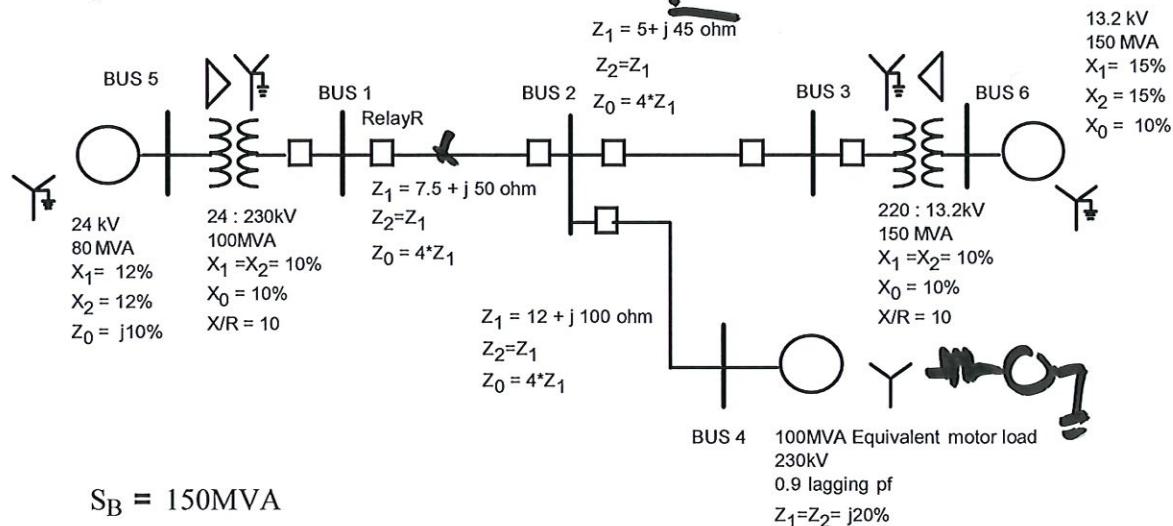


ECE 523
Symmetrical Components
Session 9

ECE 523: Homework #3

Due Session 13 (October 3)

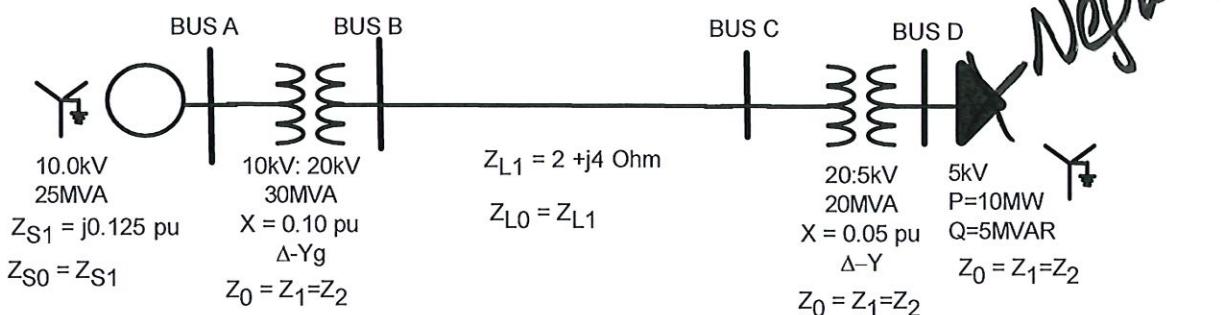
1. Create positive, negative and zero sequence Ybus and Zbus matrices for the system below to study faults on the line between BUS 1 and BUS 2. Use M=0.45 relative to Bus 1



Start voltage bases using rated voltage for the generator at BUS 5

2. Analyze the following faults. Use Sbase=25 MVA and a voltage base of 5kV at BUS D. You can neglect load current in your fault current calculations. Treat all buses as being at 1.0pu magnitude prior to the fault. Calculate by hand and with a fault program

