

# Numerical Oscillations in EMTP-Like Programs

## 1 Causes of Numerical Oscillations

The Electromagnetic transients program and its variants all use the the “trapezoidal rule” numerical integration method. The trapezoidal rule is a second order numerical integration technique that is simple to implement, astable (numerically stable in “stiff” systems), and fast. However, it is also susceptible to numerical oscillations when differentiating step changes in voltage or current. The program user should be aware of the potential for these oscillations when simulating circuits and systems using emtp-like programs.

The most common events leading to numerical oscillations are a step change in current through and inductor or a step change in voltage across a capacitor. The cause if this problem can be seen by looking at the differential equations for the inductor and the capacitor.

$$\text{Inductor Voltage} \quad v = L \frac{di}{dt} \quad (1)$$

$$\text{Capacitor Current} \quad i = C \frac{dv}{dt} \quad (2)$$

One can think of the numerical oscillations as resulting from forcing a large  $di/dt$  across and inductor or a large  $dv/dt$  on a capacitor. The best solution is to produce a more accurate circuit model by including parasitic capacitances internal resistances in a capacitor, but this isn't always reasonable, especially with power converters.

### 1.1 Mathematical Representation

The problem can be represented by modeling the equation

$$x = \frac{dy}{dt} \quad (3)$$

using the trapezoidal rule. We can integrate both sides of the equation, resulting in the following:

$$y(t) = \int_{t-\Delta t}^t x dt + y(t - \Delta t) \quad (4)$$

Next we rewrite the equation using the trapezoidal rule. Remember, we are finding an approximation of the area of the trapezoid bounded by  $x$ ,  $t$  and  $t - \Delta t$ .

$$y(t) = y(t - \Delta t) + \frac{\Delta t}{2}(x(t) + x(t - \Delta t)) \quad (5)$$

Now we can solve this equation for  $x(t)$ , which could be either the current through a capacitor or the voltage across an inductor.

$$x(t) = -x(t - \Delta t) + \frac{2}{\Delta t}(y(t) - y(t - \Delta t)) \quad (6)$$

Start out with  $x(t - \Delta t) = 0$ . We will now have  $y$  undergo a step change from  $y(t - \Delta t) = 0$  to  $y(t) = 1.0$ . The function will stay at 1.0 for the rest of the time period of interest.

We would expect  $y(t)$  and  $x(t)$  to behave as shown in Figure 1. Note that we would expect  $x(t)$  to be an impulse (Dirac Delta Function) at at  $t = t$  and zero everywhere else.

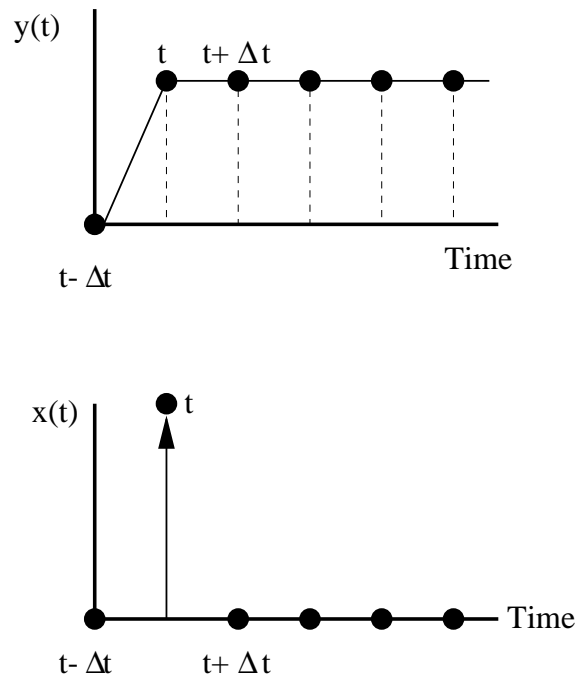


Figure 1: Plots showing  $y(t)$  and expected  $x(t)$  for example case.

