ECE 524

TRANSIENTS IN
POWER SYSTEMS

SESSION no. 23
Three phase TRV transients

\[ MVAR := 1000 \text{kW} \quad m_{\text{m}} := 10^{-6} \text{sec} \]

Consider the three phase system shown below:

**ATP Implementation**

\[ V_{LL} := 34.5 \text{kV} \quad V_m := \sqrt{\frac{2}{3}} \cdot V_{LL} \quad V_m = 28.17 \text{kV} \quad Z_s := j \cdot 10^{-5} \text{ohm} \]

\[ Q_L := 12 \text{MVAR} \quad X_L := \frac{V_{LL}^2}{Q_L} \quad X_L = 99.19 \Omega \]

\[ L_{ph} := \frac{X_L}{2 \cdot \pi \cdot 60 \text{Hz}} \quad L_{ph} = 263.1 \text{mH} \]

\[ C_{ph} := 2 \text{nF} \]

\[ f_n := \left( \frac{1}{2 \cdot \pi} \right) \cdot \frac{1}{\sqrt{L_{ph} \cdot C_{ph}}} \quad f_n = 6938.13 \frac{1}{\text{Hz}} \]

\[ T_n := \frac{1}{f_n} \quad T_n = 144.13 \cdot \mu\text{s} \]

Choose simulation time step:

\[ \frac{T_n}{30} = 4.8 \cdot \mu\text{s} \]
Case 1:  $Z_n := \infty \text{ ohm}$

Breaker currents

Neutral to ground voltage:

Phase voltages
Clearing Three Phase Reactor

A 132kV, 50MVAR three phase reactor is connected in a Y ungrounded configuration. Each phase can be represented using a pi equivalent, with capacitances of 2000pF and an inductance suitable for the MVAR rating.

\[
\text{Breaker} \\
\begin{array}{c}
V_{ag} \\
V_{cg} \\
V_{bg}
\end{array}
\begin{array}{c}
L_{\phi} \\
C_{\phi}
\end{array}
\begin{array}{c}
A' \\
L_{\phi} \\
C_{n}
\end{array}
\begin{array}{c}
B' \\
L_{\phi}
\end{array}
\begin{array}{c}
C_{\phi}
\end{array}
\begin{array}{c}
C_{\phi}
\end{array}
\]

a) Compute the natural frequencies of the reactor when the first phase clears and then when the subsequent phases clear.

\[
\begin{align*}
\text{MVA} & := 1000\text{kW} \\
\text{MVAR} & := \text{MVA} \\
\mu & := \frac{s}{10^6} \\
\mu_{\text{s}} & := \frac{s}{10^3} \\
\mu_{1} & := 1 \\
V_{LL} & := 132\text{kV} \\
V_{m} & := \sqrt{\frac{2}{3}} \cdot V_{LL} \\
V_{m} & = 107.78\text{-kV} \\
Q_{L} & := 50\text{MVAR} \\
\omega & := 2 \cdot \pi \cdot 60\text{Hz} \\
X_{L} & := \frac{V_{LL}^2}{Q_{L}} \\
X_{L} & = 348.48\Omega \\
L_{\phi} & := \frac{X_{L}}{\omega} \\
L_{\phi} & = 924.37\text{-mH} \\
C_{\phi} & := 2000\text{pF} \\
C_{n} & := 3 \cdot C_{\phi} \\
C_{n} & = 6000\text{-pF}
\end{align*}
\]
Assume that phase A opens first (we will use this in the simulations later on)

- Treat the voltage sources as short circuits at the resonant frequency.
- The circuit simplifies to:

![Circuit Diagram]

- Since the phase B and phase C to ground capacitances are shorted to ground, leaving two inductors in parallel. Then the circuit can further be reduced to the following.

\[ L_{\text{para}} := \frac{L_\phi}{2} \quad L_{\text{para}} = 462.19\,\text{mH} \]

![Circuit Diagram]

The resonant frequencies can be found by solving for the root of:

\[ 0 = s_\omega^4 + \left( \frac{1}{L_{\text{para}} \cdot C_n} + \frac{1}{L_\phi \cdot C_\phi} + \frac{1}{L_\phi \cdot C_n} \right) s_\omega^2 + \frac{1}{L_{\text{para}} \cdot C_n \cdot L_\phi \cdot C_\phi} \]

Define constants:

\[ C_1 := \frac{1}{L_{\text{para}} \cdot C_n} + \frac{1}{L_\phi \cdot C_\phi} + \frac{1}{L_\phi \cdot C_n} \]

\[ C_2 := \frac{1}{L_{\text{para}} \cdot C_n \cdot L_\phi \cdot C_\phi} \]

