$

$$= \underline{[\bar{V}_{ABC}]^T [\underline{I}_{ABC}]^*}$$

$$\Downarrow$$

$$= [A_{012} \cdot \underline{V}_{012}]^T [[A_{012}] \underline{I}_{012}]^*$$

$$= [V_{012}]^T \underbrace{[A_{012}]^T [A_{012}]^*}\ \underline{I}_{012}^*$$

$$\underline{V}_{012}^T [A_{012}] * 3 (A_{012})^{-1} \underline{I}_{012}^*$$

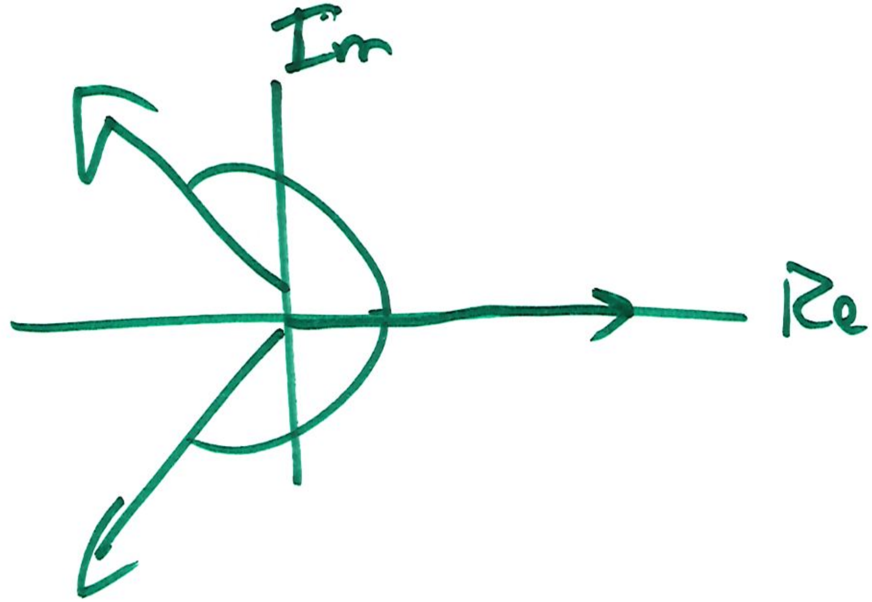
$$[A_{012}] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} = [A_{012}]^T$$

$$[A_{012}]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = 3[A_{012}]^{-1}$$

$$a = 1\angle 120^\circ = \cos 120^\circ + j\sin 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = (a^2)^*$$

$$a^2 = 1\angle 240^\circ = \cos 240 + j\sin 240^\circ = -\frac{1}{2} - j\sqrt{3}/2 = a^*$$

L7 4/10

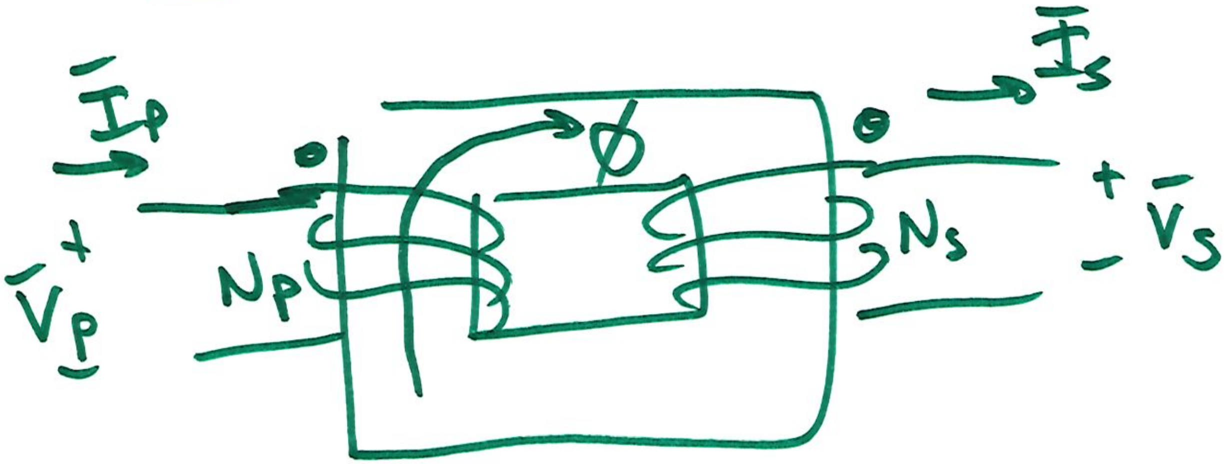


$$\bar{S}_{ABC} = \bar{S}_{012} = 3\,\underline{V}_{012}^{T}\,\underline{I}_{012}^{*}$$

$$= 3\left[\bar{V}_0 \bar{I}_0^{*} + \bar{V}_1 \bar{I}_1^{*} + \bar{V}_2 \bar{I}_2^{*}\right]$$

L7 5/20

Flux / Flux Linkages?



$$I_p \cdot N_p + I_s N_s = 0$$

$$\frac{V_p}{N_p} = \frac{V_s}{N_s} \qquad V = N\frac{d\phi}{dt}$$

- 3ϕ Transformers will have positive, negative and zero sequence flux components
- same with machines