Plug the know values into the equation for  $x(t)$ , resulting in:

$$x(t) = \frac{2}{\Delta t} \quad (7)$$

Now step ahead and find  $x(t + \Delta t)$ :

$$x(t + \Delta t) = -\frac{2}{\Delta t} + \frac{2}{\Delta t}(1.0 - 1.0) = -\frac{2}{\Delta t} \quad (8)$$

If we continue ahead another time step, we find that

$$x(t + 2\Delta t) = \frac{2}{\Delta t} \quad (9)$$

This oscillation will continue until circuit conditions change. So, if we plot  $x(t)$  we see the behavior shown in Figure 2. The average value of  $x(t)$  is the desired waveform.

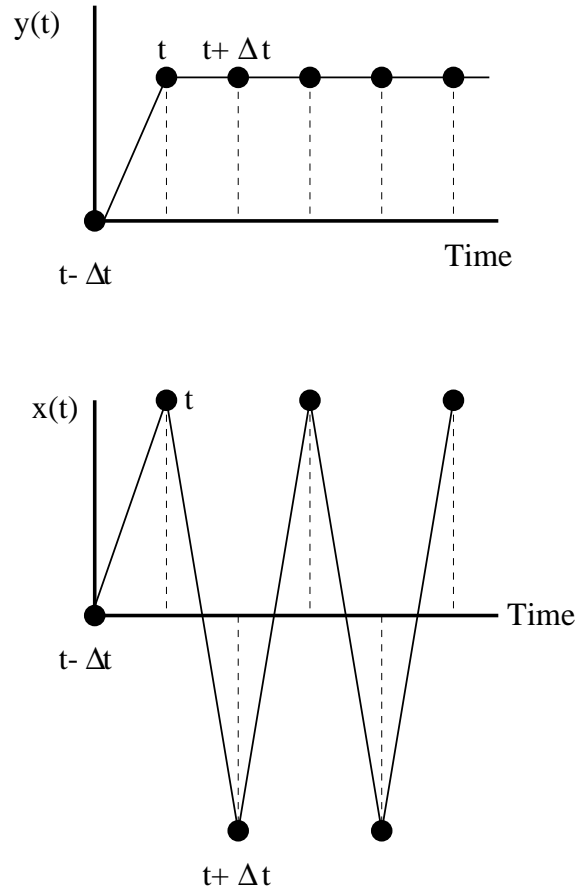


Figure 2: Actual  $x(t)$  with numerical oscillation.

If we had allowed the transition in  $y(t)$  to occur over two time steps instead of one time step, the problem would have been avoided. The following set of equations shows the results for the example above. Assume that  $y(t - \Delta t) = 0.0$ ,  $y(t) = 0.5$  and  $y(t + \Delta t) = 1.0$  and then the function stays at 1.0 for the rest of the time period.

$$x(t) = 0.5 \frac{2}{\Delta t} \tag{10}$$

$$x(t + \Delta t) = -0.5 \frac{2}{\Delta t} + \frac{2}{\Delta t} (1.0 - 0.5) = 0.0 \tag{11}$$

$$x(t + 2\Delta t) = 0.0 + \frac{2}{\Delta t} (1.0 - 1.0) = 0.0 \tag{12}$$

$$x(t + 3\Delta t) = 0.0 \tag{13}$$

⋮     ⋮

Figure 3 shows  $y(t)$  and  $x(t)$ . Notice that we see more of an impulse response. This behavior suggests that reducing the time step may in some cases reduce the problems with numerical oscillations. However, oscillations caused by switch operations will not be reduced by making a smaller time step since the ideal switch will always change in one time step.

