ECE 524: Lecture 14

Define units: MVA := MW  MVAr := MVA

\[ f := 60 \text{Hz} \quad \omega := 2 \cdot \pi \cdot f \quad \omega = 376.99 \cdot \frac{\text{rad}}{\text{s}} \]

\[ \mu \text{sec} := 10^{-6} \text{s} \quad t := 0, 1.5 \mu \text{sec} \quad \frac{1}{60} \text{sec} \]

**Back to Back Capacitor Switching Example**

For the system below find the following:

1. Worst case inrush current when closing into \( C_2 \) with \( C_1 \) in the circuit and \( C_2 \) discharged. Compute without a preinsertion resistor

\[ j X_S \]

Assume source and capacitors are \( Y \) connected.

\[ V_{LL} := 34.5 \text{kV} \quad \text{RMS} \]

Capacitor Banks:

\[ Q_1 := 18 \text{MVAr} \quad \text{at} \ 34.5 \text{kV} \]
\[ Q_2 := 10 \text{MVAr} \quad \text{at} \ 34.5 \text{kV} \]
\[ L_2 := 19.2 \mu \text{H} \quad \text{buswork between caps} \]

Source \[ I_{sc} := 25 \text{kA} \]

Determine circuit parameters:

Find capacitance using:

\[ Q = \frac{(|V_{LL}|)^2}{X_c} \]

\[ X_{c1} := \frac{V_{LL}^2}{Q_1} \quad X_{c1} = 66.13 \Omega \]
\[ X_{c2} := \frac{V_{LL}^2}{Q_2} \quad X_{c2} = 119.03 \Omega \]

\[ C_1 := \frac{1}{\omega \cdot X_{c1}} \quad C_1 = 40.11 \cdot \mu \text{F} \]
\[ C_2 := \frac{1}{\omega \cdot X_{c2}} \quad C_2 = 22.29 \cdot \mu \text{F} \]
Find source impedance using: 

\[ I_{sc} = \frac{V_{in}}{X_s} \]

\[ X_s := \frac{V_{LL}}{I_{sc}} \]

\[ X_s = 0.797 \, \Omega \]

\[ L_s := \frac{X_s}{\omega} \]

\[ L_s = 2.113 \cdot \text{mH} \]

\[ R_s := 0 \, \text{ohm} \quad \text{no source resistance} \]

Equivalent circuit:

\[ V_m := V_{LL} \cdot \sqrt{\frac{2}{3}} \quad V_m = 28.17 \cdot \text{kV} \]

\[ \phi := 90 \, \text{deg} \]

\[ v_s(t) := V_m \cdot \sin(\omega \cdot t + \phi) \]

**Approximate Solution:**

As a first pass we could look at just the transient circulating between the two capacitors.

Series combination of capacitors:

\[ C_{eq} := \frac{C_1 \cdot C_2}{C_1 + C_2} \]

\[ C_{eq} = 14.33 \cdot \mu\text{F} \]

\[ f_{01} := \frac{1}{2 \cdot \pi \cdot \sqrt{L_2 \cdot \frac{(C_1 \cdot C_2)}{(C_1 + C_2)}}} \]

\[ f_{01} = 9596.15 \frac{1}{\text{s}} \]

Likely to be damped severely by a resistance.
\[ f_{02} := \frac{1}{2 \cdot \pi \sqrt{L_s \cdot (C_1 + C_2)}} \quad f_{02} = 438.26 \frac{1}{s} \]

**True double frequency solution**

Determine natural frequencies from characteristic equation

\[ 0 = s_\omega^4 + \left( \frac{1}{L_s \cdot C_1} + \frac{1}{L_{1} \cdot C_2} + \frac{1}{L_{1} \cdot C_1} \right) s_\omega^2 + \frac{1}{L_s \cdot C_1 \cdot L_{1} \cdot C_2} \]

Define constants:

\[ C1 := \frac{1}{L_s \cdot C_1} + \frac{1}{L_{2} \cdot C_2} + \frac{1}{L_{2} \cdot C_1} \]
\[ C2 := \frac{1}{L_s \cdot C_1 \cdot L_{2} \cdot C_2} \]

So:

\[ 0 = s_\omega^4 + C1 \cdot s_\omega^2 + C2 \]

\[ s_{\omega1SQ} := \frac{-C1 + \sqrt{C1^2 - 4C2}}{2} \]
\[ s_{\omega2SQ} := \frac{-C1 - \sqrt{C1^2 - 4C2}}{2} \]

\[ s_{\omega1SQ} = -7.57 \times 10^6 \frac{1}{s^2} \]
\[ s_{\omega2SQ} = -3.64 \times 10^9 \frac{1}{s^2} \]

Recall that: \( \sqrt{s} = j \omega \)

\[ \omega_1 := \text{Im}(\sqrt{s_{\omega1SQ}}) \]
\[ \omega_2 := \text{Im}(\sqrt{s_{\omega2SQ}}) \]

\[ \omega_1 = 2752.07 \frac{\text{rad}}{s} \]
\[ \omega_2 = 6.03 \times 10^4 \frac{\text{rad}}{s} \]

\[ f_1 := \frac{\omega_1}{2 \cdot \pi} \quad f_1 = 438.01 \frac{1}{s} \]
\[ f_2 := \frac{\omega_2}{2 \cdot \pi} \quad f_2 = 9601.72 \frac{1}{s} \]
Compare to: \[ f_{02} = 438.26 \frac{1}{s} \quad f_{01} = 9596.15 \frac{1}{s} \]

So the approximation used above is close, it helps that there is a wide separation in frequencies.

