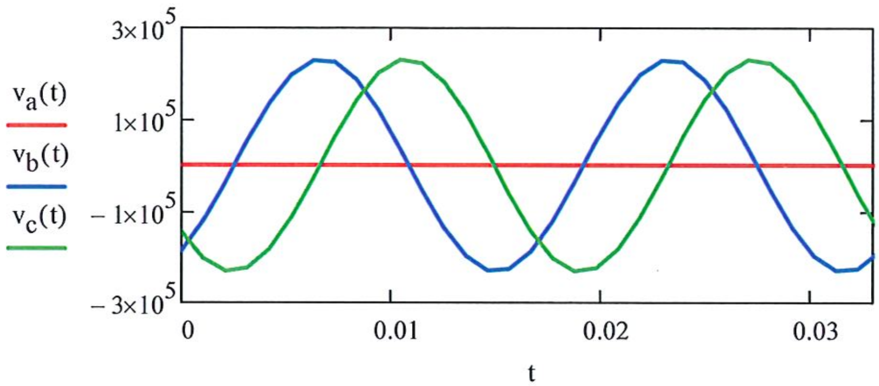
ECE 523
Symmetrical Components

Session 8

$$v_a(t) := \sqrt{2}\cdot\left|V_{ABC_0}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_0}\right)\right)$$

$$v_b(t) := \sqrt{2}\cdot\left|V_{ABC_1}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_1}\right)\right)$$

$$v_c(t) := \sqrt{2}\cdot\left|V_{ABC_2}\right|\cdot\cos\left(\omega\cdot t + \arg\left(V_{ABC_2}\right)\right)$$



- Convert **a** and **a**^{2} to units of time

$$T_a := \frac{\arg(a)}{360\text{deg}\cdot 60\text{Hz}} \qquad T_a = 5.556\cdot\text{ms}$$ -120°

$$T_{a\_sq} := \frac{\arg\left(a^2\right) + 360\text{deg}}{360\text{deg}\cdot 60\text{Hz}} \qquad T_{a\_sq} = 11.111\cdot\text{ms}$$ -240°

Note that: $I_1 = \left(I_A + a\cdot I_B + a^2\cdot I_C\right)/3$

- This equation involves rotating I_B and I_C by positive angles, which would mean advancing in time.
- We can only delay measurements, not advance them
- Recall that:

$$a = a^{-2} \qquad a - a^{-2} = 0$$

$$a^2 = a^{-1} \qquad a^2 - a^{-1} = 0$$

- Instead we will use the following equations for I1 and I2 with time delays.

$I_A + aI_B + a^2I_C$

$$I_{1\_alt} := \frac{\left(I_{ABC_0} + a^{-2}\cdot I_{ABC_1} + a^{-1}\cdot I_{ABC_2}\right)}{3}$$

delay delay

$$I_{2\_alt} := \frac{\left(I_{ABC_0} + a^{-1} \cdot I_{ABC_1} + a^{-2} \cdot I_{ABC_2}\right)}{3}$$

As a check

$$I_{1\_alt} - I_{012_1} = \left(-4.547 \times 10^{-13} - 2.274i \times 10^{-13}\right) A$$

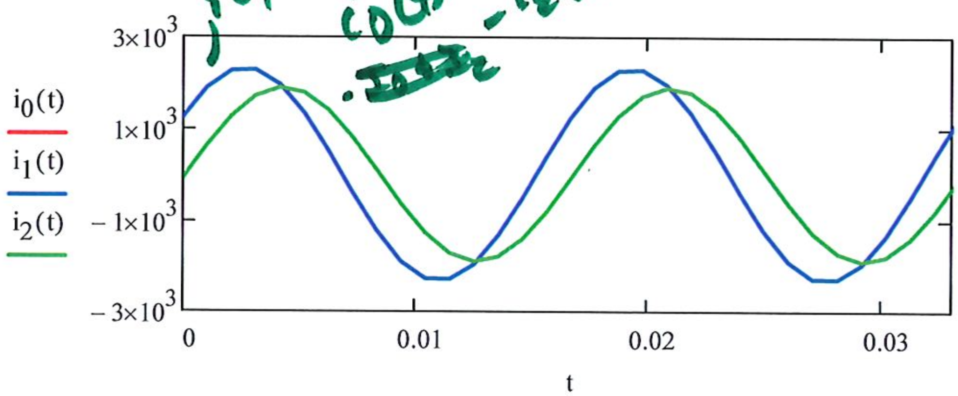
$$I_{1\_alt} - I_{012_1} = \left(-4.547 \times 10^{-13} - 2.274i \times 10^{-13}\right) A$$

- Find sequence current treaking angle rotations as time delays

$$i_0(t) := \frac{\left(i_a(t) + i_b(t) + i_c(t)\right)}{3}$$

$$i_1(t) := \frac{\left(i_a(t) + i_b\left(t - T_{a\_sq}\right) + i_c\left(t - T_a\right)\right)}{3}$$

$$i_2(t) := \frac{\left(i_a(t) + i_b\left(t - T_a\right) + i_c\left(t + T_a\right)\right)}{3}$$