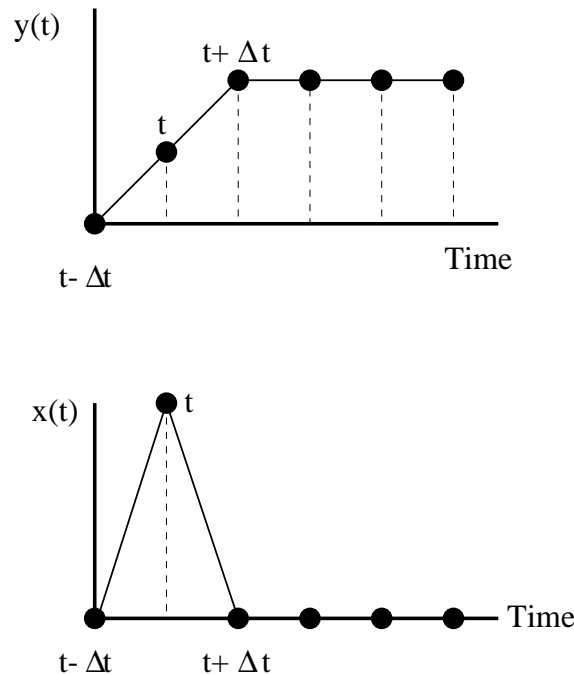


Figure 3: Plots of  $y(t)$  and  $x(t)$  with slower rise time

## 1.2 Examples

### 1.2.1 Inductor Current

The first example is a single phase system with a 10 kV (peak) voltage source supplying a 25 Ohm resistor through an inductance of 20 mH. A switch connected between the L and the R opens at 1 msec. into the simulation, resulting in the voltage oscillations shown in Figure 4. The first plot shows the node voltage on the switch pole. The second plot zooms in on the voltage, showing the numerical oscillations. The third plot shows the switch current. Notice that it goes to zero and stays at zero. The EMTP data file is provided below.

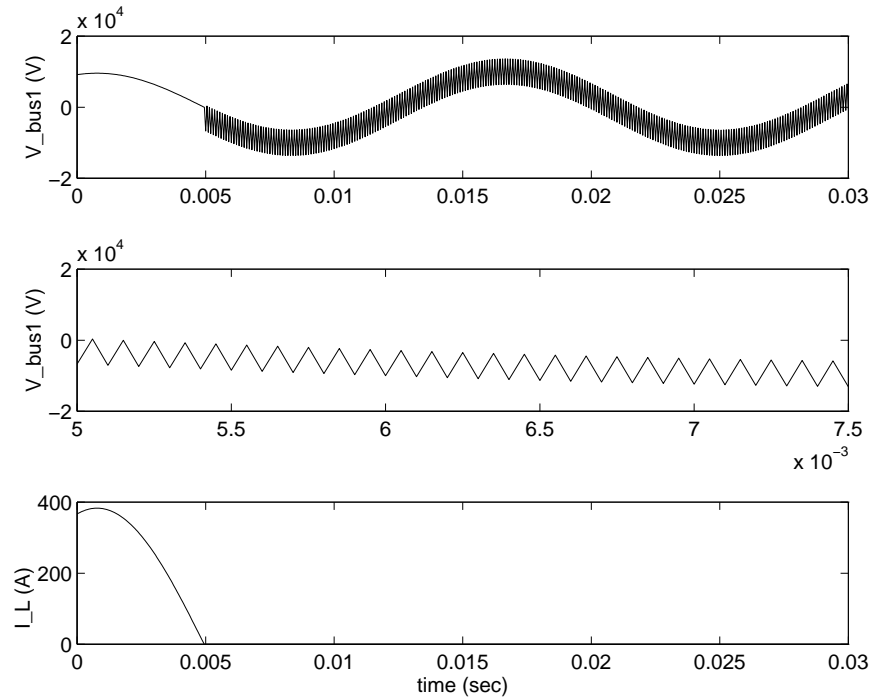


Figure 4: Example with switch in series with inductor

### 1.2.2 Capacitor Voltage

The second example is a single phase system with a sinusoidal voltage source supplying a pi circuit. A switch connecting the source to the pi circuit closes at 16.667 msec, causing a step changing in the capacitor voltage on the capacitor near the source. Figure 5 shows EMTP simulation results. The first plot shows the current through the capacitor connected to the switch. Notice the large amplitude of the oscillations in this case. The second plot zooms in on the oscillations themselves and the third plot shows that the voltage across the capacitor is not impacted.

## 2 Options to Eliminate Numerical Oscillations

### 2.1 Adding Circuit Elements

The numerical oscillations can be viewed as arising due to inadequate detail in the system model. For example, oscillations arising from a switch opening in series with an inductor would actually have an arc impedance. However, arc impedance modeling is fairly complex. Another option is include parasitic capacitance as shown in Figure 6. Instead of numerical oscillations the circuit will possible see LC oscillations that would occur in practice.

Similarly, the change in voltage across a capacitor is missing the effective series resistance of the capacitor and other loss terms as shown below when damping is added.