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5 leg, 3 phase core

$$V_0 \rightarrow \bar{V}_{0A} = \bar{V}_{0B} = \bar{V}_{0C}$$

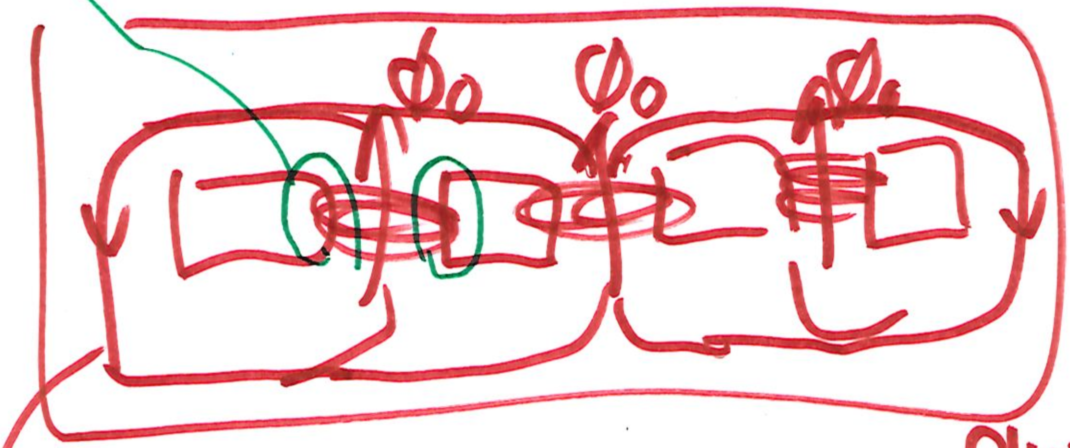
$$\Rightarrow \bar{\phi}_{0A} = \bar{\phi}_{0B} = \bar{\phi}_{0C}$$