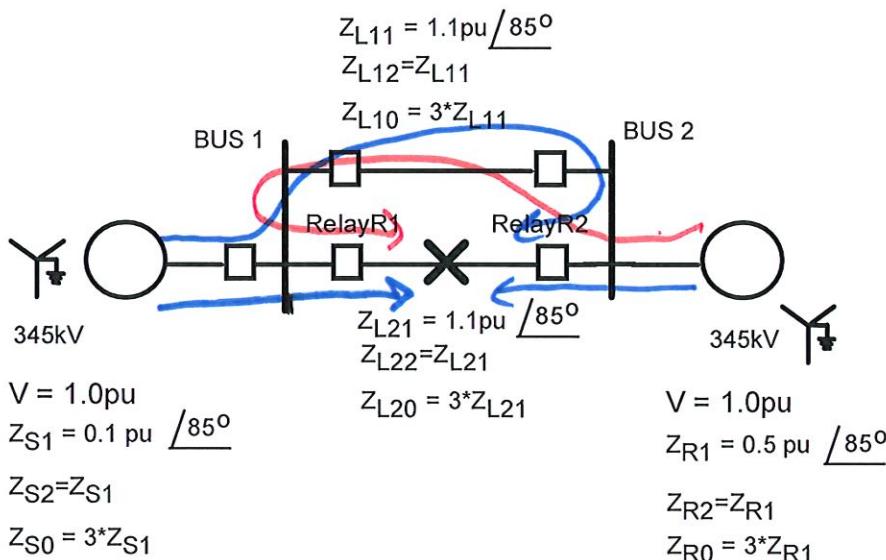
- a. Three phase fault at Bus C. Find V and I at the fault location and at BUS A
- b. SLG fault with $R_f=0$ at Bus C. Find V and I at the fault location and at BUS A
- c. LL fault with $R_f=0$ at Bus C. Find V and I at the fault location and at BUS A
- d. DLG fault with $R_f=R_g=0$ at Bus C, Find V and I at the fault location and at BUS A
- e. Compare the fault current magnitudes and voltages between each of the different fault types
- f. Using your fault program, repeat parts a-e for faults at BUS B, and comment on how the results change with the fault location.

