Symmetrical Components Example

$MVA := MW \qquad a := 1 \cdot e^{j \cdot 120deg}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \qquad Inv\_A_{012} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$S_B := 100MVA \qquad V_{BLL} := 230kV \qquad I_B := \frac{S_B}{\sqrt{3} \cdot V_{BLL}} \qquad I_B = 251.022\,A$$

- Voltages and currents from a single line to ground fault with and load flow on weak 230 kV system

$Z_1 := (3.408 + j \cdot 38.956)ohm \qquad Z_2 := Z_1 \qquad Z_2 = (3.408 + 38.956i)\,\Omega$

$Z_0 := 3 \cdot Z_1 \qquad Z_0 = (10.224 + 116.868i)\,\Omega$

$$V_S := \frac{230kV}{\sqrt{3}} \cdot e^{j \cdot 0deg} \qquad V_R := \frac{230kV}{\sqrt{3}} \cdot e^{-j \cdot 16deg}$$

$$I_{A\_prefault} := \frac{(V_S - V_R)}{Z_1} \qquad \left|I_{A\_prefault}\right| = 945.198\,A \qquad arg\left(I_{A\_prefault}\right) = -3 \cdot deg$$

- Prefault current from EMT simulation

$magI_A := 945.1A \qquad \theta_{IA} := -3.03deg$

- Fault currents and voltages from EMT simulation

$$I_{ABC} := \begin{pmatrix} 4144A \cdot e^{-j \cdot 79.74deg} \\ 945.1A \cdot e^{-j \cdot 122.9deg} \\ 945.1A \cdot e^{j \cdot 117.1deg} \end{pmatrix} \qquad V_{ABC} := \begin{pmatrix} 10^{-5} V \cdot e^{j \cdot 0deg} \\ 164.3kV \cdot e^{-j \cdot 144.0deg} \\ 164.2kV\, e^{j \cdot 128.2deg} \end{pmatrix}$$

$$I_{012} := A_{012}^{-1} \cdot I_{ABC} \qquad \overrightarrow{\left|I_{012}\right|} = \begin{pmatrix} 1345.057 \\ 1643.583 \\ 1345.057 \end{pmatrix} \cdot A \qquad \overrightarrow{arg\left(I_{012}\right)} = \begin{pmatrix} -92.923 \\ -57.822 \\ -92.923 \end{pmatrix} \cdot deg$$

$$V_{012} := A_{012}^{-1} \cdot V_{ABC} \qquad \overrightarrow{\left|V_{012}\right|} = \begin{pmatrix} 78.9 \\ 105.205 \\ 26.305 \end{pmatrix} \cdot kV \qquad \overrightarrow{\arg\left(V_{012}\right)} = \begin{pmatrix} 172.117 \\ -7.905 \\ 172.03 \end{pmatrix} \cdot deg$$

- Comparing our model to the simulation results

$$V_{0f} := 0 - I_{012_0} \cdot Z_0 \qquad \left|V_{0f}\right| = 157.795 \cdot kV \qquad \arg\left(V_{0f}\right) = 172.077 \cdot deg$$

$$V_{1f} := \frac{230kV}{\sqrt{3}} - I_{012_1} \cdot \left(Z_1\right) \qquad \left|V_{1f}\right| = 81.114 \cdot kV \qquad \arg\left(V_{1f}\right) = -21.218 \cdot deg$$

$$V_{2f} := 0 - I_{012_2} \cdot \left(Z_2\right) \qquad \left|V_{2f}\right| = 52.598 \cdot kV \qquad \arg\left(V_{2f}\right) = 172.077 \cdot deg$$

$$V_{ABCf} := A_{012} \cdot \begin{pmatrix} V_{0f} \\ V_{1f} \\ V_{2f} \end{pmatrix} \qquad \overrightarrow{\left|V_{ABCf}\right|} = \begin{pmatrix} 132.77 \\ 214.366 \\ 197.876 \end{pmatrix} \cdot kV \qquad \overrightarrow{\arg\left(V_{ABCf}\right)} = \begin{pmatrix} -179.846 \\ -158.72 \\ 133.554 \end{pmatrix} \cdot deg$$

Now look at behavior in the time domain.

