Clearing Three Phase Reactor

A 132kV, 50MVAR three phase reactor is connected in an Y ungrounded configuration. Each phase can be represented using a pi equivalent, with capacitances of 2000pF and an inductance suitable for the MVAR rating.

\[
\begin{align*}
\text{Breaker} \\
\end{align*}
\]

A' \quad \text{C'} \quad B'

\[
\begin{align*}
V_{ag} \quad V_{bg} \quad V_{cg} \\
L_\phi \quad L_\phi \quad L_\phi \\
C_\phi \quad C_\phi \quad C_\phi \\
\end{align*}
\]

\[
\begin{align*}
&\text{a) Compute the natural frequencies of the reactor when the first phase clears and then when the subsequent phases clear.} \\
&MVA := 1000kW \quad \text{MVAR} := \text{MVA} \quad \mu s := \frac{s}{10^6} \quad \text{ms} := \frac{s}{10^3} \quad \text{pu} := 1 \\
&V_{LL} := 132kV \quad V_m := \sqrt[3]{\frac{2}{3}} \cdot V_{LL} \quad V_m = 107.78\cdot kV \\
&Q_L := 50\text{MVAR} \quad \omega := 2\cdot \pi \cdot 60\text{Hz} \\
&X_L := \frac{V_{LL}^2}{Q_L} \quad X_L = 348.48\Omega \quad L_\phi := \frac{X_L}{\omega} \\
&L_\phi = 924.37\cdot \text{mH} \\
&C_\phi := 2000\text{pF} \quad C_n := 3 \cdot C_\phi \quad C_n = 6000\cdot \text{pF}
\end{align*}
\]
Assume that phase A opens first (we will use this in the simulations later on)

- Treat the voltage sources as short circuits at the resonant frequency.
- The circuit simplifies to:

\[
L_{\text{para}} := \frac{L_{\phi}}{2} \quad \text{L}_{\text{para}} = 462.19 \cdot \text{mH}
\]

The resonant frequencies can be found by solving for the root of:

\[
0 = s^{4} + \left(\frac{1}{L_{\text{para}} \cdot C_{n}} + \frac{1}{L_{\phi} \cdot C_{\phi}} + \frac{1}{L_{\phi} \cdot C_{n}}\right) \cdot s^{2} + \frac{1}{L_{\text{para}} \cdot C_{n} \cdot L_{\phi} \cdot C_{\phi}}
\]

Define constants:

\[
C_{1} := \frac{1}{L_{\text{para}} \cdot C_{n}} + \frac{1}{L_{\phi} \cdot C_{\phi}} + \frac{1}{L_{\phi} \cdot C_{n}}
\]

\[
C_{2} := \frac{1}{L_{\text{para}} \cdot C_{n} \cdot L_{\phi} \cdot C_{\phi}}
\]

So:

\[
0 = s^{4} + C_{1} \cdot s^{2} + C_{2}
\]
Two approaches:

Approach 1:

\[ s_{\omega 1SQ} := \frac{-C_1 + \sqrt{C_1^2 - 4C_2}}{2} \]
\[ s_{\omega 2SQ} := \frac{-C_1 - \sqrt{C_1^2 - 4C_2}}{2} \]

Recall that: \( \sqrt{s} = j\omega \)

\[ \omega_1 := \text{Im}(\sqrt{s_{\omega 1SQ}}) \]
\[ \omega_2 := \text{Im}(\sqrt{s_{\omega 2SQ}}) \]

\[ \omega_1 = 1.51 \times 10^4 \text{ rad/s} \]
\[ \omega_2 = 2.92 \times 10^4 \text{ rad/s} \]

\[ f_1 := \frac{\omega_1}{2\pi} \quad f_1 = 2406.42 \text{ Hz} \]
\[ f_2 := \frac{\omega_2}{2\pi} \quad f_2 = 4648.85 \text{ Hz} \]

Approach 2:

Initial Guesses:

\[ \omega_{1b} := 2000 \text{ rad/s} \]
\[ \omega_{2b} := 5000 \text{ rad/s} \]

