Calculating Modal Parameters from Unbalanced Line Constants

Line Constants:

345kV line, 75 miles, 2 conductors per bundle

$\text{freq} := 60\text{Hz}$

Inductance

$\mu o := 4 \cdot \pi \cdot 10^{-7} \frac{H}{m}$ $\rho := 100\text{ohm} \cdot m$

Conductor GMR from table:

$D_s := 0.04476\text{ft}$ at 60Hz $\text{Dbundle} := 18\text{in}$

$GMR_L := \left(D_s \cdot \text{Dbundle}\right)^{\frac{1}{2}}$ $GMR_L = 0.26\,\text{ft}$

$\text{Decon} := 2160 \cdot \frac{\text{ft} \cdot \text{Hz}^{0.5}}{(\text{ohm} \cdot m)^{0.5}}$

$\text{De} := \text{Decon} \cdot \sqrt{\frac{\rho}{\text{freq}}}$ $\text{De} = 2788.55\,\text{ft}$

$\text{Lcon} := \frac{\mu o}{2 \cdot \pi}$

Spacing: $D_{AB} := 35\text{ft}$ $D_{BC} := 35\text{ft}$ $D_{CA} := 70\text{ft}$

$$L' := Lcon \cdot \begin{pmatrix} \ln\left(\frac{De}{GMR_L}\right) & \ln\left(\frac{De}{D_{AB}}\right) & \ln\left(\frac{De}{D_{CA}}\right) \\ \ln\left(\frac{De}{D_{AB}}\right) & \ln\left(\frac{De}{GMR_L}\right) & \ln\left(\frac{De}{D_{BC}}\right) \\ \ln\left(\frac{De}{D_{CA}}\right) & \ln\left(\frac{De}{D_{BC}}\right) & \ln\left(\frac{De}{GMR_L}\right) \end{pmatrix}$$

$$L' = \begin{pmatrix} 2.9882 & 1.4091 & 1.186 \\ 1.4091 & 2.9882 & 1.4091 \\ 1.186 & 1.4091 & 2.9882 \end{pmatrix} \frac{mH}{mi}$$

Note that this line is not balanced (flat spacing).

Capacitance

$diameter := 1.108in$ $\quad radius := \frac{diameter}{2}$ $\quad radius = 0.55\,in$

$GMR_c := (radius \cdot Dbundle)^{\frac{1}{2}}$ $\quad GMR_c = 0.26\,ft$

Calculate capacitance:

$ε\_o := 8.854 \cdot 10^{-12} \frac{F}{m}$

$HeightA := 55ft$ $\quad HeightB := HeightA$ $\quad HeightB = 55\,ft$

$HeightC := 55ft$

$HA\_Aimage := 2 \cdot HeightA$ $\quad HA\_Aimage = 110\,ft$

$HB\_Bimage := 2 \cdot HeightB$ $\quad HB\_Bimage = 110\,ft$

$HC\_Cimage := 2 \cdot HeightA$ $\quad HC\_Cimage = 110\,ft$

$$HA\_Bimage := \sqrt{(HeightA + HeightB)^2 + D_{AB}^{\;2}} \qquad HA\_Bimage = 115.43\,ft$$

$$HB\_Aimage := HA\_Bimage$$

$$HB\_Cimage := HA\_Bimage$$

$$HC\_Bimage := HA\_Bimage$$

$$HA\_Cimage := \sqrt{(HeightA + HeightC)^2 + D_{CA}^{\;2}} \qquad HA\_Cimage = 130.38\,ft$$

$$HC\_Aimage := HA\_Cimage$$

$$P := \frac{1}{2\cdot\pi\cdot\varepsilon\_o}\cdot\begin{pmatrix} \ln\left(\frac{HA\_Aimage}{GMR_c}\right) & \ln\left(\frac{HA\_Bimage}{D_{AB}}\right) & \ln\left(\frac{HA\_Cimage}{D_{CA}}\right) \\ \ln\left(\frac{HB\_Aimage}{D_{AB}}\right) & \ln\left(\frac{HB\_Bimage}{GMR_c}\right) & \ln\left(\frac{HB\_Cimage}{D_{BC}}\right) \\ \ln\left(\frac{HC\_Aimage}{D_{CA}}\right) & \ln\left(\frac{HC\_Bimage}{D_{BC}}\right) & \ln\left(\frac{HC\_Cimage}{GMR_c}\right) \end{pmatrix}$$