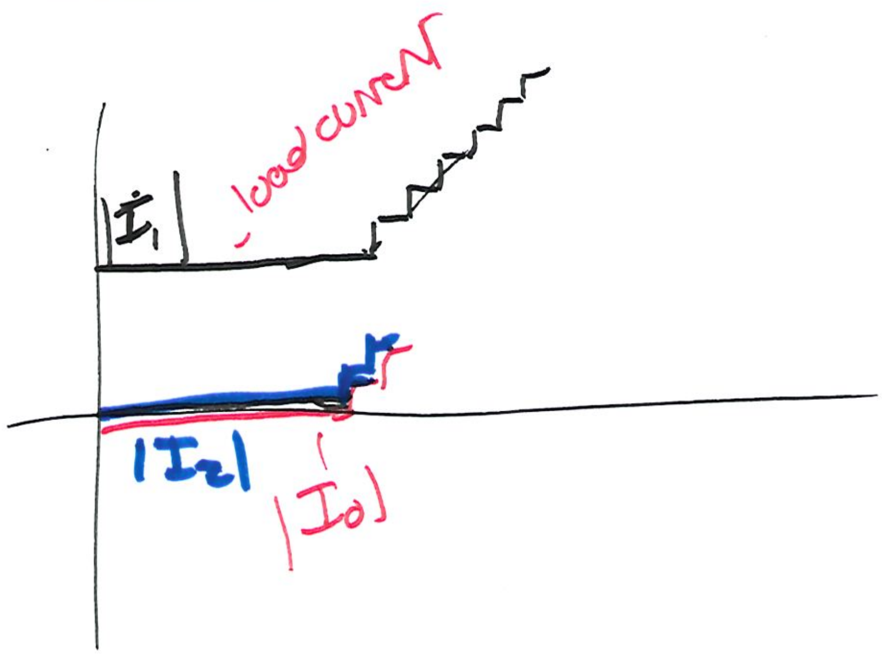


$$i_{0\_check}(t) := i_0(t) - \sqrt{2} \cdot \left|I_{012_0}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_0}\right)\right)$$

$$i_{1\_check}(t) := i_1(t) - \sqrt{2} \cdot \left|I_{012_1}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_1}\right)\right)$$

$$i_{2\_check}(t) := i_2(t) - \sqrt{2} \cdot \left|I_{012_2}\right| \cdot \cos\left(\omega \cdot t + \arg\left(I_{012_2}\right)\right)$$

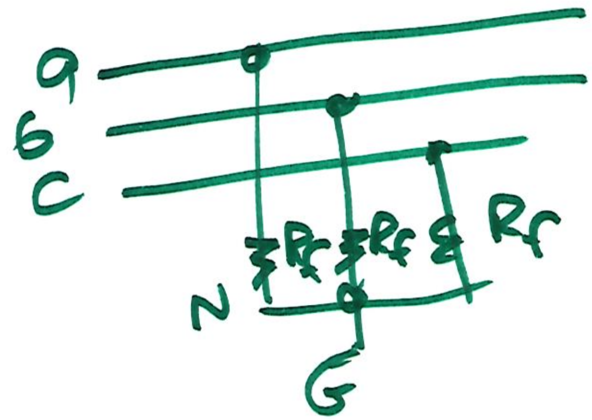
Processing measurements with digital filters



Analyzing Shunt & Series Unbalanced Conditions ⇒ FAULTS

(1) Shunt faults

(1) 3 phase fault



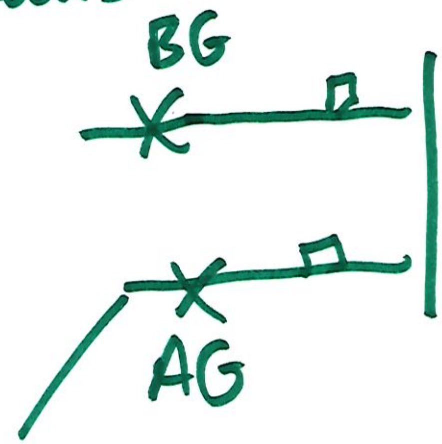
- Generally analyzed as balanced
- If balanced just positive sequence
- worst case - $R_f = 0$ ⇒ Bolted Fault

(2) Single line to ground faults → (80-85% of faults overhead lines) (single phase to ground) (ground faults)

(3) Line to line (phase to phase)

(4) Double line to ground - often evolve from single line to ground

(5) Simultaneous shunt faults at different locations:



Series unbalance (series faults)

(1) Single phase open

→ Failure cases: (1) Broken conducter → usually has conduct fall to ground

(2) Breaker pole fails to close

→ controlled cases

→ Single pole tripping

- trip both ends

(2) Two phase open → one breaker pole fails to open

(3)