$$\bar{V}_{1A} + \bar{V}_{1B} + \bar{V}_{1C} = 0$$

$$\Rightarrow \bar{\phi}_{1A} + \bar{\phi}_{1B} + \bar{\phi}_{1C} = 0$$

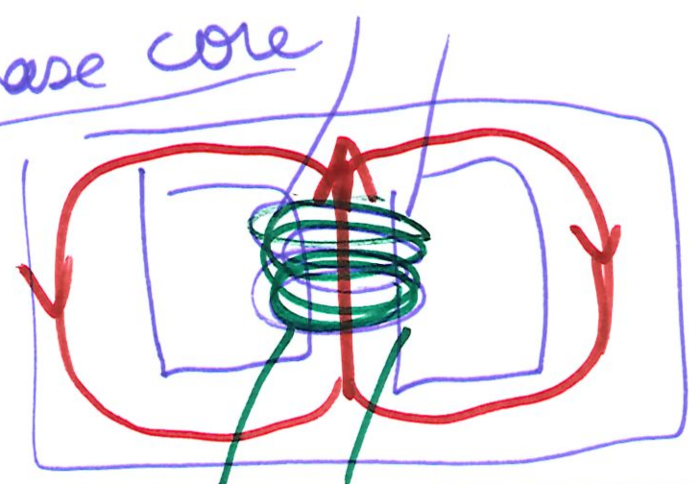
L7 8/20

leakage flux



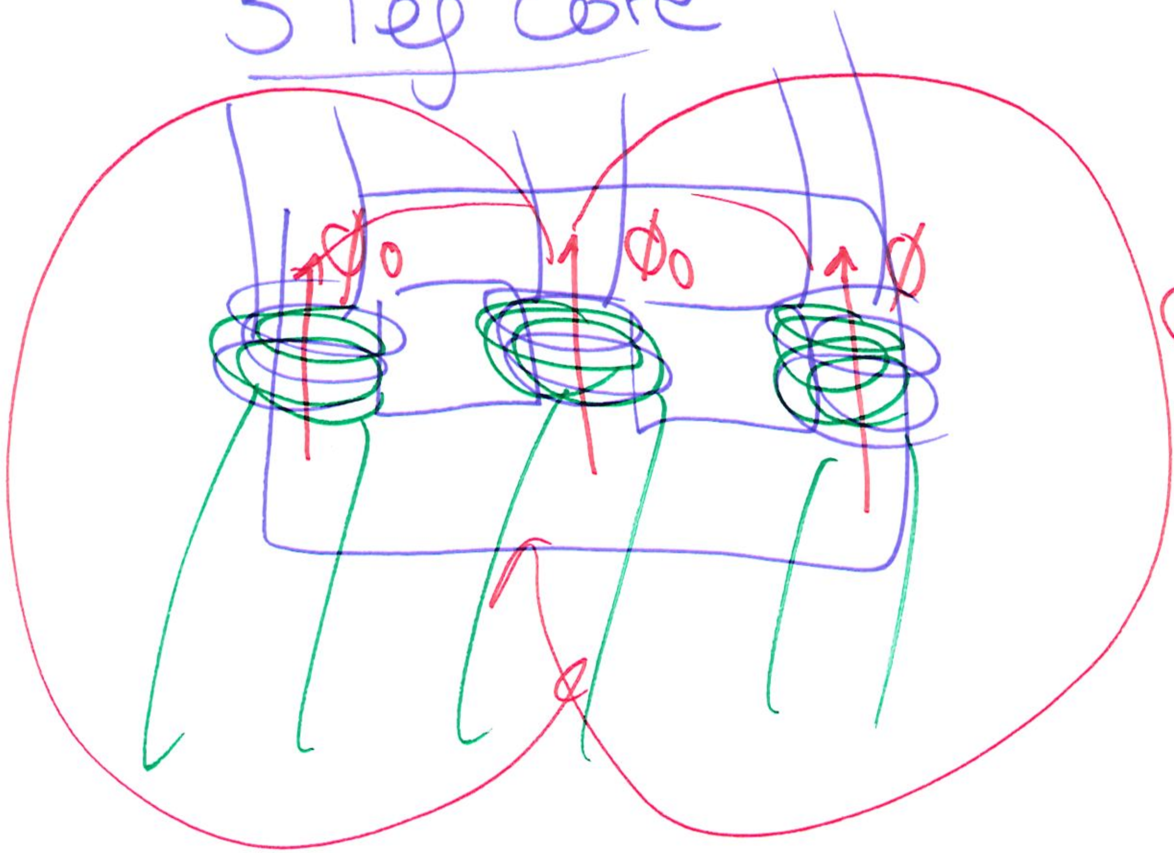
return path for zero sequence flux

Single phase core



L7 9/20

3 leg core



ϕ_0 - through Air

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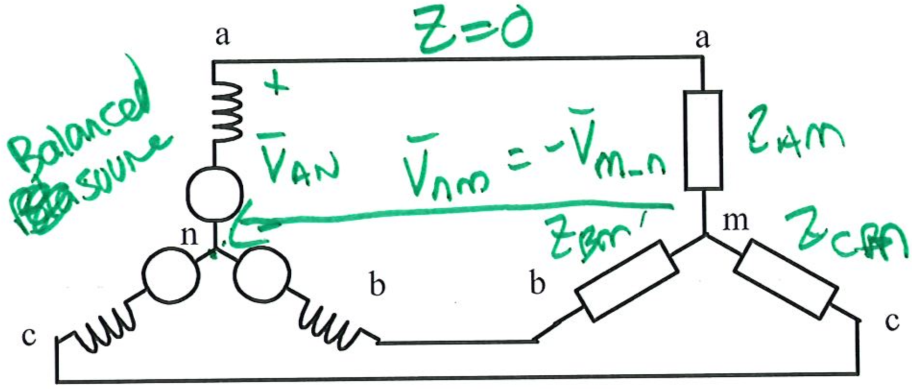
- effectively tank behaves like a low quality tertiary winding (Δ connected)

L7 11/20

ECE 523: Symmetrical Components Examples

If a load is unbalanced, its neutral, m, will not be at the same potential as the source neutral, n. Derive the between the neutral shift V_{mn} and the zero sequence voltage. V_{am0} for the system shown below. Similar 2.11. Hint, consider the line to ground voltages, line to neutral voltages and neutral to ground voltages.



$V_{m\_n} = V_{m\_ground} - V_{n\_ground}$

$V_{an} = V_{a\_ground} - V_{n\_ground}$

and $V_{am} = V_{a\_ground} - V_{m\_ground}$

so, substituting $V_{m\_n} = V_{m\_ground} - V_{n\_ground} = \left(\cancel{V_{a\_ground}} - V_{am}\right) - \left(\left(\cancel{V_{a\_ground}} - \bar{V}_{an}\right)\right)$

Then rearranging terms:

$$V_{m\_n} = V_{an} - V_{am}$$

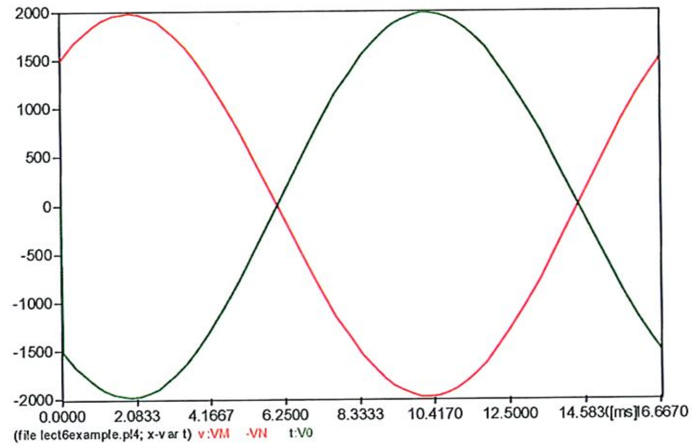
similarly

$$V_{m\_n} = V_{bn} - V_{bm}$$

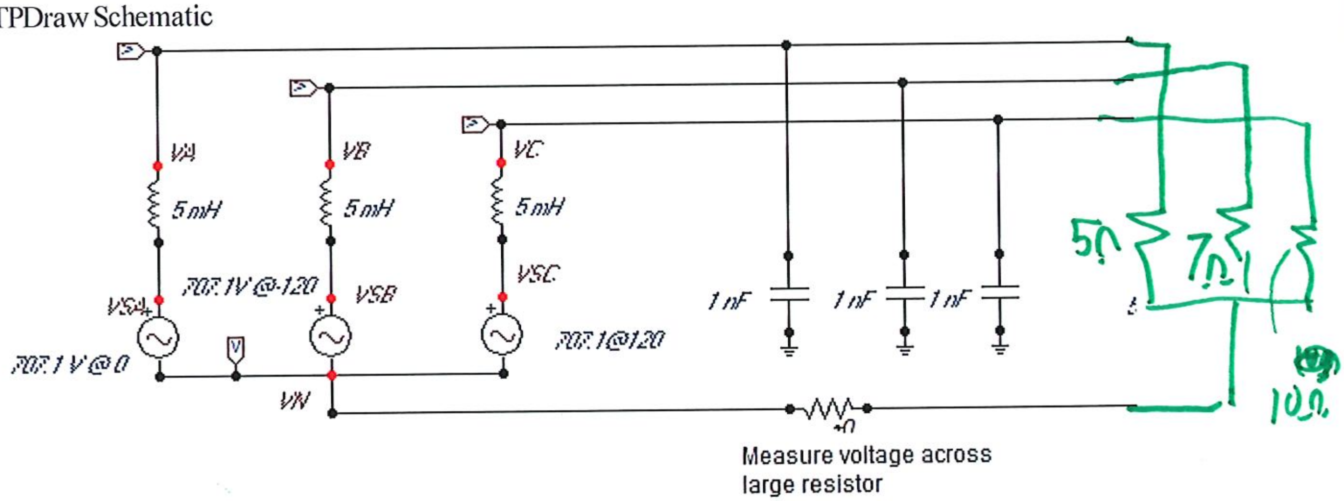
$$V_{m\_n} = V_{cn} - V_{cm}$$

L7 12/20

- Next plot instantaneous values of Vmn and V0 (note that V0 is simply 1/3*(Vam + Vbm + Vcm). Nc degree phase difference.