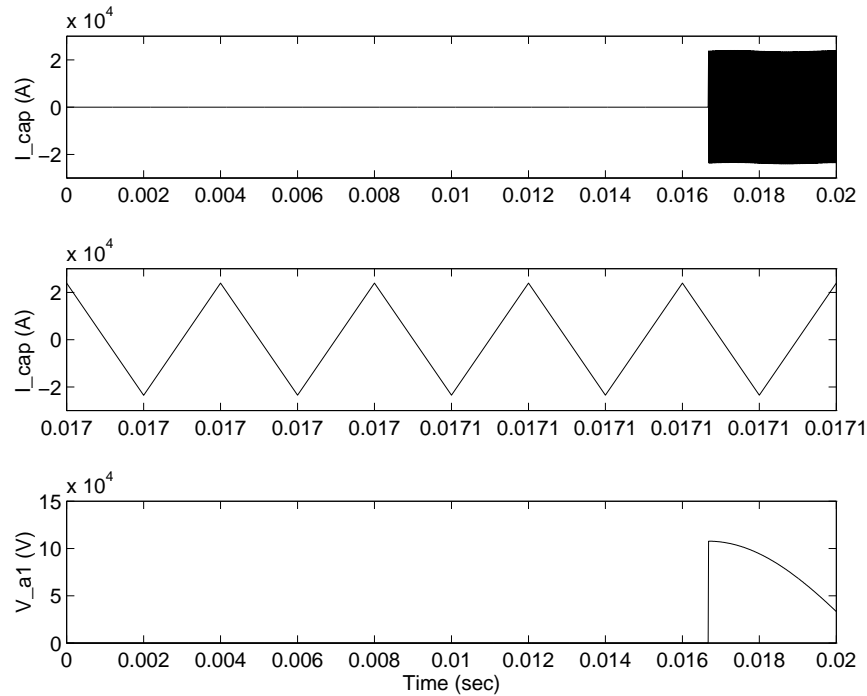


Figure 5: Capacitor example

The inductor current example shown earlier was modified to include a parasitic capacitance of  $100\text{ nF}$  connected from the inductor side of the switch to ground. In addition, the series resistance of the inductor was added. When the circuit was simulated with EMTP as shown in Figure 7, oscillations still occur, except this time they are now due to the LC resonance. The first plot shows the voltage oscillations. The second plot zooms in to show that these oscillations are now sinusoidal. The third plot shows the inductor current, which now has a small sinusoidal oscillation as well. Since the circuit has a very small resistance, these oscillations die slowly.

## 2.2 Snubber Circuits

However, it isn't always practical to include these missing parasitic elements. This is often the case modeling power electronic circuits. Many power electronic circuits have some form of passive (and sometimes active) circuit added across the switch to limit  $di/dt$  or  $dv/dt$  seen by the device. In this case we want to use  $di/dt$  snubbers, as shown in Figure 8. The power electronic switch is capable of interrupting the current too quickly, forcing a steep  $di/dt$  in the circuit inductances. The high peak voltages that result can damage circuit components and decrease insulation life. The snubber provides an alternate current path for the largely inductive current resulting in a damped RLC circuit. The current will decrease to zero when the capacitor is charged. The same effect could be created with only a capacitor, but the resistor will provide energy dissipation and damping. Several other forms of snubber circuits are used in power electronic circuits.

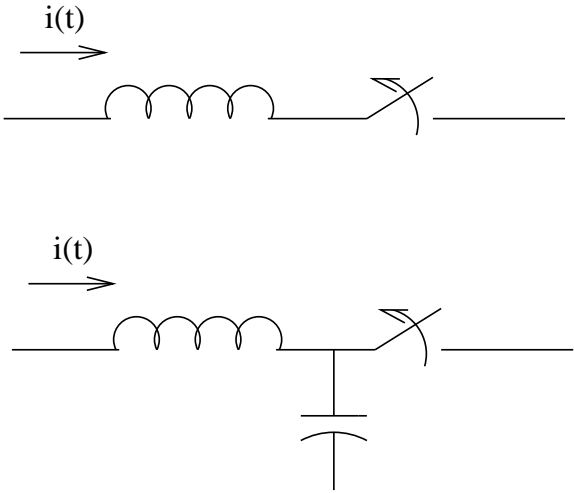


Figure 6: Switch in series with an inductor with and without parasitic capacitance.