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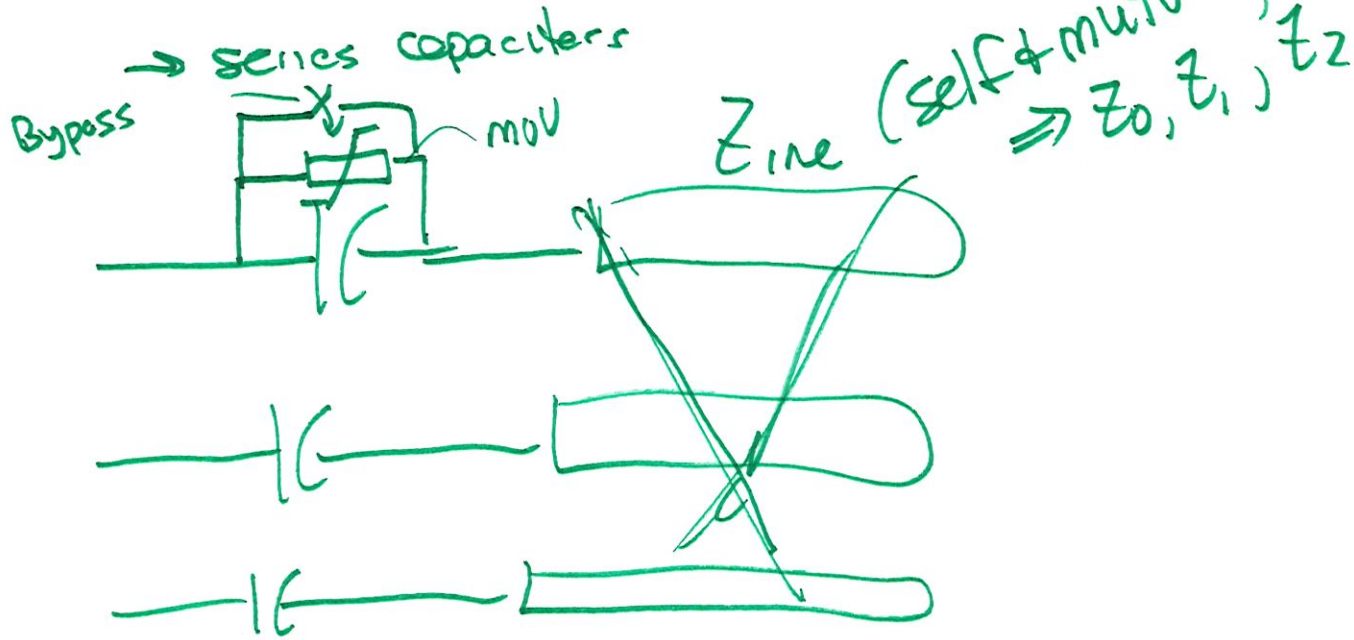
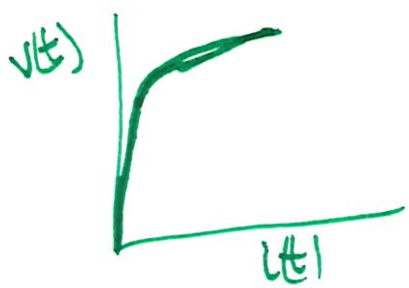
③ Transient series imbalance