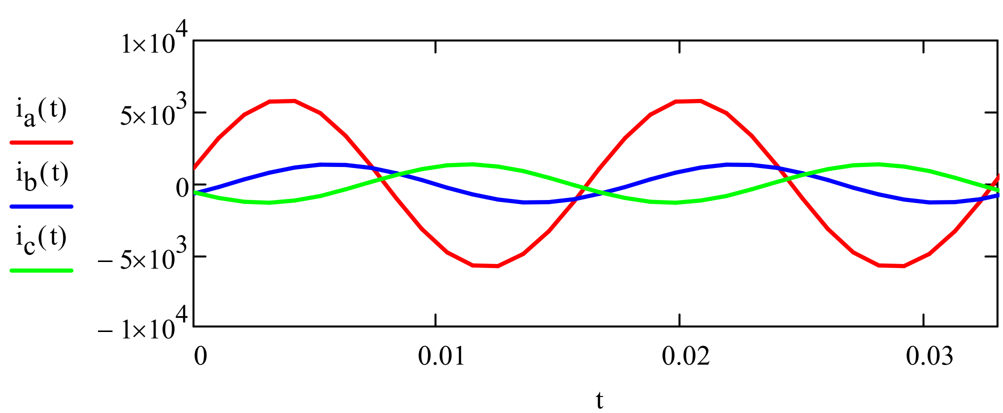
- Set up an array 2 cycles long, with 16 samples per cyle

$$t := 0sec, \frac{1}{16 \cdot 60Hz} .. \frac{2}{60Hz} \qquad \omega := 2 \cdot \pi \cdot 60Hz$$

$$i_a(t) := \sqrt{2} \cdot \left|I_{ABC_0}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{ABC_0}\right)\right)$$

$$i_b(t) := \sqrt{2} \cdot \left|I_{ABC_1}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{ABC_1}\right)\right)$$

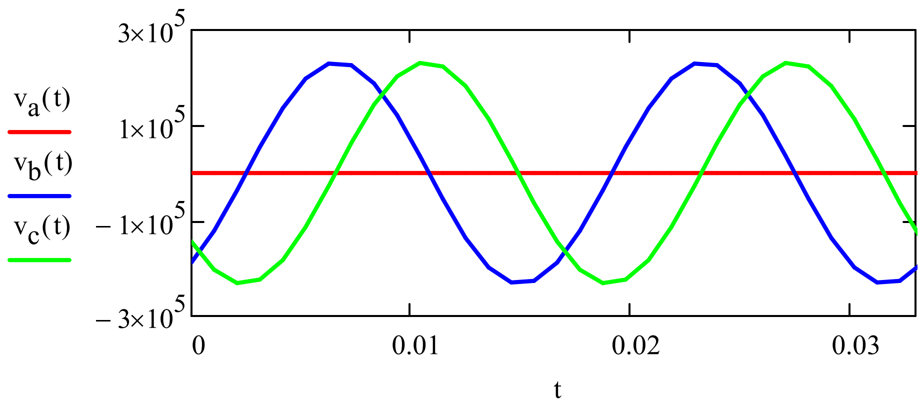
$$i_c(t) := \sqrt{2} \cdot \left|I_{ABC_2}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{ABC_2}\right)\right)$$



$$v_a(t) := \sqrt{2}\cdot\left|V_{ABC_0}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_0}\right)\right)$$

$$v_b(t) := \sqrt{2}\cdot\left|V_{ABC_1}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_1}\right)\right)$$

$$v_c(t) := \sqrt{2}\cdot\left|V_{ABC_2}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_2}\right)\right)$$



- Convert **a** and **a**^{2} to units of time

$$T_a := \frac{\arg(a)}{360\text{deg}\cdot 60\text{Hz}} \qquad T_a = 5.556\cdot\text{ms}$$

$$T_{a\_sq} := \frac{\arg\left(a^2\right) + 360\text{deg}}{360\text{deg}\cdot 60\text{Hz}} \qquad T_{a\_sq} = 11.111\cdot\text{ms}$$

Note that: $I_1 = \dfrac{\left(I_A + a\cdot I_B + a^2\cdot I_C\right)}{3}$

- This equation involves rotating I_B and I_C by positive angles,which would mean advancing in time.
- We can only delay measurements, not advance them
- Recall that:

$$a = a^{-2} \qquad a - a^{-2} = 0$$

$$a^2 = a^{-1} \qquad a^2 - a^{-1} = 0$$

- Instead we will use the following equations for I1 and I2 with time delays.