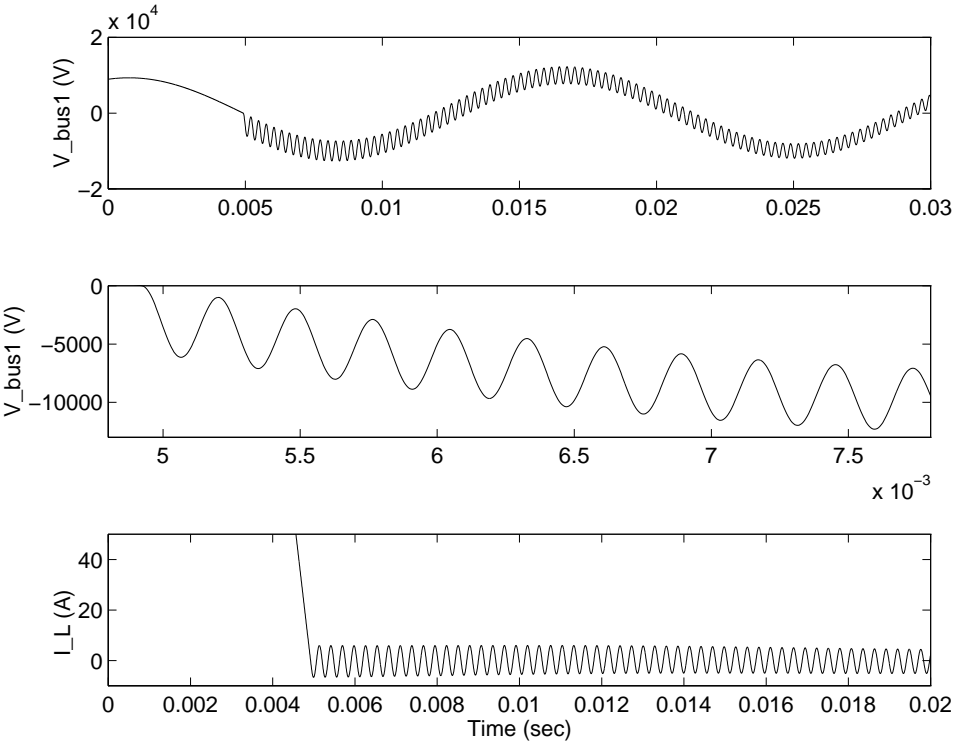


Figure 7: Simulation with parasitic capacitance added

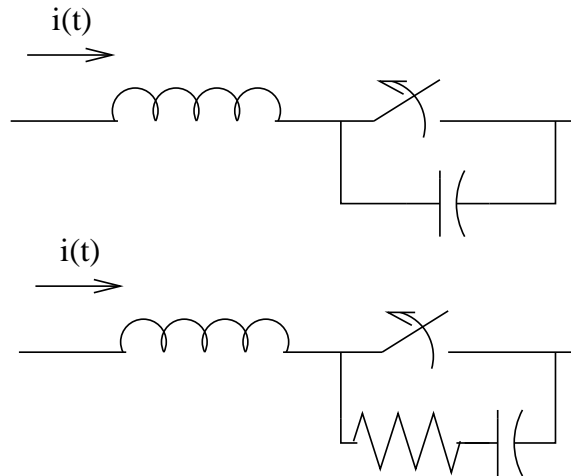


Figure 8: Snubber Options

These snubbers will also remove or reduce the numerical oscillation problem. Since the snubber circuit provides a bypass path for the switch, the numerical oscillations are avoided, and a more accurate model of the power electronic circuit is used.

However, some circuit topologies do need snubbers (diode rectifiers for example). In addition, switches with large safe operating areas (SOA) such as IGBT's and MOSFET's don't need snubbers.

In these cases the program user can add numerical snubbers. These numerical snubbers can also be added for conventional switches as well, although they have less of a basis in reality. If a capacitor alone is used, a capacitance value of roughly 1-2  $nF$  will suffice. If a RC snubber is used as shown in Figure 8, the time constant for the RC snubber must still be greater than the simulation time step to avoid problems. The resistance value should be chosen so the RC time constant is a minimum of 2-3 times the simulation time step  $\Delta t$ . Performance of the RC snubber will vary with the circuit, and the user may need to vary R and C values for best performance.