ATPDraw Schematic



- The capacitors were added to provide a ground reference. The capacive reactances are quite large anc results otherwise.
- Control modelling language TACS used to calculate V0

L7 13/20

- Adding these three expressions, results in: $$3V_{m\_n} = (V_{an} + V_{bn} + V_{cn}) - (V_{am} + V_{bm} + V_{cm})$$

- Applying the Symmetrical Components transformation:

$$3V_{an0} = (V_{an} + V_{bn} + V_{cn}) = 0$$ Since the source is still balanced

Similarly

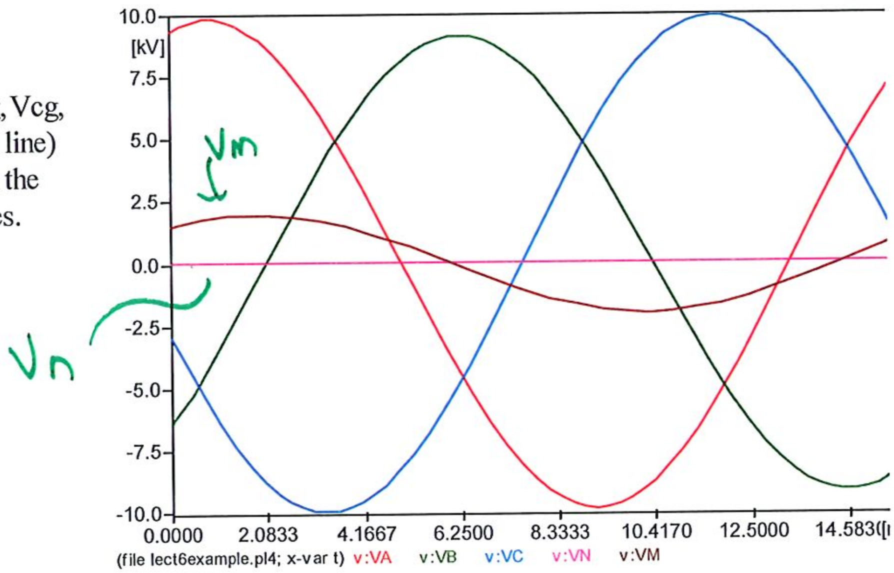
$$3V_{am0} = (V_{am} + V_{bm} + V_{cm})$$ This does not sum to 0, since the load is unbalanced

Therefore

$$3V_{m\_n} = 3V_{an0} - 3V_{am0} = -3V_{am0}$$ $$\boxed{V_{m\_n} = -V_{am0}}$$

As a check, the circuit was simulated with ATPDraw, which was also used to determine the symmetrical instantaneous quantities.

- First we see Vag, Vbg, Vcg, Vng and Vmg (brown line)
- Note the unbalance in the line-to-ground voltages.