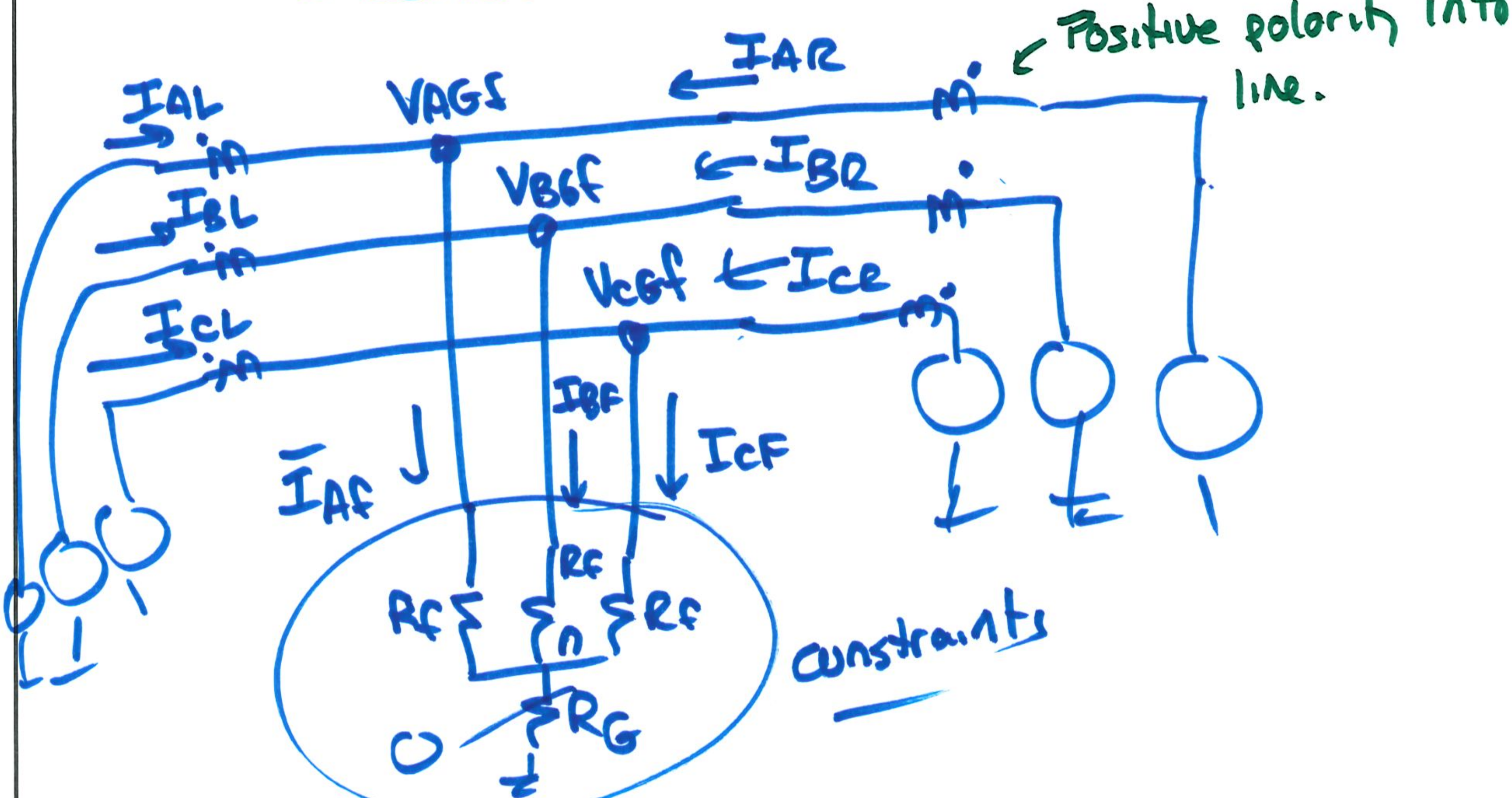
→ series capaciters





3 phase fault

- constraints at fault point



constraints

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Constraints at fault point

$$\bar{V}_{AGf} = R_f \cdot \bar{I}_{Af}$$

$$\bar{V}_{BGf} = R_f \cdot \bar{I}_{Bf}$$

$$\bar{V}_{CGf} = R_f \cdot \bar{I}_{Cf}$$

Manipulate

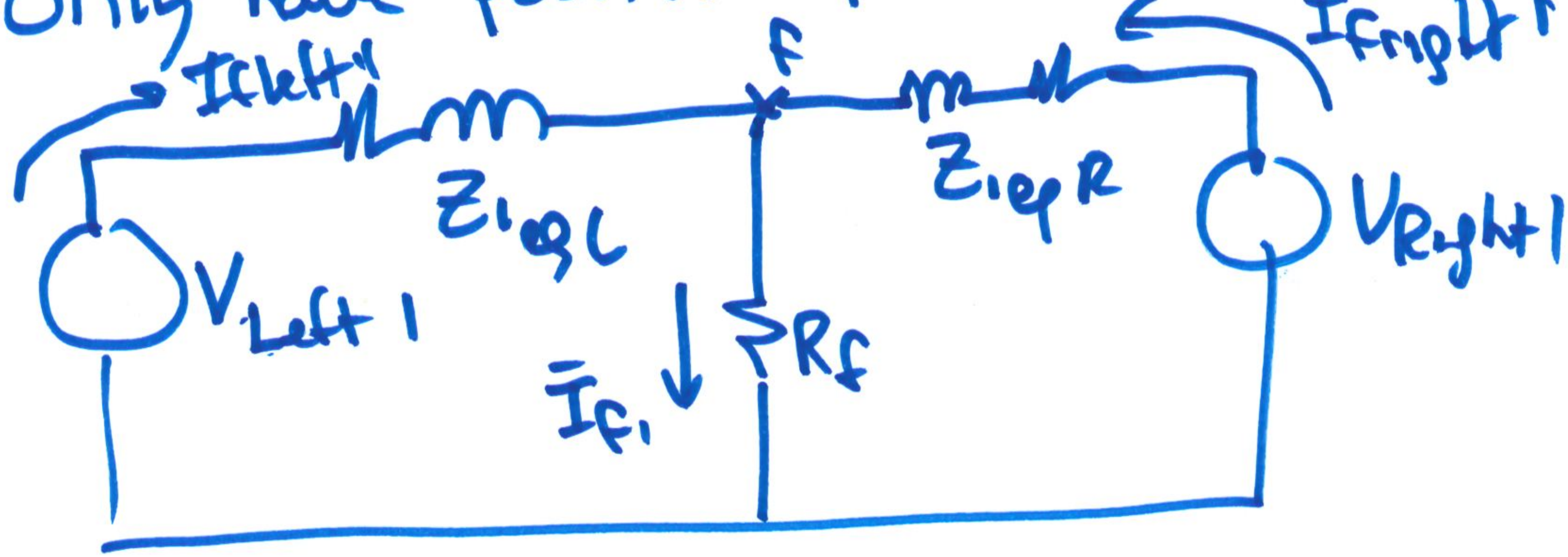
Balanced if system impedance/sources balanced

