Switch Opening Transients

- Interruption & current
  with breakers

- Voltage across contacts after clear

\[ \Delta V = L \frac{\Delta i}{\Delta t} \]
we want to calculate worst case

- Rate of Rise of Recover Voltage (RRRV)
in CB rating
Reduce TRV...
$V_C(0) = 0$
ECE 524: Lecture 17

Derivation for Undamped TRV

\[ V_m \cdot \cos(\omega \cdot t) - v_c(0) = L \cdot \frac{d}{dt} i(t) + \frac{1}{C} \int i(t) \, dt + v_c(0) \]

- Substitute the following for \( i(t) \) in the equation above

\[ i(t) = C \left( \frac{d}{dt} v_c(t) \right) \Rightarrow i_L = i_c \]

- Resulting equation:

\[ V_m \cdot \cos(\omega \cdot t) - v_c(0) = L \cdot C \cdot \frac{d^2}{dt^2} v_c(t) + v_c(t) + v'_c(0) \]

Where \( v'_c(0) \) is the derivative of \( v_c(t) \) at \( t = 0 \) (due to substitution into the integral)

- We know that:

\[ L \cdot C = \frac{1}{\omega_0^2} \]

- Divide by sides by \( L \cdot C \), but and substitute with \( \omega_0^2 \) and rearrange

\[ \frac{d^2}{dt^2} v_c(t) + \omega_0^2 \cdot v_c(t) = \omega_0^2 \cdot V_m \cdot \cos(\omega \cdot t) - \omega_0^2 \cdot v_c(0) - \omega_0^2 \cdot v'_c(0) \]

- Now take LaPlace Transform

\[ s^2 \cdot V_c(s) + \omega_0^2 \cdot V_c(s) = \omega_0^2 \cdot \frac{V_m \cdot s}{s^2 + \omega^2} - \omega_0^2 \cdot s \cdot V_c(0) - \omega_0^2 \cdot v'_c(0) \]

- From initial conditions:

\[ v_c(0) = 0 \quad \text{bolted fault, same at } t = 0- \text{ and } t = 0+ \text{ since capacitor voltage} \]

\[ i_L(0) = i_c(0) = 0 \quad \text{inductor current zero when breaker clears, and current will flow though capacitor. As a result} \]
\[ i_c(0) = C \left( \frac{d}{dt} v_c(t) \right) \] at \( t=0 \) is 0, so \( v'_c(0)=0 \)

These also map to 0 in the LaPlace domain. So the equation simplifies to:

\[ s^2 \cdot V_c(s) + \omega_0^2 \cdot V_c(s) = \frac{\omega_0^2 \cdot V_m \cdot s}{s^2 + \omega^2} \]

- Solve for \( V_c(s) \), which results in:

\[
V_c(s) = V_m \left[ \frac{\omega_0^2 \cdot s}{\left( s^2 + \omega^2 \right) \left( \frac{1}{s^2 + \omega^2} \right)} \right]
\]

- Then take partial fraction expansion and then simplify, which results in:

\[
V_c(s) = V_m \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right) \left[ \frac{s}{s^2 + \omega^2} - \frac{s}{s^2 + \omega_0^2} \right]
\]

- Take inverse LaPlace transform:

\[
v_c(t) = V_m \left( \frac{\omega_0^2}{\omega_0^2 - \omega^2} \right) \left( \cos(\omega \cdot t) - \cos(\omega_0 \cdot t) \right)
\]

So \( \frac{\omega}{\omega_0} < 1 \) resonant, power flow.

\[ \omega_0 \gg \omega \]