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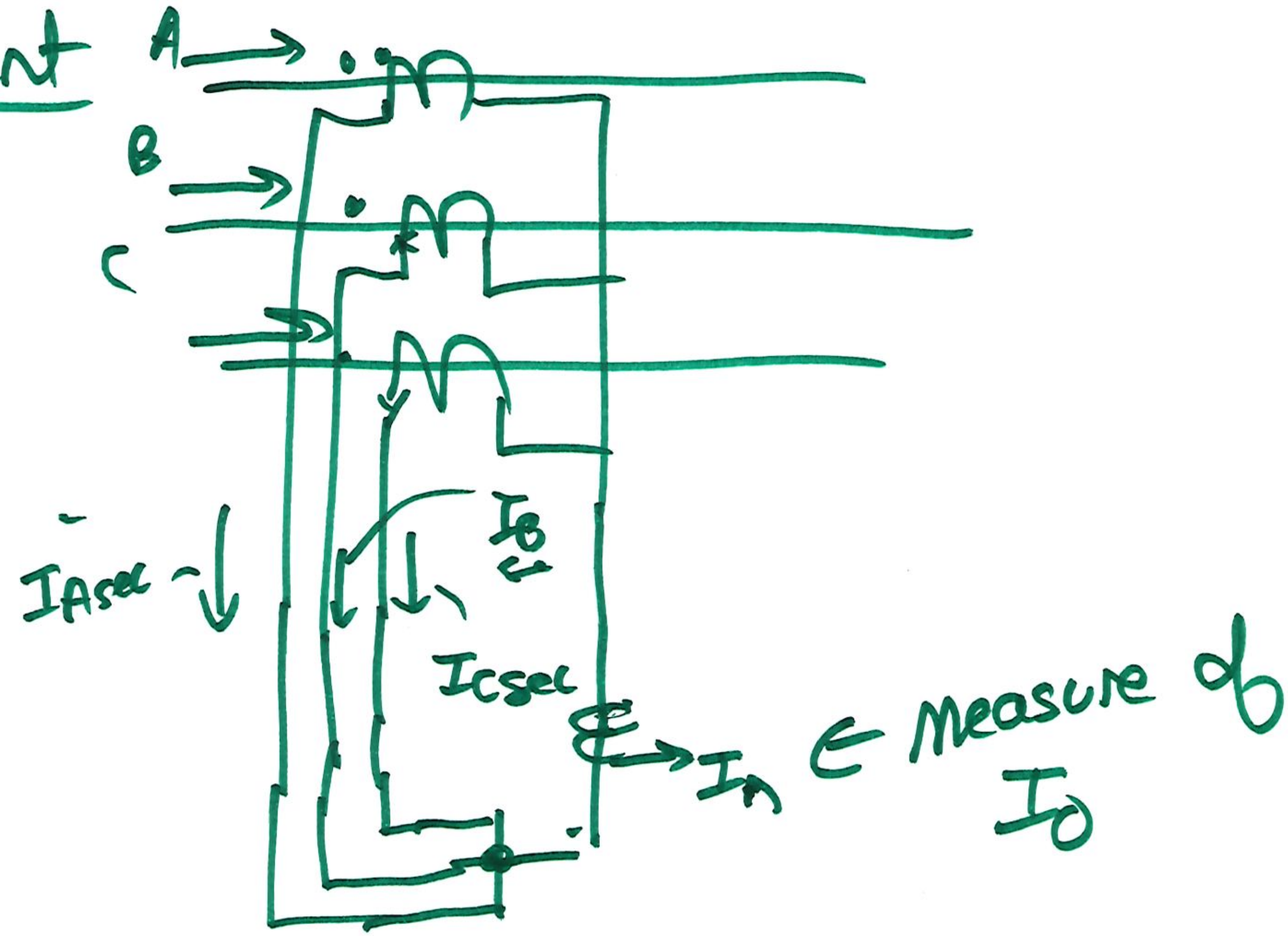
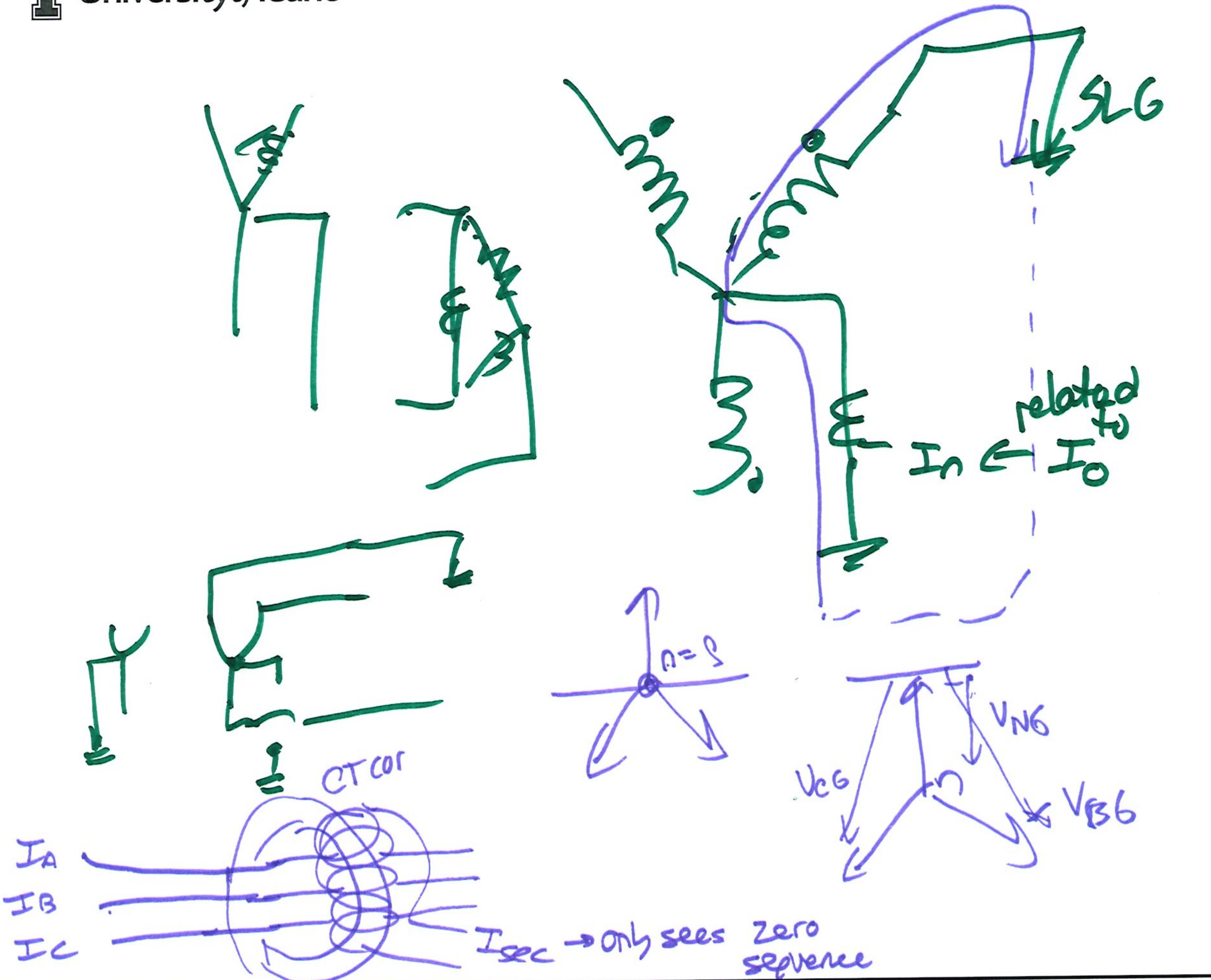
- Measurement of sequence voltages and currents?