$$I_{1\_alt} := \frac{\left(I_{ABC_0} + a^{-2}\cdot I_{ABC_1} + a^{-1}\cdot I_{ABC_2}\right)}{3} \qquad I_{1\_alt} = (875.296 - 1391.122i)\ \text{A}$$

$$I_{2\_alt} := \frac{\left(I_{ABC_0} + a^{-1} \cdot I_{ABC_1} + a^{-2} \cdot I_{ABC_2}\right)}{3} \qquad I_{2\_alt} = (-68.593 - 1343.307i)\,A$$

As a check

$$I_{1\_alt} - I_{012_1} = 0\,A$$

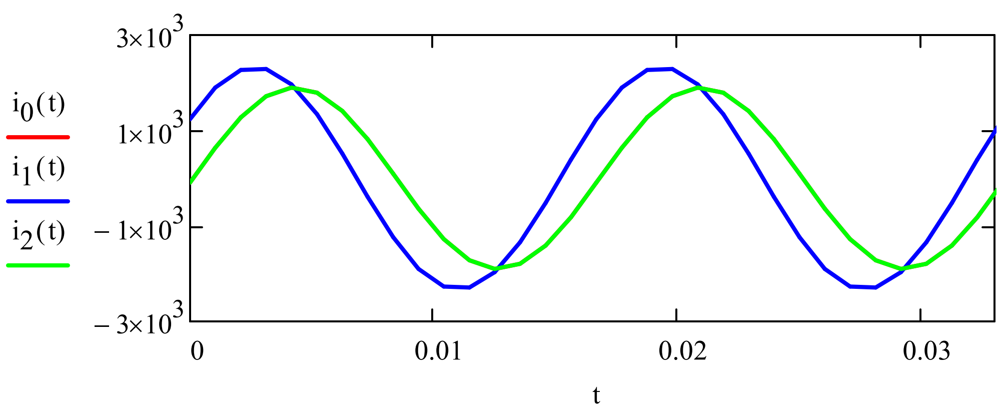
$$I_{1\_alt} - I_{012_1} = 0\,A$$

- Find sequence current treaking angle rotations as time delays

$$i_0(t) := \frac{\left(i_a(t) + i_b(t) + i_c(t)\right)}{3}$$

$$i_1(t) := \frac{\left(i_a(t) + i_b\left(t - T_{a\_sq}\right) + i_c\left(t - T_a\right)\right)}{3}$$

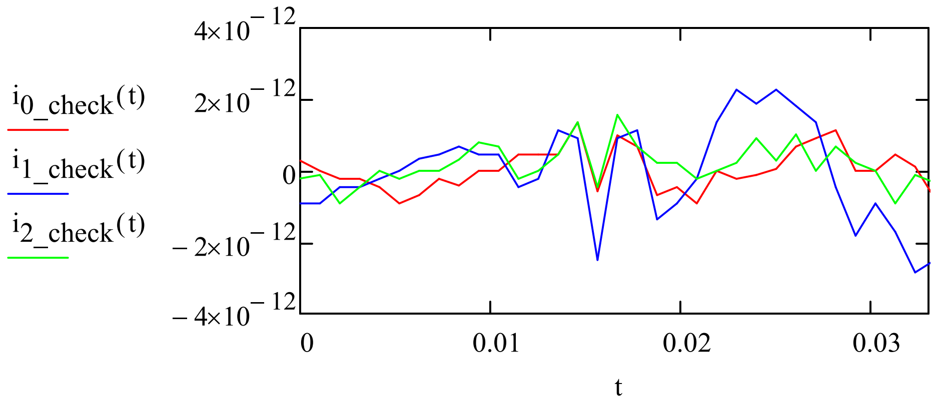
$$i_2(t) := \frac{\left(i_a(t) + i_b\left(t - T_a\right) + i_c\left(t + T_a\right)\right)}{3}$$



$$i_{0\_check}(t) := i_0(t) - \sqrt{2} \cdot \left|I_{012_0}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_0}\right)\right)$$

$$i_{1\_check}(t) := i_1(t) - \sqrt{2} \cdot \left|I_{012_1}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_1}\right)\right)$$