Given

\[ \omega_{1b}^2 + \omega_{2b}^2 = C_1 \]
\[ \omega_{1b}^2 \cdot \omega_{2b}^2 = C_2 \]

\[ \omega_{sol} := \text{Minerr}(\omega_{1b}, \omega_{2b}) \quad \text{Using Minerr function due to convergence problems} \]

\[ f_{1b} := \frac{\omega_{sol0}}{2\pi} \quad f_{1b} = 2406.42 \text{ Hz} \]

Same as above.....

\[ f_{2b} := \frac{\omega_{sol1}}{2\pi} \quad f_{2b} = 4648.85 \text{ Hz} \]
- Note that these two frequencies are close enough together that the approximation we used in problem 3 of homework 2 will not be accurate in this case:

\[
f_{1\text{approx}} := \frac{1}{2\cdot\pi\sqrt{L_{\text{para}}\left(C_{\phi} + C_n\right)}} \quad f_{1\text{approx}} = 2617.38 \text{ Hz} \quad \text{too high}
\]

\[
f_{2\text{approx}} := \frac{1}{2\cdot\pi\sqrt{L_{\phi} \left(\frac{1}{C_{\phi}} + \frac{1}{C_n}\right)^{-1}}} \quad f_{2\text{approx}} = 4274.16 \text{ Hz} \quad \text{too low}
\]

Both phases B and C will clear simultaneously, so the resulting circuit will be:

There are basically two resonant circuit path options (chapter 6 in the Greenwood book implies there is only one).

- One that passes through only the phase inductances and capacitances:

\[
f_{1c} := \frac{1}{2\cdot\pi\sqrt{L_{\phi}C_{\phi}}} \quad f_{1c} = 3701.53 \text{ Hz}
\]

Note that the \(L_{\phi}\) and \(C_{\phi}\) terms should be scaled appropriately for series or parallel combinations. The resonant path at a minimum will pass through two of these LC elements in series (\(2L_{\phi} \cdot C_{\phi}/2\)) or one LC element in series with 2 in parallel).
- The other resonant path is completed through the neutral capacitor in series with the parallel combination of the three phase elements.

\[ f_{2c} := \frac{1}{2 \cdot \pi \cdot \sqrt{\frac{L_{\phi}}{3} \left( \frac{C_n \cdot 3C_{\phi}}{C_n + 3C_{\phi}} \right)}} \]

\[ f_{2c} = 5234.76 \text{ Hz} \]

Note that this corrects for the three terms in parallel:

\[ \frac{L_{\phi}}{3} = 308.12 \text{ mH} \quad 3C_{\phi} = 6 \text{ nF} \]

b) Estimate the worst case voltage across the circuit breaker contacts due to the breaker actions.

A reasonable to estimate would be as follows, starting from phasor diagrams:

Before first phase clears

- \( V_{n'} = V_{g} \)
- \( V_{b'} = V_{b'}g \)
- \( V_{c'} = V_{c'}g \)

Phase A clears first

- \( V_{a'} = V_{a'}g = 0.5V_{\phi} \)
- \( V_{b'} = V_{b'}g \)
- \( V_{c'} = V_{c'}g \)

If the neutral shift alone was only concern then:

\[ V_{a'a} = 2V_{ag} + V_{n'g} = 2V_{m} + 0.5 \cdot V_{m} = 2.5 \cdot V_{m} \]

- Where the \( 2V_{ag} \) is the normal worst case difference in solidly grounded system
- However, the since there are capacitances involved, there will be an overshoot equal to the amount of the shift.

\[ V_{a'a} = 2V_{ag} + 2V_{n'g} = 2V_{m} + 2 \left( 0.5 \cdot V_{m} \right) = 3 \cdot V_{m} \]

Therefore first phase to clear will have the largest voltage (although in reality this may not occur before the other phases clear) of approximately:

\[ V_{AA'} := V_{m} + 2 \cdot V_{m} \quad V_{AA'} = 323.33 \text{ kV} \quad \text{Sufficient for this assignment} \]
If we instead wanted to calculate the exact voltage (not required for this homework):