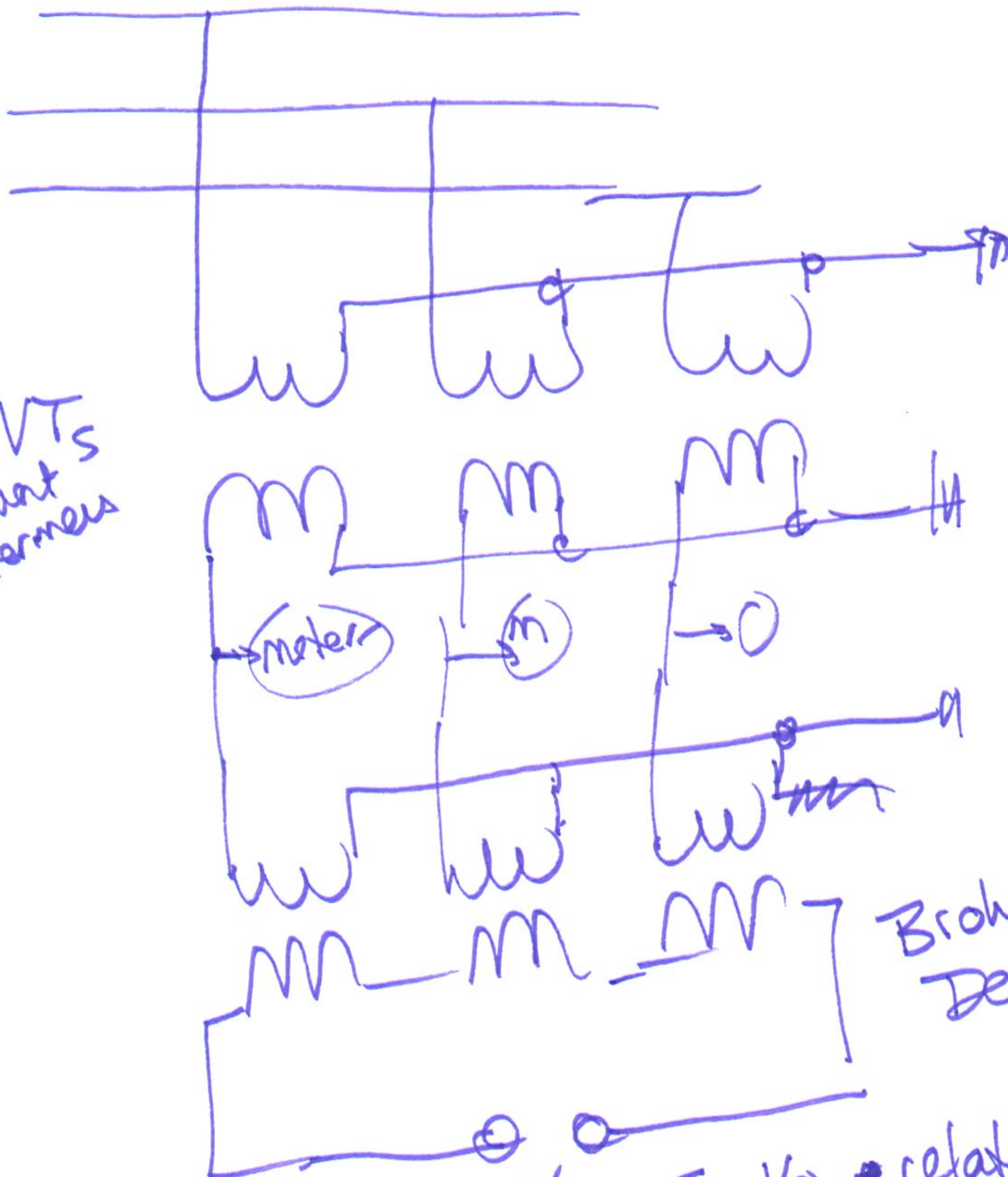
Current



L7 15/20


SLG
related to
$I_n \leftarrow I_0$
$n = 0$
V_{NG}
V_{CG}
V_{BG}
CT cor
I_A
I_B
I_C
I_{sec} → only sees zero sequence

VTs
measurement transformers



Broken Delta Windn connection

$+V_m- \Leftarrow V_0$ related quantity

L7 17/20

Symmetrical Components Example

$\mathrm{MVA} := \mathrm{MW}$ $\quad a := 1 \cdot e^{j \cdot 120\deg}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \qquad Inv\_A_{012} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

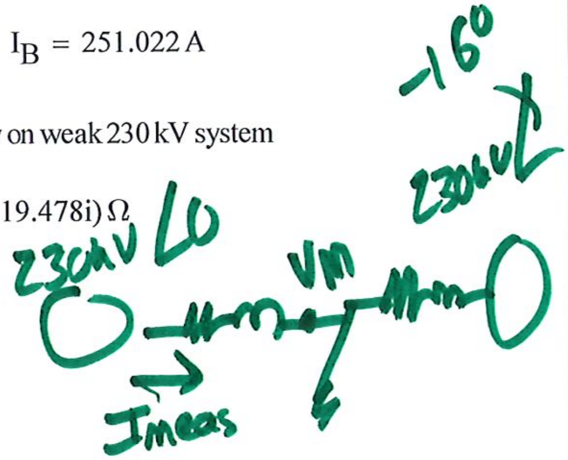
$S_B := 100\mathrm{MVA}$ $\quad V_{BLL} := 230\mathrm{kV}$ $\quad I_B := \dfrac{S_B}{\sqrt{3} \cdot V_{BLL}}$ $\quad I_B = 251.022\,\mathrm{A}$