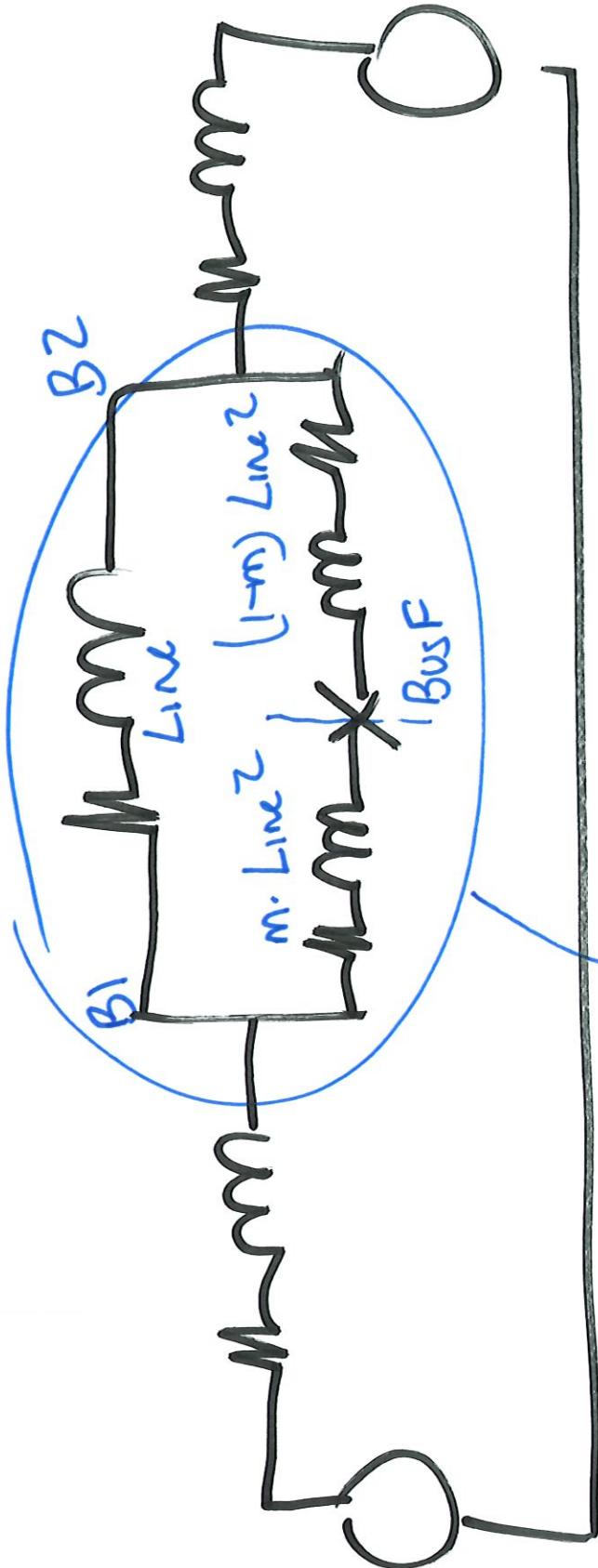


3. Do the following for the circuit below. Also check your results with a commercial fault program and show comparison in tables.

- Calculate and sketch the positive, negative and zero sequence equivalent circuits based on a fault 40% of the way down line 2 (the lower of the two lines). Get the Ybus matrices from your fault program.
- Calculate the voltages and currents at RelayR1 and RelayR2, for SLG, LL, and DLG faults with $R_f = 0$. I recommend using Zbus matrix methods.
- Repeat the part (b) for a SLG fault, LL, and DLG with $R_f = 0.75$ pu. For the DLG put the fault resistance in the neutral to ground path.