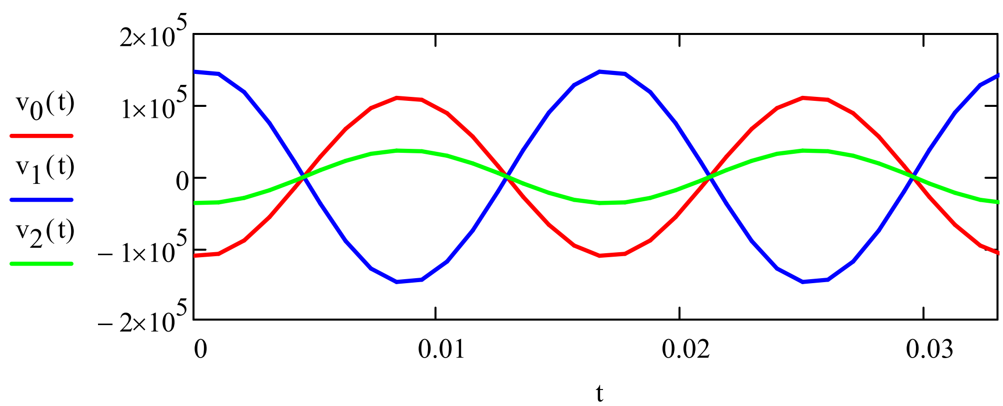
$$i_{2\_check}(t) := i_2(t) - \sqrt{2} \cdot \left|I_{012_2}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_2}\right)\right)$$



$$v_0(t) := \frac{\left(v_a(t) + v_b(t) + v_c(t)\right)}{3}$$

$$v_1(t) := \frac{\left(v_a(t) + v_b\left(t + T_a\right) + v_c\left(t + T_{a\_sq}\right)\right)}{3}$$

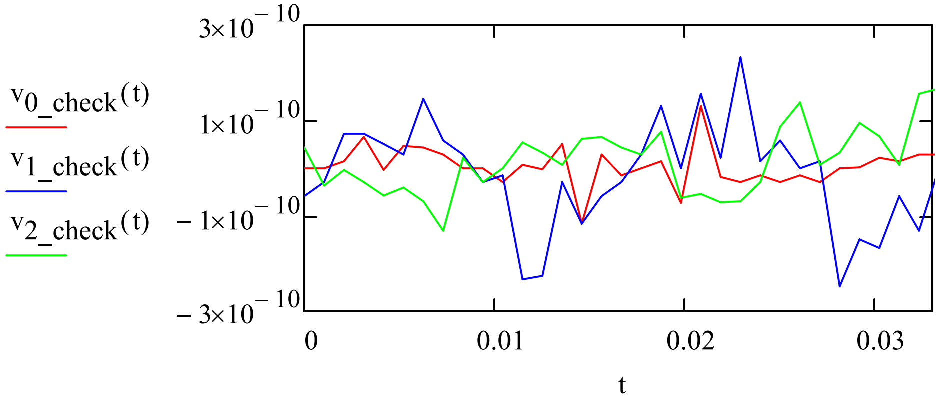
$$v_2(t) := \frac{\left(v_a(t) + v_b\left(t + T_{a\_sq}\right) + v_c\left(t + T_a\right)\right)}{3}$$



$$v_{0\_check}(t) := v_0(t) - \sqrt{2}\cdot\left|V_{012_0}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{012_0}\right)\right)$$

$$v_{1\_check}(t) := v_1(t) - \sqrt{2}\cdot\left|V_{012_1}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{012_1}\right)\right)$$

$$v_{2\_check}(t) := v_2(t) - \sqrt{2}\cdot\left|V_{012_2}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{012_2}\right)\right)$$



- Implemented same scheme in ATP.

$$I_0 := 1345A \cdot e^{-j \cdot 93.02deg}$$

$$I_1 := 1644A \cdot e^{-j \cdot 57.94deg}$$

$$I_2 := 1345A \cdot e^{-j \cdot 93deg}$$

- Above we calculated

$$\overrightarrow{\left|I_{012}\right|} = \begin{pmatrix} 1345.057 \\ 1643.583 \\ 1345.057 \end{pmatrix} A$$

$$\overrightarrow{\arg\left(I_{012}\right)} = \begin{pmatrix} -92.923 \\ -57.822 \\ -92.923 \end{pmatrix} \cdot deg$$