$$\underbrace{\bar{V}_{AG} + \bar{V}_{BG} + \bar{V}_{CG}}_{0} = R_f \underbrace{(I_{Af} + I_{Bf} + I_{Cf})}_{0}$$

$$3I_0 = 0$$

Since ~~[illegible]~~ balanced $\bar{V}_2$ & $\bar{I}_2$ are zero

Only have positive sequence current



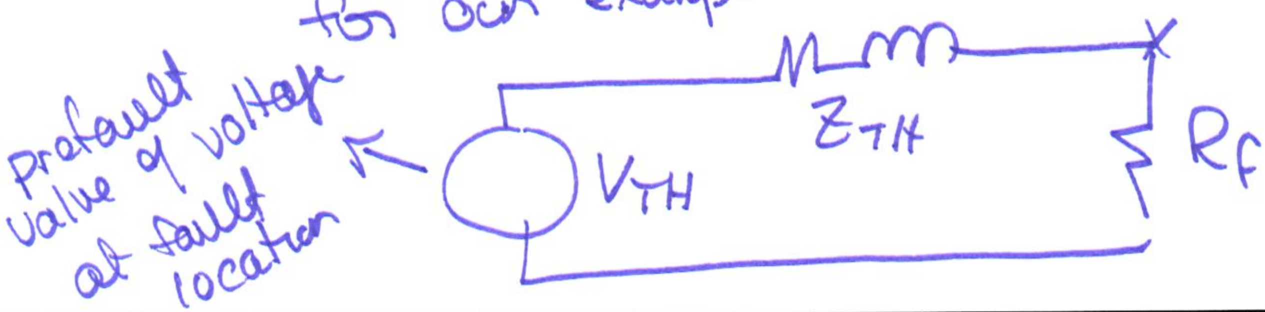
Solution approach - General (not just 3φ)

(1) Build per unit equivalent circuits in pos, neg & zero sequence for the system.

(2) Thevenin equivalent impedances looking from fault location

$\bar{Z}_{TH0}, \bar{Z}_{TH1}, \bar{Z}_{TH2}$

for our example



(3) find I_{1f}, I_{2f}, I_{0f}

(4) use current divider (or other) to find **sequence** current from left + right = convert sequence currents to ABC if needed

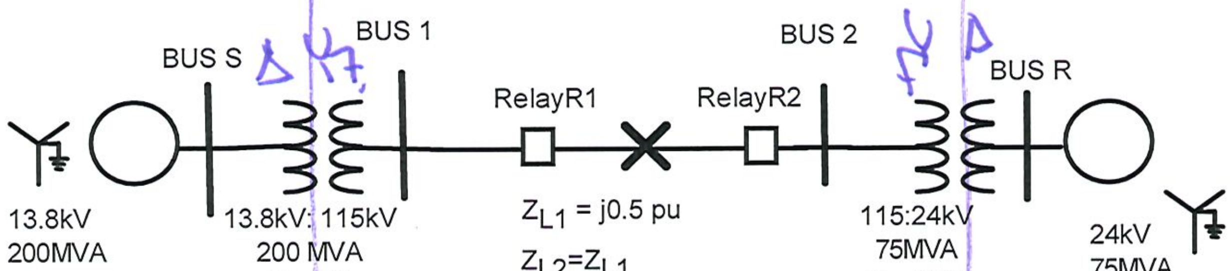
(5) Find sequence voltages at buses of interest - convert to ABC

Fault Analysis Techniques to Find Voltages and Currents at Other Buses With Transformer Phase Shifts

Define units: MVA := MW

MVAR := MVA   pu := 1

System Description:



13.8kV
200MVA

Z_{S1} = j0.1 pu

Z_{S2}=Z_{S1}

Z_{S0} = 5.0 + j0.05pu

13.8kV: 115kV
200 MVA
X = 5%
X/R = 25
Δ-Yg

Z_{L1} = j0.5 pu

Z_{L2}=Z_{L1}

Z_{L0} = j1.8 pu

B_{c1}= 0.02pu

B_{c2} = B_{c1}

B_{c0} = 0.0133 pu

on 100MVA and 115 kV

115:24kV
75MVA
X = 10%
X/R = 25
Yg-Δ

24kV
75MVA

Z_{R1} = j0.2 pu

Z_{R2}=Z_{R1}

Z_{R0} = 10 + j0.05pu

ground resistance

Per Unit Change of Base Calculations (hidden, to view double click on the arrow)

Set bases (Use the line voltage of 115kV as the reference):

$S_b := 100MVA$ $V_{b2} := 115kV$

$V_{b1} := V_{b2} \cdot \frac{13.8kV}{115kV}$ $V_{b1} = 13.8 \cdot kV$ $V_{b3} := V_{b2} \cdot \frac{24kV}{115kV}$ $V_{b3} = 24 \cdot kV$

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Change of Base Calculations:

Generator 1: $S_{g1} := 200MVA$ $\quad X_{G11old} := 0.1pu$ $\quad Z_{G10old} := 5.0 + j \cdot 0.05pu$