The inductor current example from earlier was simulated with a RC snubber across the switch, as shown in Figure 9. The first plot shows the resulting voltage. Notice that there are no oscillations. The second plot zooms in on the voltage waveform. Note that the voltage sees a decaying exponential from overdamped RLC response. The third plot shows the snubber current. Note also that the time step in the datafile might need to be smaller to capture the LC resonance.

### 2.3 Reducing the Time-step

In some cases, reducing the simulation time step can also eliminate numerical oscillations. However, this is not a general purpose solution. It depends on the presence of resistances in the circuit, preferably in parallel with inductances or in series with the capacitances. In many cases the required time step is also far too small for practical simulation. Also, since the amplitude of the oscillation varies with  $1/\Delta t$ , the smaller time step may also make the oscillations larger.



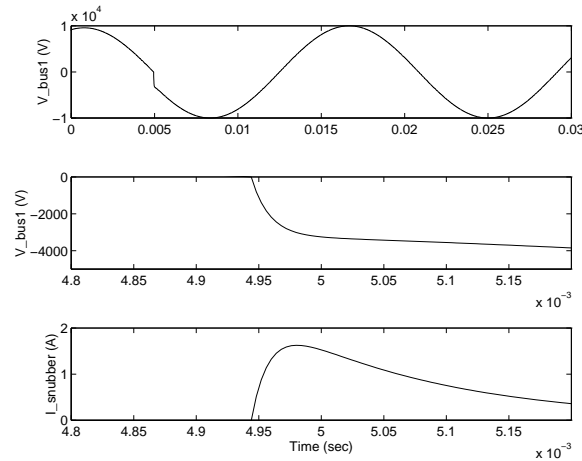


Figure 9: Inductor current example with snubber added

## 2.4 Introducing Damping

A more practical alternative to remove numerical oscillations is the addition of a damping resistor to the circuit. The damping resistor for the inductive switching case should be connected in parallel with the inductor, as shown in Figure 10, providing a conduction path for the inductor current when the switch opens. The current in the RL circuit will decay with time. Careful choice of the resistance value will lead to rapid damping of the oscillations. However, if the resistance value is too small, the presence of the resistor will impact circuit behavior. Since the RL time constant is:

$$\tau = \frac{L}{R_d} \quad (14)$$

We can base the choice of the time constant as related to the time step  $\Delta t$ .

$$R_d = \frac{L}{\alpha \Delta t} \quad (15)$$

Where  $\alpha$  is referred to as the damping factor. If  $\alpha = 0$ , we have the basic trapezoidal rule model for the inductor. Analysis of percent error in the discrete model of the inductor shows that choosing a value of  $\alpha = 0.15$  gives a good trade-off for damping versus error relative to inductor solved in the continuous time. In fact, it is actually more accurate for some values of the time step. Remember, the trapezoidal rule representation of the inductor is an approximation and is not perfect. Choosing  $\alpha = 0.15$ , sets RL time constant at  $6.67\Delta t$ .

ATPDraw implements this option as well. The inductor dialog box has an entry for  $Kp$ , where:

$$R_d = \frac{2KpL}{\Delta t} \quad (16)$$

So  $K_p = 1/2\alpha$ . The ATPDraw help file recommends  $K_p = 7.5$  as a typical value. This corresponds to  $\alpha = 0.0666$ . And a value of  $\alpha = 0.15$  corresponds to  $K_p = 3.333$ , and provides better damping. However the parallel resistor is smaller, and is more likely to impact the circuit behavior all of the time.

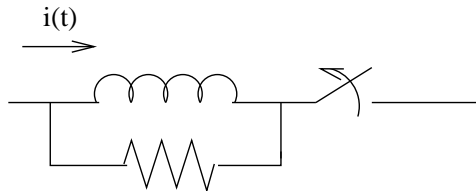


Figure 10: Inductor with Damping Resistor

A similar development can be performed for the capacitor. In this case the resistance is connected in series with the capacitor as shown in Figure 11. The step change in voltage is first dropped across the resistor and the voltage on the capacitor changes through a series RC circuit response. Similar to the RL case, an RC time constant of  $6.667\Delta t$  results in a good trade-off between accuracy and damping performance. This results in choosing  $R = \frac{6.667\Delta t}{C}$ . However, this makes the Resistor large enough to have a significant impact on the circuit. Choose a smaller number, in the 0.1 to 0.2 range is a compromise.