-compare to what you calculate





Simplify

$$B_1 \xrightarrow{Z_{L1}} \xrightarrow{(1-m)Z_{L2}} B_2$$

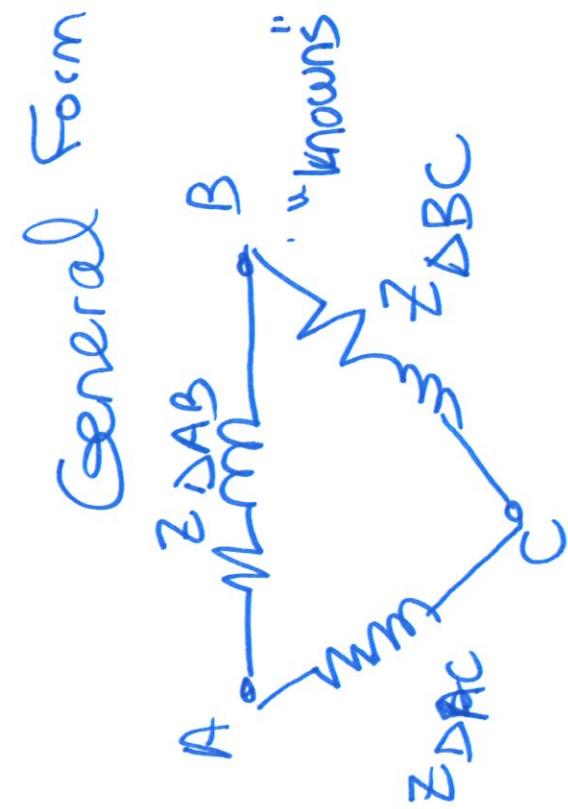
$m Z_{L2}$ $\xrightarrow{\text{Bus F}}$

unbalanced Δ

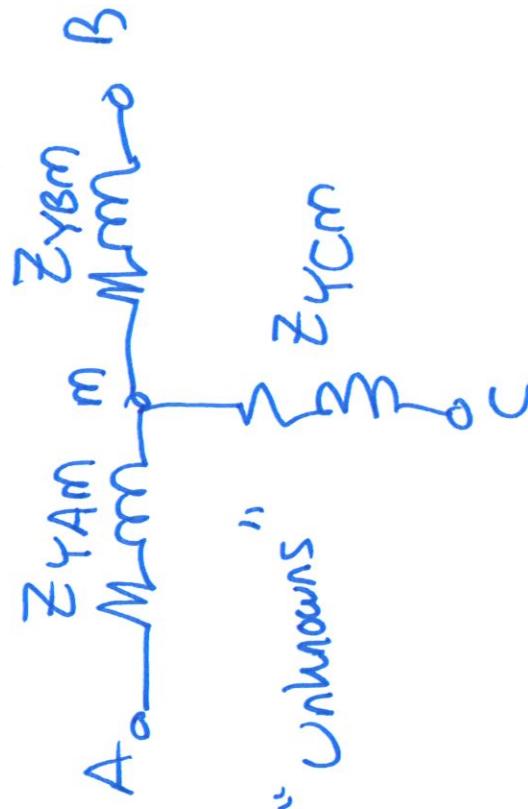
$$B_1 \xrightarrow{Z_{1mY}} \xrightarrow{(1-m)Z_{2mY}} B_2$$

$Z_{FmY} \xrightarrow{\text{Bus F}}$

unbalanced Δ -equiv.



$\Delta - \gamma$ conversion



$$Z_{eq\Delta AB} = Z_{\Delta AB} \parallel (Z_{\Delta AC} + Z_{\Delta BC})$$

$$= \frac{(Z_{\Delta AB})(Z_{\Delta AC} + Z_{\Delta BC})}{Z_{\Delta AB} + Z_{\Delta AC} + Z_{\Delta BC}}$$

$$Z_{eq\Delta BC} = Z_{\Delta BC} \parallel (Z_{\Delta AB} + Z_{\Delta AC})$$

$$Z_{eq\Delta AC} = Z_{\Delta AC} \parallel (Z_{\Delta AB} + Z_{\Delta BC})$$

3 equations, 3 unknowns

$$Z_{eq\Delta AB} = Z_{eq\Delta BC} = Z_{eq\Delta AC}$$

As a check, if balanced ..

$$Z_D, Z_Y$$

$$Z_{\text{op}ABD} = \cancel{\frac{Z_D(2Z_D)}{3Z_D}}$$

$$\cancel{\frac{2Z_D}{3}} = Z_Y$$

$$Z_{\text{op}ABY} = 2Z_Y$$

$$\frac{Z_D}{3} = Z_Y$$

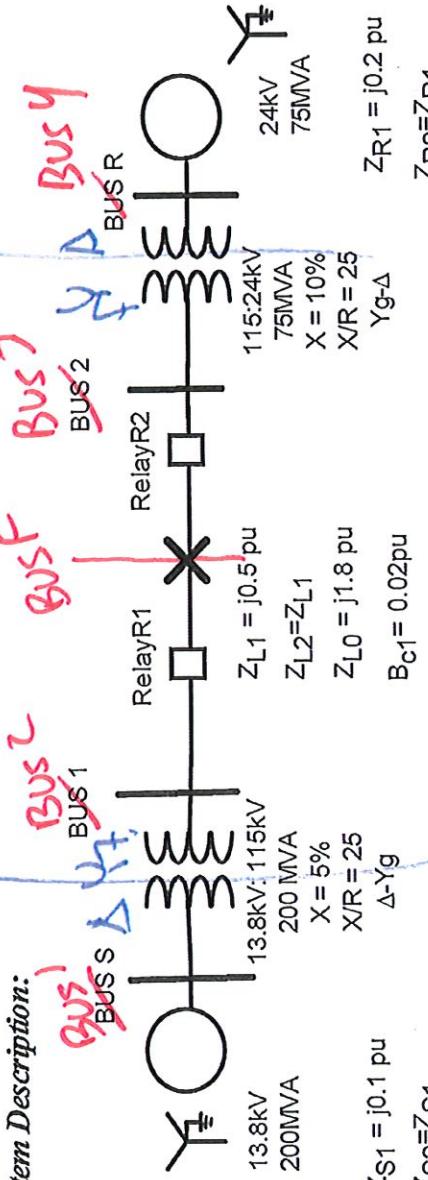
Fault Analysis Techniques to Find Voltages and Currents at Other Buses With Transformer Phase Shifts