Using Equation 3.4.14 from the textbook by Greenwood:

\[ A_1 := \frac{1}{L_{\text{para}}C_nL_\phi C_\phi} \]

\[ A_1 = 1.95 \times 10^{17} \frac{1}{\text{s}^4} \]

\[ B_1 := \frac{1}{A_1 C_\phi (L_{\text{para}} + L_\phi)} \]

\[ B_1 = 1.85 \times 10^{-9} \frac{2}{\text{s}^2} \]

As a check:

\[ \frac{A_1}{\omega_1^2 \cdot \omega_2^2} = 1 \]

Define additional constants so results fit on a page better:

\[ K_{\omega_1} := \frac{1 - \omega_1^2 \cdot B_1}{\omega_1^2 \left( \omega_1^2 - \omega_2^2 \right)} \]

\[ A_1 \cdot V_m \cdot K_{\omega_1} = -85 \cdot \text{kV} \]

\[ K_{\omega_2} := \frac{1 - \omega_2^2 \cdot B_1}{\omega_2^2 \left( \omega_1^2 - \omega_2^2 \right)} \]

\[ A_1 \cdot V_m \cdot K_{\omega_2} = 22.78 \cdot \text{kV} \]

We can estimate the peak magnitude of the switch voltage with:

\[ V_m + \frac{A_1 \cdot V_m}{\omega_1^2 \cdot \omega_2^2} + \left| A_1 \cdot V_m \cdot K_{\omega_1} \right| + \left| A_1 \cdot V_m \cdot K_{\omega_2} \right| = 323.33 \cdot \text{kV} \]

Which matches our estimate

Now as a further check, lets try plotting the waveforms using equation 3.4.14 against simulation results

Define a time period of 3 cycles \( t := 0 \text{sec}, 1 \cdot 10^{-6} \text{sec}.. 50 \cdot 10^{-3} \text{sec} \)

- Add a slight modification to equation 3.4.14, and include the 60Hz variation in the first term

\[ V_{A_g}(t) := -A_1 \cdot V_m \cdot \frac{\cos(\omega_1 \cdot t)}{\omega_1^2 \cdot \omega_2^2} + K_{\omega_1} \cdot \cos(\omega_1 \cdot t) - K_{\omega_2} \cdot \cos(\omega_2 \cdot t) \]

\[ V_{Ag}(t) := V_m \cdot \cos(\omega \cdot t) \]

\[ V_{\text{switch}}(t) := V_{Ag}(t) - V_{A_g}(t) \]
Plot the first cycle of the source voltage and the voltage on the load side of the breaker.

Voltage across the switch, again one 60Hz cycle.

Now load the plot data from ATP (with only phase A opened and the other two phases staying closed)

\[
\text{data := } \ldots\backslash\text{prob2data.txt}
\]

Time vector: \( tt := \text{data}^{(0)} \)  
Switch voltage: \( V_{sa_a} := \text{data}^{(1)} \)

Load voltage: \( V_{a1} := \text{data}^{(2)} \)  
Source voltage \( V_{sa} := \text{data}^{(3)} \)
The source voltages match, as one would expect

\[ V_{sa} = V_{Ag}(t) \]

However, the line to ground voltage on the load side of the breaker does not match well at all.

\[ V_{al} = V_{Ag'}(t) \]

As a result, neither does the voltage across the breaker contacts

\[ V_{sa_a} = V_{switch}(t) \]
Now, think about what might be wrong in the equation:

- First, the 60Hz variation should have an amplitude of 0.5, since the $V_{a'}$ should be at $0.5*(V_b+V_c)$ if the ringing is ignored and only the 60Hz shift is included.
- The voltage driving the two terms at the resonant frequencies should be $1.5*V_m$ not $V_m$ if the shift in the neutral (and $V_{a'}$ is included)

As a check:

$$V_m + A_1 \cdot V_m\left(\frac{0.5}{\omega_1^2 \cdot \omega_2^2} + 1.5 \left| K_{\omega_1} \right| + 1.5 \left| K_{\omega_2} \right| \right) = 323.33 \text{ kV}$$

Correct total voltage

Then

$$V_A'_{g\_fix}(t) := -A_1 \cdot V_m\left(\frac{0.5 \cos(\omega_1 t)}{\omega_1^2 \cdot \omega_2^2} + 1.5 \cdot K_{\omega_1} \cdot \cos(\omega_1 t) - 1.5 \cdot K_{\omega_2} \cdot \cos(\omega_2 t)\right)$$

$$V_{\text{switch\_fix}}(t) := V_{A\_g}(t) - V_{A'_{g\_fix}}(t)$$

Now repeat the comparison plots over 3 60Hz cycles (again with phase B and C breakers staying closed):

Now we have an exact match after the simulation started.
Again, we have an exact match. Notice that largest differences occur a cycle after the first phase clears, so we should see a smaller peak if the trip signal is sent to all three phases together.