So:

\[ 0 = s_\omega^4 + C_1 \cdot s_\omega^2 + C_2 \]
Phase voltage after first switch clears:

Phase voltage after remaining switches clear:
Breaker voltages:

\[ V_{\text{maxA}} = 85.586\text{kV} \quad 3V_m \]

RRRV \rightarrow \frac{3V_m}{\sqrt{2}} \quad -T_0 \text{ is natural } f-

Case 2: Repeat with \( Z_n = 0\text{ohm} \)

Breaker currents:
Neutral current

Phase voltages:

\[ V_{\text{max}} := 57.295\text{kV} \]

\[ 2 \cdot V_m \]
Two approaches:

**Approach 1:**

\[
s_{ω1SQ} := \frac{-C1 + \sqrt{C1^2 - 4C2}}{2}
\]
\[
s_{ω2SQ} := \frac{-C1 - \sqrt{C1^2 - 4C2}}{2}
\]

Recall that: \( \sqrt{s} = jω \)

\[
ω_1 := \text{Im}(\sqrt{s_{ω1SQ}})
\]
\[
ω_1 = 1.51 \times 10^4 \text{ rad/s}
\]

\[
f_1 := \frac{ω_1}{2\cdotπ} \quad f_1 = 2406.42 \text{ Hz}
\]

\[
ω_2 := \text{Im}(\sqrt{s_{ω2SQ}})
\]
\[
ω_2 = 2.92 \times 10^4 \text{ rad/s}
\]

\[
f_2 := \frac{ω_2}{2\cdotπ} \quad f_2 = 4648.85 \text{ Hz}
\]

**Approach 2:**

Initial Guesses:

\[
ω_{1b} := 2000 \frac{\text{rad}}{s}
\]
\[
ω_{2b} := 5000 \frac{\text{rad}}{s}
\]

Given

\[
ω_{1b}^2 + ω_{2b}^2 = C1
\]
\[
ω_{1b}^2 \cdot ω_{2b}^2 = C2
\]

\[ω_{sol} := \text{Minerr}(ω_{1b}, ω_{2b})\] Using Minerr function due to convergence problems

\[
f_{1b} := \frac{ω_{sol0}}{2\cdotπ} \quad f_{1b} = 2406.42 \text{ Hz}
\]

\[
f_{2b} := \frac{ω_{sol1}}{2\cdotπ} \quad f_{2b} = 4648.85 \text{ Hz}
\]

*Note: This approach is marked with a red note indicating that it should be reviewed.*
Case 3: Repeat with $C_n := 3 \cdot C_{ph}$

Breaker currents:

Neutral current after first phase clears:

Neutral current when all phases clear:
Neutral voltage:

Phase voltages:
Breaker voltages:

\[ V_{\text{max}} := 78.846 \text{kV} \]

\[ \frac{V_{\text{max}}}{V_{\text{m}}} = 2.8 \]

Case 4: high resistance ground: \(3 \cdot R_{\text{gr}} = X_{c0}\)

\[ X_{\text{cph}} := \frac{1}{2 \cdot \pi \cdot 60 \text{Hz} \cdot C_{\text{ph}}} \]

\[ X_{\text{cph}} = 1.33 \times 10^6 \Omega \]

\[ R_{\text{gr}} := \frac{X_{\text{cph}}}{3} \]

\[ R_{\text{gr}} = 442.1 \text{k}\Omega \]

\[ V_{\text{max}} := 84.462 \text{kV} \]

\[ \frac{V_{\text{max}}}{V_{\text{m}}} = 3 \]
Case 5: Add grounding resistance (high resistance ground—limit In to 20A):

\[ I_{n_{\text{max}}} := 20\text{A} \]

\[ R_{\text{gr}} := \frac{V_{LL}}{\sqrt{3}I_{n_{\text{max}}}} \]

\[ R_{\text{gr}} = 995.93\Omega \quad \text{use 1000 ohm} \]

Breaker voltages:

\[ V_{\text{max}} := 60.018\text{kV} \]

\[ \frac{V_{\text{max}}}{V_m} = 2.13 \]