- Voltages and currents from a single line to ground fault with and load flow on weak 230 kV system

update

$Z_1 := \cancel{2(0.852 + j \cdot 9.739)\mathrm{ohm}}$ $\quad Z_2 := Z_1$ $\quad Z_1 = (1.704 + 19.478i)\,\Omega$

$Z_0 := 3 \cdot Z_1$ $\quad Z_0 = (5.112 + 58.434i)\,\Omega$



$V_S := \dfrac{230\mathrm{kV}}{\sqrt{3}} \cdot e^{j \cdot 0\deg}$ $\qquad V_R := \dfrac{230\mathrm{kV}}{\sqrt{3}} \cdot e^{-j \cdot 16\deg}$

$I_{A\_prefault} := \dfrac{(V_S - V_R)}{2Z_1}$ $\quad |I_{A\_prefault}| = 945.198\,\mathrm{A}$ $\quad \arg(I_{A\_prefault}) = -3 \cdot \deg$

Total Z

- Fault currents and voltages from EMT simulation

SLG

$$I_{ABC} := \begin{pmatrix} 4144\mathrm{A} \cdot e^{-j \cdot 79.74\deg} \\ 945.1\mathrm{A} \cdot e^{-j \cdot 122.9\deg} \\ 945.1\mathrm{A} \cdot e^{j \cdot 117.1\deg} \end{pmatrix} \qquad V_{ABC} := \begin{pmatrix} 10^{-5}\mathrm{V} \cdot e^{j \cdot 0\deg} \\ 164.3\mathrm{kV} \cdot e^{-j \cdot 144.0\deg} \\ 164.2\mathrm{kV}\, e^{j \cdot 128.2\deg} \end{pmatrix}$$

load current on B&C

$$I_{012} := A_{012}^{-1} \cdot I_{ABC} \qquad \overrightarrow{|I_{012}|} = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} \begin{pmatrix} 1345.057 \\ 1643.583 \\ 1345.057 \end{pmatrix} \cdot \mathrm{A} \qquad \overrightarrow{\arg(I_{012})} = \begin{pmatrix} -92.923 \\ -57.822 \\ -92.923 \end{pmatrix} \cdot \deg$$

$I_0 = I_2$

$I_1 \neq$ superposition of I_f & I_L

$$V_{012} := A_{012}^{-1} \cdot V_{ABC} \qquad \overrightarrow{|V_{012}|} = \begin{pmatrix} 78.9 \\ 105.205 \\ 26.305 \end{pmatrix} \cdot \mathrm{kV} \qquad \overrightarrow{\arg(V_{012})} = \begin{pmatrix} 172.117 \\ -7.905 \\ 172.03 \end{pmatrix} \cdot \deg$$

L7 18/20

- Comparing our model to the simulation results

source is only pos seq

$$V_{0f} := 0 - I_{012_0} \cdot (Z_0) \qquad |V_{0f}| = 78.897 \cdot kV \qquad \arg(V_{0f}) = 172.077 \cdot deg$$

$$V_{1f} := \frac{230kV}{\sqrt{3}} - I_{012_1} \cdot (Z_1) \qquad |V_{1f}| = 105.232 \cdot kV \qquad \arg(V_{1f}) = -8.018 \cdot deg$$

$$V_{2f} := 0 - I_{012_2} \cdot (Z_2) \qquad |V_{2f}| = 26.299 \cdot kV \qquad \arg(V_{2f}) = 172.077 \cdot deg$$

$$V_{ABCf} := A_{012} \cdot \begin{pmatrix} V_{0f} \\ V_{1f} \\ V_{2f} \end{pmatrix} \qquad \overrightarrow{|V_{ABCf}|} = \begin{pmatrix} 0.179 \\ 164.32 \\ 164.223 \end{pmatrix} \cdot kV \qquad \overrightarrow{\arg(V_{ABCf})} = \begin{pmatrix} -86.602 \\ -144.08 \\ 128.117 \end{pmatrix} \cdot deg$$

Now look at behavior in the time domain.

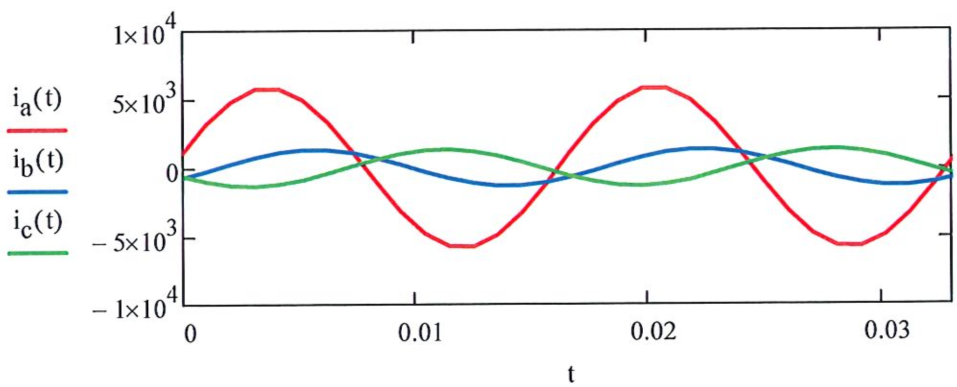
Set up an array 3 cycle long, with 16 samples per cyle

$$t := 0sec, \frac{1}{16 \cdot 60Hz} .. \frac{2}{60Hz} \qquad \omega := 2 \cdot \pi \cdot 60Hz$$

$$i_a(t) := \sqrt{2} \cdot \left| I_{ABC_0} \right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{ABC_0}\right)\right)$$

$$i_b(t) := \sqrt{2} \cdot \left| I_{ABC_1} \right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{ABC_1}\right)\right)$$

$$i_c(t) := \sqrt{2} \cdot \left| I_{ABC_2} \right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{ABC_2}\right)\right)$$