$V_{g1} := 13.8kV$

$$X_{G11} := X_{G11old} \cdot \left(\frac{V_{g1}}{V_{b1}}\right)^2 \cdot \left(\frac{S_b}{S_{g1}}\right) \qquad X_{G11} = 0.05 \cdot pu \qquad X_{G12} := X_{G11}$$

$$Z_{G10} := Z_{G10old} \cdot \left(\frac{V_{g1}}{V_{b1}}\right)^2 \cdot \left(\frac{S_b}{S_{g1}}\right) \qquad Z_{G10} = (2.5 + 0.025i) \cdot pu$$

Transformer 1: $X_{T1old} := 0.05pu$ $\quad R_{T1old} := \frac{X_{T1old}}{25}$ $\quad R_{T1old} = 0.002 \cdot pu$

- Small enough to neglect

X/R = 25

$V_{T1old} := 13.8kV$ $\quad S_{T1} := 200MVA$

$$X_{T11} := X_{T1old} \cdot \left(\frac{V_{T1old}}{V_{b1}}\right)^2 \cdot \left(\frac{S_b}{S_{T1}}\right) \qquad X_{T11} = 0.025 \cdot pu$$

$Z_{T11} := j \cdot X_{T11}$ $\quad |Z_{T11}| = 0.025 \cdot pu$ $\quad \arg(Z_{T11}) = 90 \cdot deg$

$Z_{T12} := Z_{T11}$ $\quad Z_{T10} := Z_{T11}$

Transmisson Line (no change of base needed):

$$Z_{L11} := 0.5pu \cdot e^{j \cdot 90deg} \qquad Z_{L11} = 0.5i \cdot pu$$

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$Z_{L12} := Z_{L11}$

$Z_{L10} := 1.8pu \cdot e^{j \cdot 90deg}$ $\quad Z_{L10} = 1.8i \cdot pu$

concentrate on series impedances

Transformer 2: $\quad X_{T2old} := 0.10pu \quad R_{T2old} := \frac{X_{T2old}}{25} \quad R_{T2old} = 0.004 \cdot pu$

- Again, neglect

$V_{T2old} := 115kV \quad S_{T2} := 75MVA$

$X_{T21} := X_{T2old} \cdot \left(\frac{V_{T2old}}{V_{b2}}\right)^2 \cdot \left(\frac{S_b}{S_{T2}}\right) \quad X_{T21} = 0.1333 \cdot pu$

$Z_{T21} := j \cdot X_{T21} \quad |Z_{T21}| = 0.1333 \cdot pu \quad arg(Z_{T21}) = 90 \cdot deg$

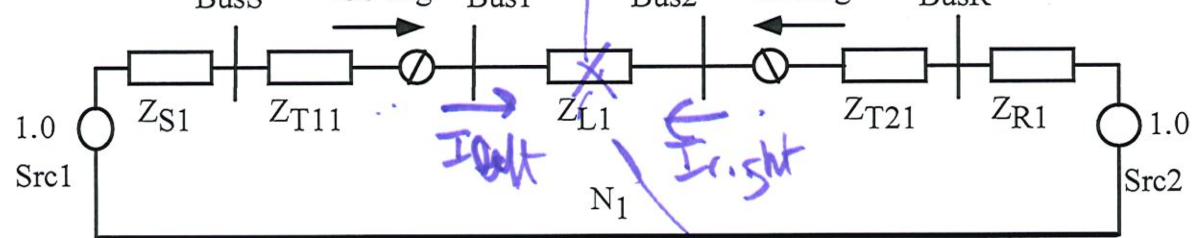
$Z_{T22} := Z_{T21} \quad Z_{T20} := Z_{T21}$

Generator 2: $\quad S_{g2} := 75MVA \quad X_{G21old} := 0.2pu \quad Z_{G20old} := 10.0 + j \cdot 0.05pu$

$V_{g2} := 24kV$

$X_{G21} := X_{G21old} \cdot \left(\frac{V_{g2}}{V_{b3}}\right)^2 \cdot \left(\frac{S_b}{S_{g2}}\right) \quad X_{G21} = 0.2667 \cdot pu \quad X_{G22} := X_{G21}$

$Z_{G20} := Z_{G20old} \cdot \left(\frac{V_{g2}}{V_{b3}}\right)^2 \cdot \left(\frac{S_b}{S_{g2}}\right) \quad Z_{G20} = (13.3333 + 0.0667i) \cdot pu$