Case 6: Now try a low resistance ground (In = 500A)

\[ I_{n_{\text{max}}} := 500\text{A} \]

\[ R_{\text{gr}} := \frac{V_{LL}}{\sqrt{3}I_{n_{\text{max}}}} \]

\[ R_{\text{gr}} = 39.84\Omega \quad \text{use 40ohm} \]

\[ V_{\text{max}} := 56.771\text{kV} \]

\[ \frac{V_{\text{max}}}{V_m} = 2.02 \]
Case 7: Now try a low inductance ground (In = 500A)

\[ I_{\text{max}} := 500A \]
\[ \text{mag}Z_{\text{gr}} := \frac{V_{\text{LL}}}{\sqrt{3} \cdot I_{\text{max}}} \quad \text{mag}Z_{\text{gr}} = 39.84 \Omega \]
\[ L_{\text{gr}} := \frac{\text{mag}Z_{\text{gr}}}{2 \cdot \pi \cdot 60\text{Hz}} \quad L_{\text{gr}} = 105.67\text{mH} \]

\[ V_{\text{max}} := 73.874\text{kV} \]
\[ \frac{V_{\text{max}}}{V_{\text{m}}} = 2.62 \]

- On first phase to clear
- A little higher after subsequent phases clear it damping neglected
• Note that these two frequencies are close enough together that the approximation we used in problem 3 of homework 2 will not be accurate in this case:

\[ f_{1\text{approx}} := \frac{1}{2\cdot\pi\cdot\sqrt{L_{\text{para}}(C_\phi + C_n)}} \]

\[ f_{2\text{approx}} := \frac{1}{2\cdot\pi\cdot\sqrt{L_{\phi}\left(\frac{1}{C_\phi} + \frac{1}{C_n}\right)^{-1}}} \]

\[ f_{1\text{approx}} = 2617.38\text{-Hz too high} \]

\[ f_{2\text{approx}} = 4274.16\text{-Hz too low} \]

Both phases B and C will clear simultaneously, so the resulting circuit will be:

![Circuit Diagram]

This can be simplified to:

![Simplified Circuit Diagram]

There are basically two resonant circuit path options (chapter 6 in the Greenwood book implies there is only one).

• One that passes through only the phase inductances and capacitances:

\[ f_{1c} := \frac{1}{2\cdot\pi\cdot\sqrt{L_\phi\cdot C_\phi}} \]

\[ f_{1c} = 3701.53\text{-Hz} \]

Note that the \( L_\phi \) and \( C_\phi \) terms should be scaled appropriately for series or parallel combinations. The resonant path at a minimum will pass through two of these LC elements in series (\( 2L_\phi \cdot C_\phi/2 \)) or one LC element in series with 2 in parallel.)
- The other resonant path is completed through the neutral capacitor in series with the parallel combination of the three phase elements.

\[ f_{2c} := \frac{1}{2 \cdot \pi \cdot \sqrt[3]{\frac{L_{\phi}}{3 \cdot \left( \frac{C_n \cdot 3C_{\phi}}{C_n + 3 \cdot C_{\phi}} \right)}} \]

\[ f_{2c} = 5234.76 \text{ Hz} \]

Note that this corrects for the three terms in parallel:

\[ \frac{L_{\phi}}{3} = 308.12 \text{ mH} \quad 3C_{\phi} = 6 \text{ nF} \]

b) Estimate the worst case voltage across the circuit breaker contacts due to the breaker actions.

A reasonable to estimate would be as follows, starting from phasor diagrams:

Before first phase clears

\[ V_{a'n'} = V_{a'g} \]

\[ V_{n'} = V_{g} \]

\[ V_{c'n'} = V_{c'g} \]

Phase A clears first

\[ V_{ag} \]

\[ V_{c'g} \]

\[ V_{n'g} = V_{a'g} = 0.5V_{\phi} \]

Oscillatory overshoot of \( V_{n'g} \)

If the neutral shift alone was only concern then:

\[ V_{a'a} = 2V_{ag} + V_{n'g} = 2V_m + 0.5 \cdot V_m = 2.5 \cdot V_m \]

- Where the \( 2V_{ag} \) is the normal worst case difference in solidly grounded system
- However, the since there are capacitances involved, there will be an overshoot equal to the amount of the shift.

\[ V_{a'a} = 2V_{ag} + 2V_{n'g} = 2V_m + 2 \left( 0.5 \cdot V_m \right) = 3 \cdot V_m \]

Therefore first phase to clear will have the largest voltage (although in reality this may not occur before the other phases clear) of approximately:

\[ V_{AA'} := V_m + 2 \cdot V_m \]

\[ V_{AA'} = 323.33 \text{ kV} \] Sufficient for this assignment
If we instead wanted to calculate the exact voltage (not required for this homework):

Using Equation 3.4.14 from the textbook by Greenwood:

\[ A_1 := \frac{1}{L_{\text{para}} C_n \cdot L_\phi C_\phi} \quad A_1 = 1.95 \times 10^7 \frac{1}{\text{s}^4} \]

\[ B_1 := \frac{1}{A_1 \cdot C_\phi (L_{\text{para}} + L_\phi)} \quad B_1 = 1.85 \times 10^{-9} \text{s}^2 \]