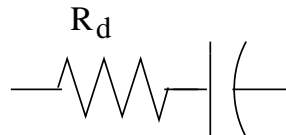


Figure 11: Capacitor with Damping Resistor

ATPDraw implements this option as well. The capacitor dialog box has an entry for  $K_s$ , where:

$$R_d = \frac{K_s \Delta t}{2C} \tag{17}$$

ATPDraw recommends a value of about  $K_s = 0.15$ , the ratio suggested above would have  $K_s = 13.3333$ . Again, it is a trade off between damping and impact on the circuit. It would be smarter to configure the circuit to prevent this circumstance by always having an impedance between an ideal source and a capacitor if you have a switch between them as well.

Both the inductor and capacitor switching examples from earlier were repeated with a damping resistor added. Figure 12 shows the results for the inductive switching case. The first plot shows the resulting voltage. The second plot zooms in on transient. Notice that a few oscillations occur and are damped.

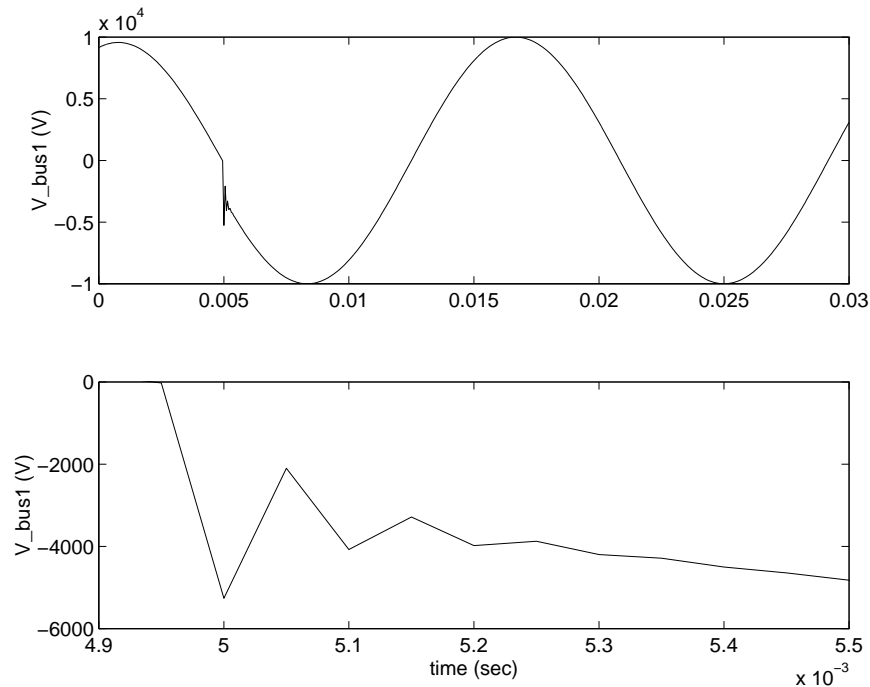


Figure 12: Inductor switching case with damping resistor

Figure 13 shows the results for the capacitor switching case. The first plot shows the resulting current. Notice that there is a significant initial overshoot as the damped numerical oscillation begins. However, it dies rapidly. The second plot zooms in on transient. Notice that essentially one oscillation occurs.

## 2.5 Interpolation

The interpolation scheme detects the step change in voltage and current that will trigger oscillations. Then moves backward in time using linear interpolation to the current zero or switch operation. The differential equations are solved at this point and the program restarts the time step loop from that point.

Netomac was probably the first of the EMTP-like programs to implement interpolation. EMTDC implements interpolation for the LC elements. In addition interpolation is also used for transmission line models and firing pulses for power electronic devices.