- Regular circuit analysis approach:
  - Calculate reduced equivalent for several fault locations
  - Use a generic location, M, and define a Mathcad function:

$$Z_{left}(M) := j \cdot X_{G11} + Z_{T11} + Z_{L11} \cdot M$$

$$Z_{right}(M) := Z_{L11} \cdot (1 - M) + Z_{T21} + j \cdot X_{G21}$$

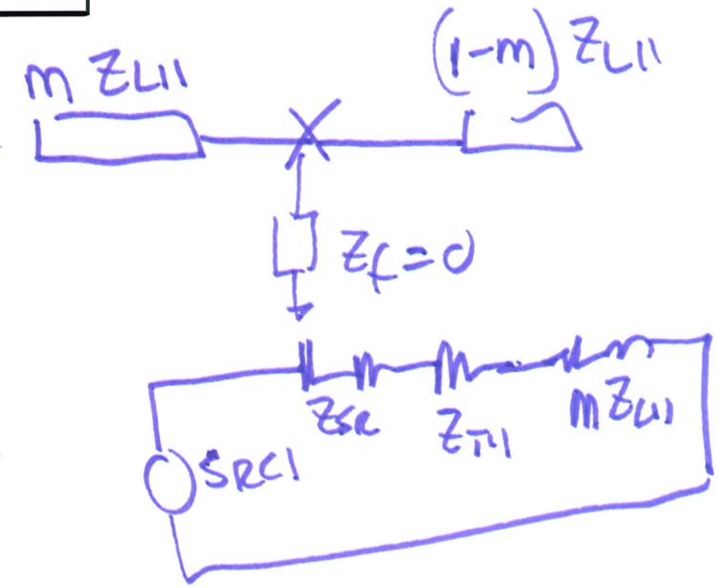
$$Z_{1equiv}(M) := \left[\frac{1}{j \cdot X_{G11} + Z_{T11} + Z_{L11} \cdot M} + \frac{1}{Z_{L11} \cdot (1 - M) + Z_{T21} + j \cdot X_{G21}} \right]^{-1}$$

- More compactly: $Z_{1thev}(M) := \left(\frac{1}{Z_{left}(M)} + \frac{1}{Z_{right}(M)} \right)^{-1}$

$V_f := 1.0pu$ → Both sources at $1\angle 0$ – referred to high side of transformers

$I_{3ph}(M) := \frac{V_f}{Z_{1equiv}(M)}$ $I_{3ph}(0.5) = -4.6154i$ Total fault current on phase A

M=0.5



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- Now use current divider to find current from left and right (through the relays)

$$I_{left}(M) := I_{3ph}(M)\cdot\frac{Z_{right}(M)}{Z_{left}(M)+Z_{right}(M)}$$ $I_{left}(0.5) = -3.0769i$

$$I_{right}(M) := I_{3ph}(M)\cdot\frac{Z_{left}(M)}{Z_{left}(M)+Z_{right}(M)}$$ $I_{right}(0.5) = -1.5385i$

- Now find voltages at bus 1 and bus 2 (start from sequence neutral plane, not fault location):

$V_S := 1.0$ $\quad$ $V_R := 1.0$ - referred to HV side

$$V_{bus2}(M) := V_S - I_{left}(M)\cdot(j\cdot X_{G11} + Z_{T11})$$ $V_{bus2}(0.5) = 0.7692$

$$V_{bus3}(M) := V_R - I_{right}(M)\cdot(j\cdot X_{G21} + Z_{T21})$$ $V_{bus3}(0.5) = 0.3846$

- same as $I_{left}(M)\cdot M\, Z_{L11}$ / $I_{right}(M)\cdot(1-M)\, Z_{L11}$

- Now find voltage and current on the generator side of the transformer (including transformer phase shift)