As a check: \[ \frac{A_1}{\omega_1^2 \cdot \omega_2^2} = 1 \]

Define additional constants so results fit on a page better:

\[ K_{\omega 1} := \frac{1 - \omega_1^2 \cdot B_1}{\omega_1^2 \cdot (\omega_1^2 - \omega_2^2)} \quad A_1 \cdot V_m \cdot K_{\omega 1} = -85 \cdot \text{kV} \]

\[ K_{\omega 2} := \frac{1 - \omega_2^2 \cdot B_1}{\omega_2^2 \cdot (\omega_1^2 - \omega_2^2)} \quad A_1 \cdot V_m \cdot K_{\omega 2} = 22.78 \cdot \text{kV} \]

We can estimate the peak magnitude of the switch voltage with:

\[ V_m + \frac{A_1 \cdot V_m}{\omega_1^2 \cdot \omega_2^2} + |A_1 \cdot V_m \cdot K_{\omega 1}| + |A_1 \cdot V_m \cdot K_{\omega 2}| = 323.33 \cdot \text{kV} \]

Which matches our estimate.

Now as a further check, lets try plotting the waveforms using equation 3.4.14 against simulation results:

Define a time period of 3 cycles \( t := 0 \text{sec}, 1 \cdot 10^{-6} \text{sec} \ldots 50 \cdot 10^{-3} \text{sec} \)

- Add a slight modification to equation 3.4.14, and include the 60Hz variation in the first term.

\[ V_{Ag}(t) := -A_1 \cdot V_m \left( \frac{\cos(\omega_1 \cdot t)}{\omega_1^2 \cdot \omega_2^2} + K_{\omega 1} \cdot \cos(\omega_1 \cdot t) - K_{\omega 2} \cdot \cos(\omega_2 \cdot t) \right) \]

\[ V_{Ag}(t) := V_m \cdot \cos(\omega_1 \cdot t) \]
Now, think about what might be wrong in the equation:

- First, the 60Hz variation should have an amplitude of 0.5, since the $V_a$ should be at $0.5(V_b + V_c)$ if the ringing is ignored and only the 60Hz shift is included.
- The voltage driving the two terms at the resonant frequencies should be $1.5V_m$ not $V_m$ if the shift in the neutral (and $V_a$ is included).

As a check:

$$V_m + A_1 \cdot V_m \left( \frac{0.5}{\omega_1^2 \omega_2^2} + 1.5 \left| K_{\omega 1} \right| + 1.5 \left| K_{\omega 2} \right| \right) = 323.33\text{-kV}$$  Correct total voltage

Then

$$V_{A'_{g \text{ fix}}} (t) := -A_1 \cdot V_m \left( \frac{0.5 \cos(\omega_1 \cdot t)}{\omega_1^2 \omega_2^2} + 1.5 \cdot K_{\omega 1} \cdot \cos(\omega_1 \cdot t) - 1.5 \cdot K_{\omega 2} \cdot \cos(\omega_2 \cdot t) \right)$$

$$V_{\text{switch fix}}(t) := V_{Ag}(t) - V_{A'_{g \text{ fix}}}(t)$$

Now repeat the comparison plots over 3 60Hz cycles (again with phase B and C breakers staying closed):

Now we have an exact match after the simulation started.
Zoomed phase B and C switch voltages on subsequent phases:

Note that there are clearly two frequencies present, although the textbook predicted one.

However, if we run the simulation with phase never closed and then open B and C we get waveforms that looks like this single frequency.

There also a small amplitude voltage on the neutral voltage, at the frequency calculated above, although that plot isn't shown here. That means that the switch voltage will see a very small oscillation at that frequency that won't be visible just by eyeballing the curve.

The resulting frequency is:

$$T_{1a} := 0.010652$$
$$T_{2a} := 0.010921$$

$$\frac{1}{T_{2a} - T_{1a}} = 3717.47$$

Which matches pretty closely with:

$$f_{1c} = 3701.53\text{-Hz}$$

Alternatively: if we run a discrete Fourier transform calculation (with a base frequency of 1 Hz, so harmonics are integer multiples of 1 Hz and simulation runs with over 1 second of data for the desired conditions) on the original waveforms the ones with phase A never closed, we find the following frequencies present.

Note that this is a slow calculation, so just have it calculate terms for a smaller desired frequency range (the default is 0-30th harmonics. You can only increase harmonics in subsequent calculations.