- So these changes give an exact match.
- This also shows that the frequencies in the simulated waveforms match the calculated values
- Note that this not a full derivation of the correct equation

c) Simulate this circuit using your emtp-type program.

**ATPDraw Implementation:**

```
BEGIN NEW DATA CASE
C ---------------------------------------------------------------------------
C Generated by ATPDRAW  March, Saturday 6, 2010
C A Bonneville Power Administration program
C by H. K. Heidalen at SEFAS/NTNU - NORWAY 1994-2008
C ---------------------------------------------------------------------------
C  dT  >< Tmax >< Xopt >< Copt >
   1.E-6     .03
C        1         2         3         4         5         6         7         8
C 345678901234567890123456789012345678901234567890123456789012345678901234567890
/BRANCH
C < n1 >< n2 ><<ref1><ref2>< R >> L >> C >
C < n1 >< n2 ><<ref1><ref2>< R >> A >> B >><Leng><><>0
   V1A     .002     3
   V1B     .002     3
   V1C     .002     3
   V1A V1N 924.37     0
   V1B V1N 924.37     0
   V1C V1N 924.37     0
   V1N    .006     2
```
Phase currents: notice that all three phases now clear, with phase A first then phases B and C together.

Switch voltages: notice that the voltage across the first switch to clear isn’t as large as the maximum value but it is still the largest. As we saw above, that would be true if we ran the case longer before opening the other phases.
Zoomed Switch Voltage on First Phase to Clear:

\[ V_{pk} = 278.43 \text{kV} \]

\[ \frac{V_{pk}}{V_m} = 2.58 \text{pu} \]

If we ran this longer without tripping phases B and C, we would eventually see 323.33kV.

- In order to determine the resonant frequencies after the first switch opens from simulation, an open command was given to one breaker, and the others were left closed. The simulation was run for just over 1 sec.
The calculated value was:

\[ f_{1b} = 2406.42 \text{ Hz} \]

Again, good match

The calculated value was:

\[ f_{2b} = 4648.85 \text{ Hz} \]

Again, good match
Neutral to ground voltage. Note the steady-state shift after the last phases clear:

Zoomed in voltage near the second switch transition.
- Two resonant frequencies plus 60Hz prior to phases B and C clearing together
- One frequency after B and C clear (clearly higher frequency).

Calculate the frequency of the neutral to ground voltage after phases B and C clear by finding time between voltage peaks (we can do this if there is one frequency present):

\[
t_1 := 0.013178 \text{sec} \\
t_2 := 0.013369 \text{sec}
\]

\[
\frac{1}{t_2 - t_1} = 5235.6 \frac{1}{s}
\]

earlier we calculated: \( f_{2c} = 5234.76 \cdot \text{Hz} \)

This is a good match, considering the approach we used.
Note that there are clearly two frequencies present, although the textbook predicted one.

However, if we run the simulation with phase never closed and then open B and C we get waveforms that look like this single frequency.

There also a small amplitude voltage on the neutral voltage, at the frequency calculated above, although that plot isn’t shown here. That means that the switch voltage will see a very small oscillation at that frequency that won’t be visible just by eyeballing the curve.

The resulting frequency is:

\[
T_{1a} := 0.010652 \\
T_{2a} := 0.010921 \\
\frac{1}{T_{2a} - T_{1a}} = 3717.47
\]

Which matches pretty closely with:

\[
f_{1c} = 3701.53\text{Hz}
\]

Alternatively: if we run a discrete Fourier transform calculation (with a base frequency of 1 Hz, so harmonics are integer multiples of 1 Hz and simulation runs with over 1 second of data for the desired conditions) on the original waveforms the ones with phase A never closed, we find the following frequencies present.