Define units: $MVA := MW$

$MVAR := MVA$

$pu := 1$

System Description:



/ ground resistor @
on 100MVA and 115 kV

Per Unit Change of Base Calculations (hidden, to view double click on the arrow)



Set bases (Use the line voltage of 115kV as the reference):

$S_b := 100 \text{MVA}$

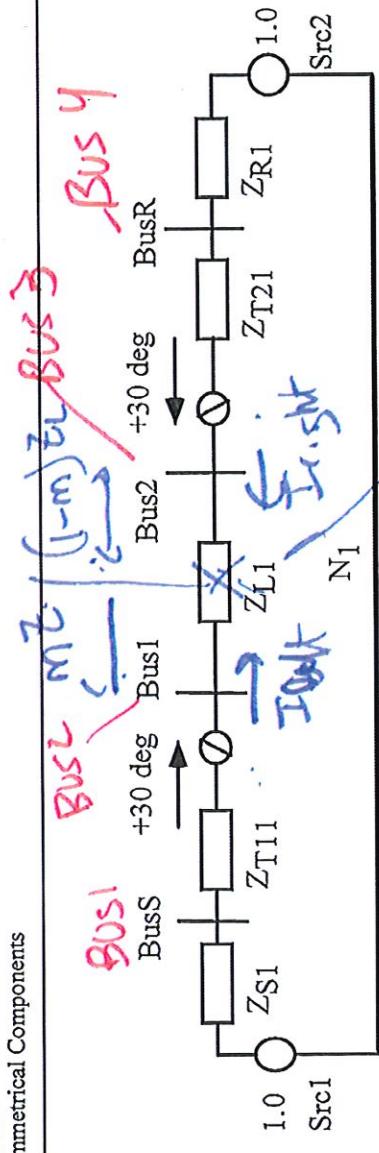
$V_{b2} := 115 \text{kV}$

$$V_{b1} := V_{b2} \cdot \frac{13.8 \text{kV}}{115 \text{kV}}$$

$$V_{b3} := V_{b2} \cdot \frac{24 \text{kV}}{115 \text{kV}}$$

$$V_{b3} = 24 \cdot \text{kV}$$

L9 7/17 t8 16/19



- Regular circuit analysis approach:

- Calculate reduced equivalent for several fault locations
- Use a generic location, M , and define a Matcad function:

$$Z_{\text{left}}(M) := j \cdot X_{G11} + Z_{T11} + Z_{L11} \cdot M$$

$$Z_{\text{right}}(M) := Z_{L11} \cdot (1 - M) + Z_{T21} + j \cdot X_{G21}$$

$$Z_{\text{equiv}}(M) := \left[\frac{1}{j \cdot X_{G11} + Z_{T11} + Z_{L11} \cdot M} + \frac{1}{Z_{L11} \cdot (1 - M) + Z_{T21} + j \cdot X_{G21}} \right]^{-1}$$

- More compactly: $Z_{1\text{thev}}(M) := \left(\frac{1}{Z_{\text{left}}(M)} + \frac{1}{Z_{\text{right}}(M)} \right)^{-1}$
- **Both sources at $M=0$ - referred to as $\text{short circuit on phase A}$**

$$I_{3\text{ph}}(M) := \frac{V_f}{Z_{1\text{equiv}}(M)}$$

$$I_{3\text{ph}}(0.5) = -4.6154i$$

Total fault current on phase A

$$I_{3\text{ph}}(M, R_f) = \frac{V_f}{Z_{1\text{equiv}}(M) + R_f}$$

If $R_f \neq 0$

$$M = 0.5 \quad \checkmark \text{ Load resistance}$$

Z_{Bus} Approach

- Bus impedance matrix
- can be solved directly from a circuit using circuit operations
- messy.