$1\angle -30^\circ$

refer to Low side

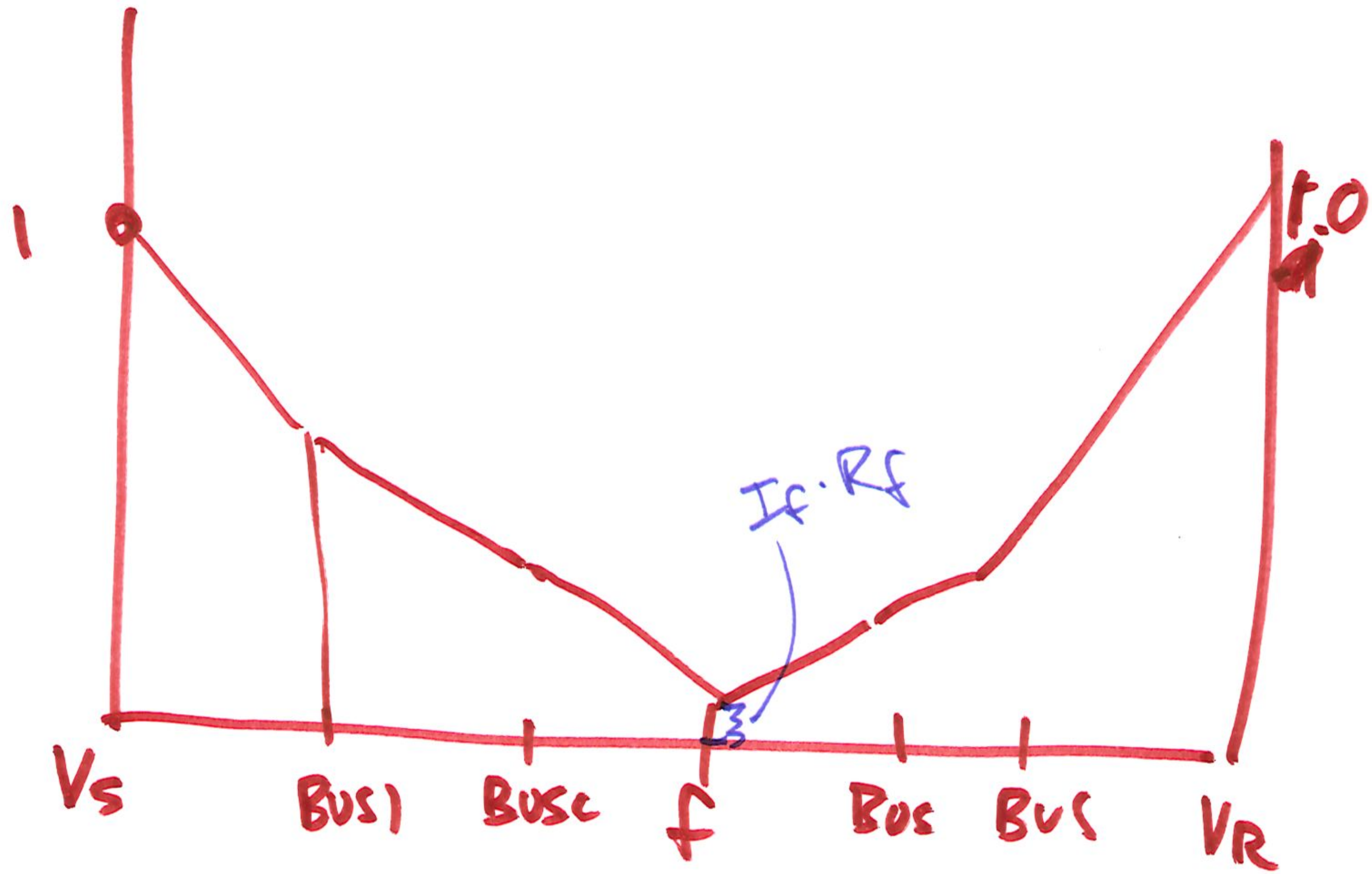
$$I_{SrcS}(M) := I_{left}(M)\cdot e^{-j\cdot 30deg}$$ $|I_{SrcS}(0.5)| = 3.0769$ $\quad$ $\arg(I_{SrcS}(0.5)) = -120\cdot deg$

$$I_{SrcR}(M) := I_{right}(M)\cdot e^{-j\cdot 30deg}$$ $|I_{SrcR}(0.5)| = 1.5385$ $\quad$ $\arg(I_{SrcR}(0.5)) = -120\cdot deg$

$$V_{bus1}(M) := V_S\cdot e^{-j\cdot 30deg} - I_{SrcS}(M)\cdot(j\cdot X_{G11})$$ $|V_{bus1}(0.5)| = 0.8462$ $\quad$ $\arg(V_{bus1}(0.5)) = -30\cdot deg$

$$V_{bus4}(M) := V_R\cdot e^{-j\cdot 30deg} - I_{SrcR}(M)\cdot(j\cdot X_{G21})$$ $|V_{bus4}(0.5)| = 0.5897$ $\quad$ $\arg(V_{bus4}(0.5)) = -30\cdot deg$

Pos sequence voltage profile



- modified Y_{BUS}

1. neglect shunt caps of lines.

2. Include internal impedances of generators & large motors

Easiest Approach is to Create Ybus Matrix. Modified for Fault Calculations: 3 Z Bus matrix Approach

Transformer Phase Shift:

$$Y_1(M) := \begin{bmatrix} \frac{1}{jX_{G11}} + \frac{1}{Z_{T11}} & \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T11}} & 0 & 0 & 0 \\ \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T11}} & \frac{1}{Z_{T11}} + \frac{1}{M\cdot Z_{L11}} & 0 & 0 & \frac{-1}{M\cdot Z_{L11}} \\ 0 & 0 & \frac{1}{Z_{T21}} + \frac{1}{(1-M)\cdot Z_{L11}} & \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T21}} & \frac{-1}{(1-M)\cdot Z_{L11}} \\ 0 & 0 & \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T21}} & \frac{1}{Z_{T21}} + \frac{1}{jX_{G21}} & 0 \\ 0 & \frac{-1}{M\cdot Z_{L11}} & \frac{-1}{(1-M)\cdot Z_{L11}} & 0 & \frac{1}{M\cdot Z_{L11}} + \frac{1}{(1-M)\cdot Z_{L11}} \end{bmatrix}$$

$$Z_1(M) := Y_1(M)^{-1}$$

$$Z_1(0.5) = \begin{pmatrix} 0.0474i & 0.0231+0.04i & 0.0103+0.0178i & 0.0137i & 0.0167+0.0289i \\ -0.0231+0.04i & 0.0692i & 0.0308i & -0.0103+0.0178i & 0.05i \\ -0.0103+0.0178i & 0.0308i & 0.2359i & -0.0786+0.1362i & 0.1333i \\ 0.0137i & 0.0103+0.0178i & 0.0786+0.1362i & 0.1937i & 0.0444+0.077i \\ -0.0167+0.0289i & 0.05i & 0.1333i & -0.0444+0.077i & 0.2167i \end{pmatrix}$$