$$P = \begin{pmatrix} 67.41 & 13.33 & 6.95 \\ 13.33 & 67.41 & 13.33 \\ 6.95 & 13.33 & 67.41 \end{pmatrix} \frac{mi}{\mu F}$$

$$C' := P^{-1}$$

$$C' = \begin{pmatrix} 0.0155 & -2.8618\times10^{-3} & -1.0321\times10^{-3} \\ -2.8618\times10^{-3} & 0.016 & -2.8618\times10^{-3} \\ -1.0321\times10^{-3} & -2.8618\times10^{-3} & 0.0155 \end{pmatrix} \frac{\mu F}{mi}$$

Imedance and admittance matrices:

$$Z' := j\cdot 2\cdot\pi\cdot 60Hz\cdot L' \qquad Z' = \begin{pmatrix} 1.13i & 0.53i & 0.45i \\ 0.53i & 1.13i & 0.53i \\ 0.45i & 0.53i & 1.13i \end{pmatrix} \frac{ohm}{mi}$$

$$Y' := j\cdot 2\cdot\pi\cdot 60Hz\cdot C' \qquad Y' = \begin{pmatrix} 5.85i\times 10^{-6} & -1.08i\times 10^{-6} & -3.89i\times 10^{-7} \\ -1.08i\times 10^{-6} & 6.02i\times 10^{-6} & -1.08i\times 10^{-6} \\ -3.89i\times 10^{-7} & -1.08i\times 10^{-6} & 5.85i\times 10^{-6} \end{pmatrix} \frac{mho}{mi}$$

Matrix Products for Eigenvector Calculations:

$$ZY := Z'\cdot Y' \qquad ZY = \begin{pmatrix} -5.84\times 10^{-6} & -1.5\times 10^{-6} & -1.6\times 10^{-6} \\ -1.68\times 10^{-6} & -5.63\times 10^{-6} & -1.68\times 10^{-6} \\ -1.6\times 10^{-6} & -1.5\times 10^{-6} & -5.84\times 10^{-6} \end{pmatrix} \frac{1}{mi^2}$$

$$YZ := Y'\cdot Z' \qquad YZ = \begin{pmatrix} -5.84\times 10^{-6} & -1.68\times 10^{-6} & -1.6\times 10^{-6} \\ -1.5\times 10^{-6} & -5.63\times 10^{-6} & -1.5\times 10^{-6} \\ -1.6\times 10^{-6} & -1.68\times 10^{-6} & -5.84\times 10^{-6} \end{pmatrix} \frac{1}{mi^2}$$

$LC := L' \cdot C'$

$$LC = \begin{pmatrix} 4.11\times10^{-11} & 1.06\times10^{-11} & 1.13\times10^{-11} \\ 1.18\times10^{-11} & 3.96\times10^{-11} & 1.18\times10^{-11} \\ 1.13\times10^{-11} & 1.06\times10^{-11} & 4.11\times10^{-11} \end{pmatrix} \frac{s^2}{mi^2}$$

$CL := C' \cdot L'$

$$CL = \begin{pmatrix} 4.11\times10^{-11} & 1.18\times10^{-11} & 1.13\times10^{-11} \\ 1.06\times10^{-11} & 3.96\times10^{-11} & 1.06\times10^{-11} \\ 1.13\times10^{-11} & 1.18\times10^{-11} & 4.11\times10^{-11} \end{pmatrix} \frac{s^2}{mi^2}$$

Matrix of Eigenvectors (Te) from voltage equation:

$T_e := \text{eigenvecs}(ZY)$

$$T_e = \begin{pmatrix} 0.57 & 0.71 & 0.38 \\ 0.58 & 0 & -0.84 \\ 0.57 & -0.71 & 0.38 \end{pmatrix} \frac{1}{m^2} \qquad \text{eigenvals}(ZY) = \begin{pmatrix} -3.46\times10^{-12} \\ -1.64\times10^{-12} \\ -1.59\times10^{-12} \end{pmatrix} \frac{1}{m^2}$$