## 2.6 Changing Integration Method

The EPRI version of EMTP eliminates the numerical oscillation problem by performing two time steps using the backward Euler integration method following a discontinuity with time step cut in half. The half-length time steps for the backward Euler stages result in the program restarting

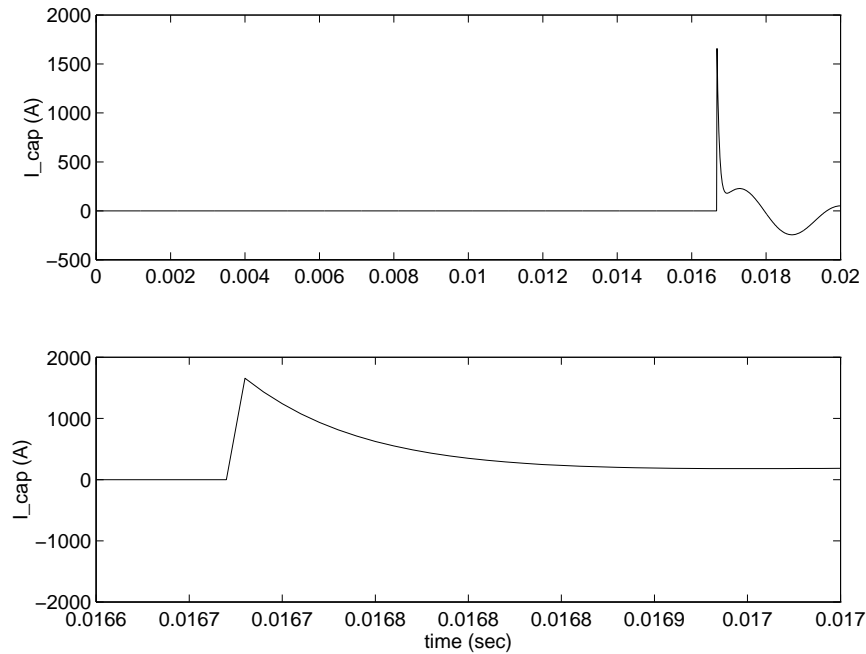


Figure 13: Capacitor switching case with damping resistor

with trapezoidal rule integration one timestep after the transition.

Remember, this is only available in the DCG EPRI version of EMTP.

The case discussed earlier will now be re-done using the critical damping adjustment. Again, start with:

$$x = \frac{dy}{dt} \tag{18}$$

And then integrate both sides of the equation, resulting in the following:

$$y\left(t + \frac{\Delta t}{2}\right) = \int_t^{t + \frac{\Delta t}{2}} x dt + y(t) \tag{19}$$

Next we rewrite the integral equation using the backward Euler, and a time step of  $1/2 \Delta t$  as is used for the CDA method.

$$y\left(t + \frac{\Delta t}{2}\right) = y(t) + \frac{\Delta t}{2} \left[ x\left(t + \frac{\Delta t}{2}\right) \right] \tag{20}$$

Solve this equation for  $x\left(t + \frac{\Delta t}{2}\right)$ , which could be either the current through a capacitor or the voltage across an inductor.

$$x(t + \frac{\Delta t}{2}) = \frac{2}{\Delta t} \left[ y(t + \frac{\Delta t}{2}) - y(t) \right] \quad (21)$$

Similarly,  $x(t + \Delta t)$  can be written as:

$$x(t + \Delta t) = \frac{2}{\Delta t} \left[ y(t + \Delta t) - y(t + \frac{\Delta t}{2}) \right] \quad (22)$$

The trapezoidal rule integration resumes starting from this point and the intermediate step is not used.

Start out with  $x(t) = 0$ . We will now have  $y$  undergo a step change from  $y(t) = 0$  to  $y(t + \frac{\Delta t}{2}) = 1.0$  and then  $y(t)$  stays at 1.0.

$$\begin{aligned} y(t) &= 0.0 \\ x(t) &= 0.0 \\ y(t + \frac{\Delta t}{2}) &= 1.0 \\ x(t + \frac{\Delta t}{2}) &= \frac{\Delta t}{2}(1.0 - 0) = \frac{\Delta t}{2} \\ y(t + \Delta t) &= 1.0 \\ x(t + \Delta t) &= \frac{\Delta t}{2}(1.0 - 1.0) = 0.0 \end{aligned}$$

Note that interpolation or CDA eliminate the numerical oscillations. But they might not accurately represent the circuit response if the numerical oscillations were caused by a deficient model.