$$\Rightarrow Z = \begin{bmatrix} Z_{TH1} & & \\ & Z_{TH2} & \\ & & Z_{TH3} \end{bmatrix} \dots \begin{bmatrix} Z_{THN} & & \\ & Z_{THN-1} & \\ & & Z_{TH1} \end{bmatrix}$$

Transistor
Impedance
 Z_1, Z_2, \dots, Z_N

$$\Delta V_{BUS1} = Z_{12T} \cdot I_{A1}$$

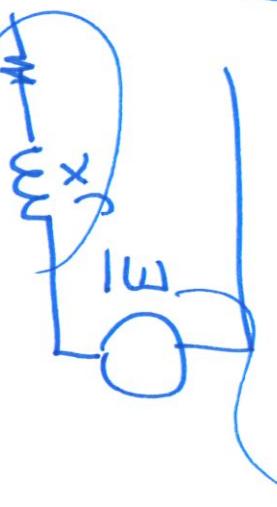
- Get the Z_{bus} matrix by inverting \mathbf{Y}_{bus} matrix
- \mathbf{Y}_{bus} is a sparse matrix

\rightarrow Partial sparse inverse to get the elements needed.

Symmetric, non-zero

$\left[\begin{array}{cccccc} X & X & X & & & \\ X & X & X & X & X & X \\ X & X & X & X & X & X \\ & & & X & X & X \\ & & & X & X & X \\ & & & & X & X \end{array} \right]$

ψ_{BUS} : for fault analysis. [Does not consider sequence matrices]

1. Include internal impedances of generators (rotating machinery) - BUS and large motors
 2. most programs neglect line capacitance (shunt) calculate based on pre-fault load current
 3. model mutual coupling of parallel lines (zero sequence) - later in the course
- Inverter losses - later
resonant currents

4. $\Delta - \frac{Y_1 Y_2}{Z}$ (or other) transforms
connections - in sequence domain -
(3 winding) transformer
Tap changes
phase shifter



Diagonal elements of V_{bus}

$$V_{ii} \quad i \in [1, n]$$

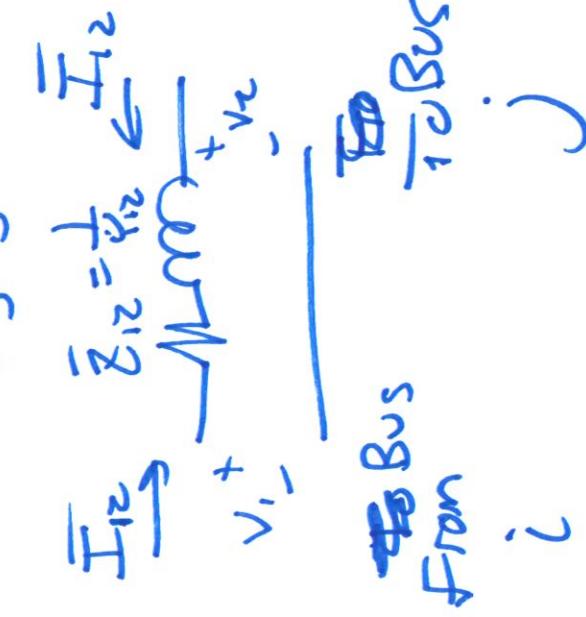
V_{ii} $i \in [1, n]$
→ sum of admittances connected
to that bus

→ Again: includes internal
impedances of sources

Off diagonal elements

\mathcal{Y}_{ij} ($i \neq j$)