Note that this is a slow calculation, so just have it calculate terms for a smaller desired frequency range (the default is 0-30th harmonics. You can only increase harmonics in subsequent calculations.)
Since the resonance frequency isn't exactly an integer multiple of 1Hz, there is energy in the side bands too. But the central peak shows a resonant frequency close to 3701Hz, which matches analytical values.

Since the resonance frequency isn't exactly an integer multiple of 1Hz, there is energy in the side bands too. But the central peak shows a resonant frequency close to 5234Hz, which matches analytical values.
Case without phase A energized at all:

Again, we have the expected center frequency. Note the change in amplitude since the initial voltage conditions have changed.

So there actually is a peak at 5234Hz. The amplitude is much smaller since the amplitude of the neutral voltage is small in this case. So we really seeing two frequencies.
**PSCAD/EMTDC solution:**

Switches set to have Rclosed=0

- **Line currents**

![Graph showing line currents](image)

Phase A clears first, then B and C clear together.
- Switch voltages

Notice that the voltage across the first switch to clear isn't as large as the maximum value but it is still the largest. As we saw above, that would be true if we ran the case longer before opening the other phases.

Zoom in on the first phase to clear. Notice that there are two frequencies.
If we ran this longer without tripping phases B and C, we would eventually see 323.33kV.

\[ V_{pk} := 278.39\text{kV} \quad \frac{V_{pk}}{V_m} = 2.58 \]

Neutral to ground voltage. Note the steady-state shift after the last phases clear:
- Two resonant frequencies plus 60Hz prior to phases B and C clearing together
- One frequency after B and C clear (clearly higher frequency).

Calculate the frequency of the neutral to ground voltage after phases B and C clear by finding time between voltage peaks (we can do this if there is one frequency present):

\[ t_1 := 0.08401\text{sec} \]
\[ t_2 := 0.08420\text{sec} \]
\[ \frac{1}{t_2 - t_1} = 5235.6 \frac{1}{s} \]

earlier we calculated: \[ f_2c = 5234.76 \frac{1}{s} \]

This is a good match, considering the approach we used.

Zoomed phase B and C switch voltages on subsequent phases. Note that there are clearly two frequencies present, although the text book predicted one.
However, if we run the simulation with phase never closed and then open B and C we get waveforms that looks like this single frequency.

There also a small amplitude voltage on the neutral voltage, at the frequency calculated above, although that plot isn't shown here. That means that the switch voltage will see a very small oscillation at that frequency that won't be visible just by eyeballing the curve.

The resulting frequency is:  
\[ T_{1a} := 0.083468 \]
\[ T_{2a} := 0.083738 \]
\[ \frac{1}{T_{2a} - T_{1a}} = 3703.7 \]

Which matches pretty closely with:
\[ f_{1c} = 3701.53 \frac{1}{s} \]

As final check, plot the phase A switch voltage from PSCAD/EMTDC versus the analytical one above, for the case where the other two switches do not open.

Now load the plot data from PSCAD/EMTDC (with only phase A opened and the other two phases staying closed)

PSCADout :=
\[ ..\plotdata_01.ou \]
Time vector: \[ \text{tt} := \text{PSCADout}(0) \]

Phase A source voltage: \[ V_{sa} := \text{PSCADout}(1) \]

Phase A load voltage: \[ V_{a1} := \text{PSCADout}(4) \]

Phase A switch voltage: \[ V_{sa_a} := \text{PSCADout}(7) \]

Column information retrieved from *.inf file.

Now we will need to offset time and scale by 1000 to align data. Look at the PSCAD/EMTDC plots and see where the time instant where the switch voltage starts to increase.

\[ t_{\text{offset}} := \text{tt} - 0.079172 \]

The source voltages match, as one would expect.

\[ 1000 \cdot V_{sa} \]
\[ V_{Ag}(t) \]
Now we have an exact match after the simulation started.

Again, we have an exact match. Notice that largest differences occur a cycle after the first phase clears, so we should see a smaller peak if the trip signal is sent to all three phases together.