$Te := \text{eigenvecs}(LC)$

$$Te = \begin{pmatrix} -0.71 & 0.57 & 0.38 \\ 0 & 0.58 & -0.84 \\ 0.71 & 0.57 & 0.38 \end{pmatrix} \frac{s^2}{m^2} \qquad \text{eigenvals}(LC) = \begin{pmatrix} 2.98\times10^{-11} \\ 6.3\times10^{-11} \\ 2.9\times10^{-11} \end{pmatrix} \frac{s^2}{mi^2}$$

Note that LC and ZY have the same matrix of eigenvectors, but different eigenvalues

Matrix of eigenvectors (Ti) for current equation:

$$T_i := \text{eigenvecs}(YZ) \qquad T_i = \begin{pmatrix} 0.6 & 0.71 & 0.41 \\ 0.54 & 0 & -0.81 \\ 0.6 & -0.71 & 0.41 \end{pmatrix} \frac{1}{m^2} \qquad \text{eigenvals}(YZ) = \begin{pmatrix} -3.46 \times 10^{-12} \\ -1.64 \times 10^{-12} \\ -1.59 \times 10^{-12} \end{pmatrix} \frac{1}{m^2}$$

$$Ti := \text{eigenvecs}(CL) \qquad Ti = \begin{pmatrix} -0.71 & 0.6 & 0.41 \\ 0 & 0.54 & -0.81 \\ 0.71 & 0.6 & 0.41 \end{pmatrix} \frac{s^2}{m^2} \qquad \text{eigenvals}(CL) = \begin{pmatrix} 2.98 \times 10^{-11} \\ 6.3 \times 10^{-11} \\ 2.9 \times 10^{-11} \end{pmatrix} \frac{s^2}{mi^2}$$

Note that LC and ZY have a similar matrix of eigenvectors (but the last two columns are swapped), but different eigenvalues

Also note that ZY and YZ have the same eigenvalues. Likewise LC and CL have the same eigenvalues (different order)

$$T_e \cdot T_i^T = \begin{pmatrix} 1 & 0 & -7.17 \times 10^{-4} \\ 0 & 1 & 0 \\ -7.17 \times 10^{-4} & 0 & 1 \end{pmatrix} \frac{1}{m^4}$$

Note quite the identity matrix as expected.

Modal parameters:

$$L_m := T_e^{-1} \cdot L' \cdot T_i \qquad L_m = \begin{pmatrix} 5.65 & 0 & 0 \\ 0 & 1.8 & 0 \\ 0 & 0 & 1.51 \end{pmatrix} \frac{mH}{mi}$$

$$C_m := T_i^{-1} \cdot C' \cdot T_e$$

$$C_m = \begin{pmatrix} 11.16 & 0 & 0 \\ 0 & 16.54 & 0 \\ 0 & 0 & 19.23 \end{pmatrix} \frac{nF}{mi}$$

$$Z_{c0} := \sqrt{\frac{L_{m_{0,0}}}{C_{m_{0,0}}}} \qquad Z_{c0} = 711.17\,\Omega$$

$$Z_{c1} := \sqrt{\frac{L_{m_{1,1}}}{C_{m_{1,1}}}} \qquad Z_{c1} = 330.1\,\Omega$$

Note that these are similar, but not quite the same

$$Z_{c2} := \sqrt{\frac{L_{m_{2,2}}}{C_{m_{2,2}}}} \qquad Z_{c2} = 279.81\,\Omega$$

$$v_0 := \frac{1}{\sqrt{L_{m_{0,0}} \cdot C_{m_{0,0}}}} \qquad v_0 = 1.2595 \times 10^5 \frac{mi}{s}$$

$$v_1 := \frac{1}{\sqrt{L_{m_{1,1}} \cdot C_{m_{1,1}}}} \qquad v_1 = 1.8317 \times 10^5 \frac{mi}{s}$$

Again, similar but not identical

$$v_2 := \frac{1}{\sqrt{L_{m_{2,2}} \cdot C_{m_{2,2}}}} \qquad v_2 = 1.8583 \times 10^5 \frac{mi}{s}$$

Now suppose that instead the line is transposed:

Consider transposition:

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad f1 := \frac{1}{3} \quad f2 := \frac{1}{3} \quad f3 := \frac{1}{3}$$

$$L_{net} := f1 \cdot L' + f2 \cdot Rp^{-1} \cdot L' \cdot Rp + f3 \cdot Rp \cdot L' \cdot Rp^{-1}$$

$$L_{net} = \begin{pmatrix} 2.9882 & 1.3348 & 1.3348 \\ 1.3348 & 2.9882 & 1.3348 \\ 1.3348 & 1.3348 & 2.9882 \end{pmatrix} \frac{mH}{mi}$$

$$C_{net} := f1 \cdot C' + f2 \cdot Rp^{-1} \cdot C' \cdot Rp + f3 \cdot Rp \cdot C' \cdot Rp^{-1}$$

$$C_{net} = \begin{pmatrix} 15.6593 & -2.2519 & -2.2519 \\ -2.2519 & 15.6593 & -2.2519 \\ -2.2519 & -2.2519 & 15.6593 \end{pmatrix} \frac{nF}{mi}$$

Modal Transformations for transposed line:

Clark Tranformation:

$$T_{iTR} := \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 & \sqrt{2} & 0 \\ 1 & \frac{-1}{\sqrt{2}} & \sqrt{\frac{3}{2}} \\ 1 & \frac{-1}{\sqrt{2}} & -\sqrt{\frac{3}{2}} \end{pmatrix}$$

$$L_{modal} := T_{iTR}^{-1} L_{net} \cdot T_{iTR} \qquad L_{modal} = \begin{pmatrix} 5.6577 & 0 & 0 \\ 0 & 1.6534 & 0 \\ 0 & 0 & 1.6534 \end{pmatrix} \frac{mH}{mi}$$

Recall:

$$L_m = \begin{pmatrix} 5.6463 & 0 & 0 \\ 0 & 1.8021 & 0 \\ 0 & 0 & 1.5057 \end{pmatrix} \frac{mH}{mi} \qquad \frac{L_{m_{1,1}} + L_{m_{2,2}}}{2} = 1.65 \frac{mH}{mi}$$

$$C_{modal} := T_{iTR}^{-1} C_{net} \cdot T_{iTR} \qquad C_{modal} = \begin{pmatrix} 11.1554 & 0 & 0 \\ 0 & 17.9112 & 0 \\ 0 & 0 & 17.9112 \end{pmatrix} \frac{nF}{mi}$$

$$C_m = \begin{pmatrix} 11.1641 & 0 & 0 \\ 0 & 16.5382 & 0 \\ 0 & 0 & 19.2318 \end{pmatrix} \frac{nF}{mi} \qquad \frac{C_{m_{1,1}} + C_{m_{2,2}}}{2} = 17.89 \frac{nF}{mi}$$

Characteristic Impedances:

$$Z_0 := \sqrt{\frac{L_{modal_{0,0}}}{C_{modal_{0,0}}}} \qquad Z_0 = 712.16\,\Omega$$

For untransposed case we had: $Z_{c0} = 711.17\,\Omega$

$$Z_1 := \sqrt{\frac{L_{modal_{1,1}}}{C_{modal_{1,1}}}} \qquad Z_1 = 303.83\,\Omega \qquad Z_2 := Z_1$$

For untransposed case:

$$Z_{c1} = 330.1\,\Omega \qquad Z_{c2} = 279.81\,\Omega$$

$$\frac{Z_{c1} + Z_{c2}}{2} = 304.96\,\Omega$$

Propagation Velocity

$$v_{0tr} := \frac{1}{\sqrt{L_{modal_{0,0}} \cdot C_{modal_{0,0}}}} \qquad v_{0tr} = 1.2587 \times 10^5 \frac{mi}{s}$$

from above: $v_0 = 1.2595 \times 10^5 \frac{mi}{s}$

$$\nu_{1tr} := \frac{1}{\sqrt{L_{modal_{1,1}} \cdot C_{modal_{1,1}}}}$$

$$\nu_{1tr} = 1.8376 \times 10^{5} \frac{mi}{s}$$

$$\nu_{2tr} := \nu_{1tr}$$

from above

$$\nu_{1} = 1.8317 \times 10^{5} \frac{mi}{s}$$

$$\nu_{2} = 1.8583 \times 10^{5} \frac{mi}{s}$$

$$\frac{\nu_{1} + \nu_{2}}{2} = 1.845 \times 10^{5} \frac{mi}{s}$$