→ same rules as powerflow v_{bus}



$$\bar{I}_{12} = (V_1 - V_2) \bar{Y}_{12}$$

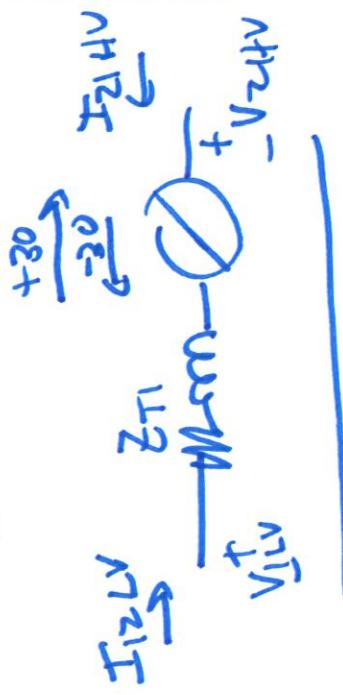
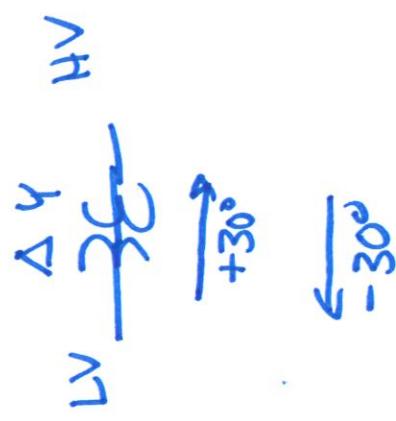
$$\bar{I}_{21} = (V_2 - V_1) \bar{Y}_{21}$$

$$\bar{Y}_{12} = \frac{1}{\bar{Y}_{21}} \text{ for most equipment}$$

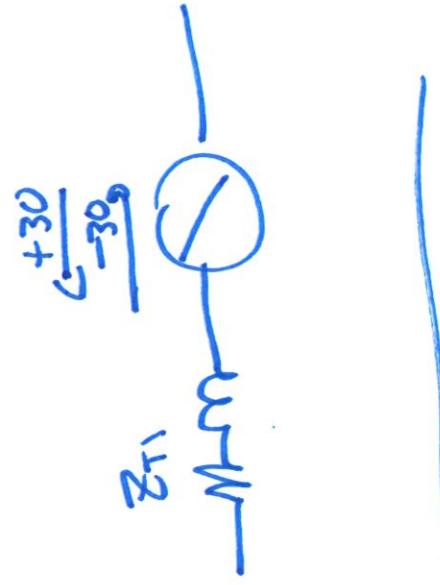
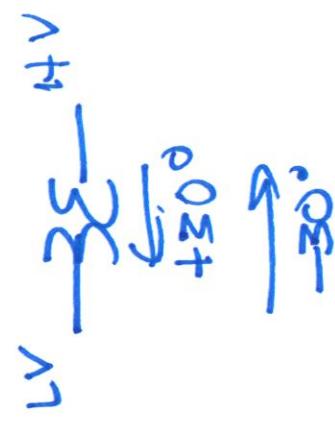
$$\begin{bmatrix} \bar{I}_{12} \\ \bar{I}_{21} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{12} & -\bar{Y}_{12} \\ -\bar{Y}_{21} & \bar{Y}_{21} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

ΔY transformer

Pos sequence



Neg sequence



Pos sequence

$\frac{1}{\sqrt{3}}$

$$\bar{I}_{121LV} = \left(\bar{V}_{1LV} - \bar{V}_{2HV} (1 \angle -30^\circ) \right) e^{j\tau_1}$$

refer to
low voltage side

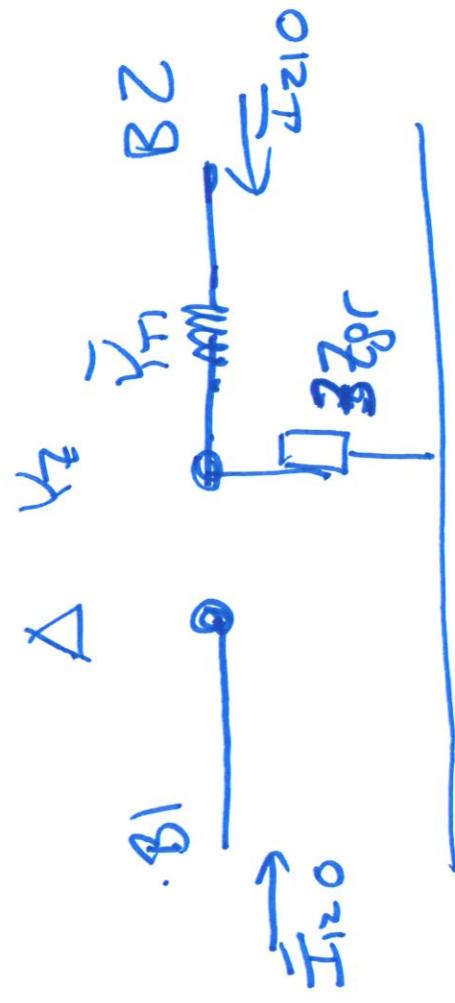
$$\bar{I}_{121HV} = \left(\bar{V}_{2HV} - \bar{V}_{1LV} (1 \angle 30^\circ) \right) e^{j\tau_1}$$

