**ECE 524: Lecture 17**

*Derivation for Undamped TRV*

\[ V_m \cos(\omega \cdot t) - v_c(0) = L \cdot \frac{d}{dt} i(t) + \frac{1}{C} \int i(t) \, dt \]

- Substitute the following for \( i(t) \) in the equation above

\[ i(t) = C \cdot \left( \frac{d}{dt} v_c(t) \right) \]

- Resulting equation:

\[ V_m \cos(\omega \cdot t) - v_c(0) = L \cdot C \cdot \frac{d^2}{dt^2} v_c(t) + v_c(t) + v_c'(0) \]

Where \( v_c'(0) \) is the derivative of \( v_c(t) \) at \( t = 0 \) (due to substitution into the integral)

- We know that:

\[ L \cdot C = \frac{1}{\omega_0^2} \]

- Divide by sides by LC, but and substitute with \( \omega_0^2 \) and rearrange terms

\[ \frac{d^2}{dt^2} v_c(t) + \omega_0^2 \cdot v_c(t) = \omega_0^2 \cdot V_m \cos(\omega \cdot t) - \omega_0^2 \cdot v_c(0) - \omega_0^2 \cdot v_c'(0) \]

- Now take LaPlace Transform

\[ s^2 \cdot V_c(s) + \omega_0^2 \cdot V_c(s) = \omega_0^2 \cdot \frac{V_m \cdot s}{s^2 + \omega^2} - \omega_0^2 \cdot s \cdot V_c(0) - \omega_0^2 \cdot V_c'(0) \]

- From initial conditions:

\[ v_c(0) = 0 \quad \text{bolted fault, same at } t = 0^- \text{ and } t = 0^+ \text{ since capacitor voltage} \]

\[ i_L(0) = i_c(0) = 0 \quad \text{inductor current zero when breaker clears, and current will flow through capacitor. As a result} \]
\[ i_c(0) = C \cdot \frac{d}{dt} v_c(t) \] at \( t = 0 \) is 0, so \( v_c'(0) = 0 \)

These also map to 0 in the LaPlace domain. So the equation simplifies to:

\[ s^2 \cdot V_c(s) + \omega_0^2 \cdot V_c(s) = \omega_0^2 \cdot \frac{V_m \cdot s}{s^2 + \omega^2} \]

- Solve for \( V_c(s) \), which results in:
  \[ V_c(s) = V_m \left[ \frac{\omega_0^2 \cdot s}{(s^2 + \omega^2)(s^2 + \omega_0^2)} \right] \]

- Then take partial fraction expansion and then simplify, which results in
  \[ V_c(s) = V_m \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right) \left[ \frac{s}{(s^2 + \omega^2)} - \frac{s}{(s^2 + \omega_0^2)} \right] \]

- Take inverse LaPlace transform:
  \[ v_c(t) = V_m \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right) \left[ \cos(\omega \cdot t) - \cos(\omega_0 \cdot t) \right] \]