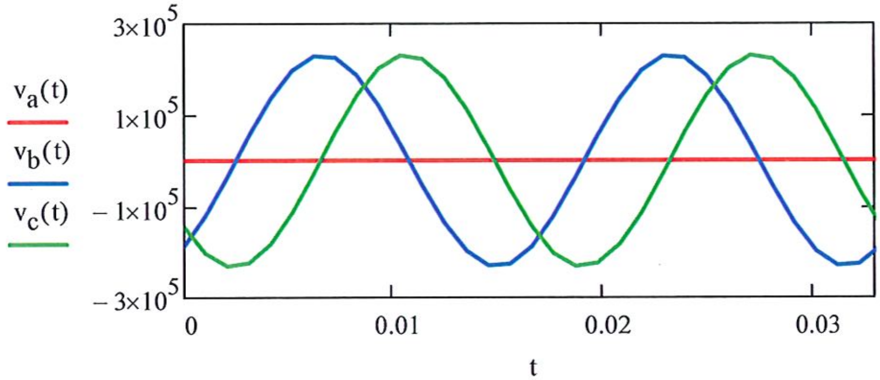


L7 9/20

$$v_a(t) := \sqrt{2}\cdot\left|V_{ABC_0}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_0}\right)\right)$$

$$v_b(t) := \sqrt{2}\cdot\left|V_{ABC_1}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_1}\right)\right)$$

$$v_c(t) := \sqrt{2}\cdot\left|V_{ABC_2}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_2}\right)\right)$$



- Convert **a** and $\mathbf{a}^2$ to units of time

$$T_a := \frac{\arg(a)}{360\text{deg}\cdot 60\text{Hz}} \qquad T_a = 5.556\cdot\text{ms}$$

-120°

$$T_{a\_sq} := \frac{\arg\left(a^2\right) + 360\text{deg}}{360\text{deg}\cdot 60\text{Hz}} \qquad T_{a\_sq} = 11.111\cdot\text{ms}$$

-240°

Note that: $I_1 = I_A + a\cdot I_B + a^2\cdot I_C$

- This equation involves rotating I_B and I_C by positive angles,which would mean advancing in time.
- We can only delay measurements, not advance them
- Recall that:

$$a = a^{-2} \qquad a - a^{-2} = 0$$

$$a^2 = a^{-1} \qquad a^2 - a^{-1} = 0$$

- Instead we will use the following equations for I1 and I2 with time delays.

$I_A + aI_B + a^2I_C$

$$I_{1\_alt} := \frac{\left(I_{ABC_0} + a^{-2}\cdot I_{ABC_1} + a^{-1}\cdot I_{ABC_2}\right)}{3}$$

delay delay

L7 20/20

$I_A - a^2 I_B + a I_c$

$$I_{2\_alt} := \frac{\left(I_{ABC_0} + a^{-1} \cdot I_{ABC_1} + a^{-2} \cdot I_{ABC_2}\right)}{3}$$

As a check I_1

$$I_{1\_alt} - I_{012_1} = \left(-4.547 \times 10^{-13} - 2.274i \times 10^{-13}\right) A$$

$$I_{1\_alt} - I_{012_1} = \left(-4.547 \times 10^{-13} - 2.274i \times 10^{-13}\right) A$$

I_2'

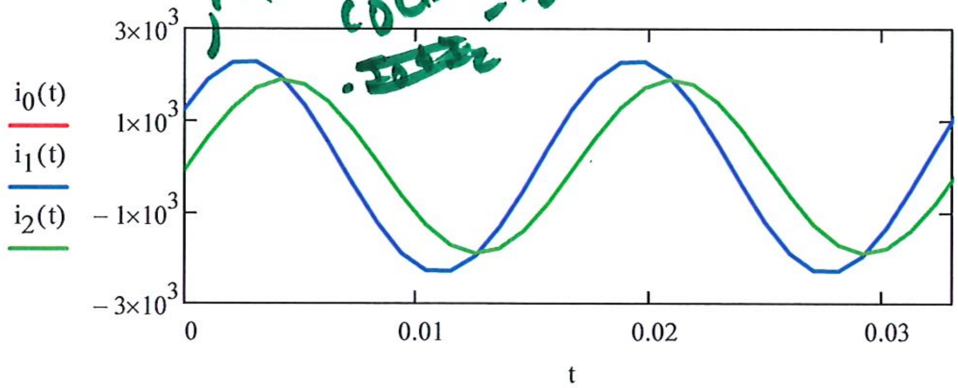
- Find sequence current treaking angle rotations as time delays

$$i_0(t) := \frac{\left(i_a(t) + i_b(t) + i_c(t)\right)}{3}$$

$$i_1(t) := \frac{\left(i_a(t) + i_b\left(t - T_{a\_sq}\right) + i_c\left(t - T_a\right)\right)}{3}$$

$$i_2(t) := \frac{\left(i_a(t) + i_b\left(t - T_a\right) + i_c\left(t + T_a\right)\right)}{3}$$

$i_1(t)$ $i_0(t)$ $- i_2(t)$



$$i_{0\_check}(t) := i_0(t) - \sqrt{2} \cdot \left|I_{012_0}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_0}\right)\right)$$

$$i_{1\_check}(t) := i_1(t) - \sqrt{2} \cdot \left|I_{012_1}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_1}\right)\right)$$

$$i_{2\_check}(t) := i_2(t) - \sqrt{2} \cdot \left|I_{012_2}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_2}\right)\right)$$