Initial Voltage Conditions:

\[ V_{c1_0} := V_m \quad V_{c2_0} := 0 \text{V} \]

\[ Z_{01} := \frac{L_2}{\sqrt{\frac{C_{eq}}{C}}} \quad Z_{01} = 1.158 \text{Ω} \]

Peak current:

\[ I_{pk1} := \frac{V_{c1_0} - V_{c2_0}}{Z_{01}} \quad I_{pk1} = 24.33 \text{kA} \quad \text{Very large current at a high frequency!} \]

**Note:** This is circulating between the capacitors, not through the source.....

For comparison, the steady-state current would be:

\[ I_{ss} := \frac{V_{LL}}{\sqrt{3}} \left\{ \frac{1}{j \cdot X_s + \left( \frac{1}{-j \cdot X_{c1}} + \frac{1}{-j \cdot X_{c2} + j \cdot \omega \cdot L_2} \right)^{-1}} \right\} - 1 \quad |I_{ss}| = 477.53 \text{A} \]

Or with just 1 capacitor (as we saw a few lectures ago):

\[ I_{ss_{1cap}} := \frac{V_{LL}}{\sqrt{3}} \left\{ \frac{1}{j \cdot X_s + -j \cdot X_{c1}} \right\} \quad |I_{ss_{1cap}}| = 304.9 \text{A} \]
Now look at the voltage at the bus:

First look at the term at the above resonant frequency:

From charge balance:

$$V_{\infty} := \frac{V_{c1,0} \cdot C_1 + V_{c2,0} \cdot C_2}{C_1 + C_2}$$

$$V_{\infty} = 18.11 \text{ kV}$$

There will be an oscillating voltage based on this:

$$\omega_{01} := 2 \cdot \pi \cdot f_{01}$$

$$v_{c2}(t) := V_{\infty} - V_{\infty} \cdot \cos(\omega_{01} \cdot t)$$

This represents the increase in voltage at that capacitor

$$v_{c1}(t) := V_{\infty} + (V_m - V_{\infty}) \cdot \cos(\omega_{01} \cdot t)$$

This represents the decrease in voltage at this capacitor

Note that:

$$(V_m - V_{\infty}) = 10.06 \text{ kV}$$

- Note that these are only correct if the high freq resonant term is considered in isolation
- The other terms correct the offsets
There will be 2 other terms in the response:

1. The 60 Hz response---just the steady-state solution
2. A term that is equivalent to the two capacitors adding in parallel, resonating with the source impedance.

\[
f_{02} := \frac{1}{2 \cdot \pi \cdot \sqrt{L_s \cdot \left( C_1 + C_2 \right)}} \quad \quad f_{02} = 438.26 \frac{1}{s}
\]

This is how the circuit would behave when the high frequency transient dies out. We looked at this part of the response last time with adding the resistor. This will add to the above response since the source circuit will respond to the change in voltage across \( C_1 \).

\[
Z_{02} := \sqrt{\frac{L_s}{C_1 + C_2}} \quad \quad Z_{02} = 5.82 \, \Omega
\]

\[
I_{2pk} := \frac{V_m - V_{\infty}}{Z_{02}} \quad \quad I_{2pk} = 1728.69 \, A
\]

The voltage at this frequency will be the same on both capacitors and it will have an amplitude of:

\[
(V_m - V_{\infty}) = 10.06 \cdot kV
\]

Capacitor C1 voltage (unzoomed)--note multiple frequencies:
Two frequencies, one is 180 degrees out of phase on the two caps and the other is in phase

Zoom in to first few cycles at high frequency:

First peak of blue line is 36.59kV, compare to 36.22kV the analytical

Initial voltage for green (Vc1) is 28.5kV compared to 28.169kV and the negative peak is 8.26kV compared to 8.21kV
Source current (compared to Vs/10)

Note that the high frequency term is barely noticeable

Zoom in on the current
Now look at the current between the capacitors (now the source voltage is not scaled)

![Graph showing current between capacitors]

Now the high frequency term dominates....

Zoom in on the current between caps: Calculated 24.33kA with high freq term alone

![Graph showing zoomed-in current]

\[
I_{pk} := 24.56\text{kA}
\]

\[
T_1 := 0.016693\text{sec} \quad T_2 := 0.016797\text{sec} \quad \frac{1}{T_2 - T_1} = 9615.38 \frac{1}{\text{s}}
\]
- **PSCAD/EMTDC Results**

- Source voltage and Vc1
- Zoom in on $V_s$, $V_{c1}$ and $V_{c2}$

Zoom in to first few cycles at high frequency:

First peak of green line is 36.72kV, compare to 36.22kV the analytical

Initial voltage for green ($V_{c1}$) is 28.57kV compared to 28.169kV and the negative peak is 8.26kV compared to 8.21kV
- Source voltage (scaled by 1/10) and current through Ls

- Source voltage (unscaled) and current through L2
- Zoom in on IL2

\[ I_{pk} := 24.64\text{kA} \]

Calculated 24.33kA with high freq term alone

\[ T_1 := 0.087526\text{sec} \quad T_2 := 0.087630\text{sec} \quad \frac{1}{T_2 - T_1} = 9615.38\text{Hz} \]