matrix form

$$\begin{bmatrix} \bar{I}_{121LV} \\ \bar{I}_{121HV} \end{bmatrix} = \begin{bmatrix} \bar{V}_{1LV} \\ \bar{V}_{2HV} \end{bmatrix} - \bar{Y}_{T1} \cdot \bar{E}^{-j30^\circ} \begin{bmatrix} \bar{V}_{1LV} \\ \bar{V}_{2HV} \end{bmatrix}$$

$\Delta \psi_2$ in zero sequence
 \rightarrow No phase shift

$\rightarrow I_0$ Circulates in a Δ



3 Bus Matrix Approach

Easiest Approach is to Create Ybus Matrix Modified for Fault Calculations:

Transformer Phase Shift:

$$\begin{aligned}
 & \text{Transformer Phase Shift: } \text{BUS 1} \quad \text{BUS 2} \quad \text{BUS 3} \quad \text{BUS 4} \quad \text{BUS F} \\
 & \left[\begin{array}{ccccc}
 \frac{1}{jX_{G1}} + \frac{1}{Z_{T11}} & \frac{-1 \cdot e^{-j \cdot 30\deg}}{Z_{T11}} & 0 & 0 & 0 \\
 \frac{-1 \cdot e^{j \cdot 30\deg}}{Z_{T11}} & \frac{1}{Z_{T11} + M \cdot Z_{L11}} & 0 & 0 & \frac{-1}{M \cdot Z_{L11}} \\
 0 & 0 & \frac{1}{Z_{T21}} + \frac{1}{(1 - M) \cdot Z_{L11}} & \frac{-1 \cdot e^{j \cdot 30\deg}}{Z_{T21}} & \frac{-1}{(1 - M) \cdot Z_{L11}} \\
 0 & 0 & \frac{-1 \cdot e^{-j \cdot 30\deg}}{Z_{T21}} & \frac{1}{Z_{T21}} + \frac{1}{jX_{G21}} & 0 \\
 0 & \frac{-1}{M \cdot Z_{L11}} & \frac{-1}{(1 - M) \cdot Z_{L11}} & 0 & \frac{1}{M \cdot Z_{L11}} + \frac{1}{(1 - M) \cdot Z_{L11}}
 \end{array} \right] \quad \text{Bus 1} \quad \text{Bus 2} \quad \text{Bus 3} \quad \text{Bus 4} \quad \text{Bus F}
 \end{aligned}$$

$$Z_1(M) := Y_1(M)^{-1}$$

$$Z_1(0.5) = \begin{pmatrix}
 0.0474i & 0.0231 + 0.04i & 0.0103 + 0.0178i & 0.0137i & 0.0167 + 0.0289i \\
 -0.0231 + 0.04i & 0.0692i & 0.0308i & -0.0103 + 0.0178i & 0.05i \\
 -0.0103 + 0.0178i & 0.0308i & 0.2359i & -0.0786 + 0.1362i & 0.1333i \\
 0.0137i & 0.0103 + 0.0178i & 0.0786 + 0.1362i & 0.1937i & 0.0444 + 0.077i \\
 -0.0167 + 0.0289i & 0.05i & 0.1333i & -0.0444 + 0.077i & 0.2167i
 \end{pmatrix}$$