ECE 524

TRANSIENTS IN
POWER SYSTEMS

SESSION no.  29
\[
\begin{bmatrix}
C'_B \\
C'_A \\
C'_C
\end{bmatrix} = 
\begin{bmatrix}
C_{AB} + C_{AC} + C_{BC} & -C_{AB} & -C_{AC} \\
-C_{AB} & C_{BG} + C_{AB} + C_{BC} & -C_{BC} \\
-C_{AC} & -C_{BC} & C_{CG} + C_{AC} + C_{BC}
\end{bmatrix}
\begin{bmatrix}
q_A \\
q_B \\
q_C
\end{bmatrix} = 
\begin{bmatrix}
V_{AB} \\
V_{BG} \\
V_{CG}
\end{bmatrix}
\]
These matrices are generally symmetric. If the line is transposed, the elements on the main diagonal of each matrix are all equal, and the off-diagonal terms of each matrix are also equal.

If the system is excited at a particular frequency, $\omega$, then the equations become:

\[
\begin{align*}
\frac{\partial E}{\partial x} &= [Z']I \\
\frac{\partial I}{\partial x} &= [Y']E
\end{align*}
\] (3) (4)

where: $Z'_{ij} = R'_{ij} + j\omega L'_{ij}$ and $Y'_{ij} = j\omega C'_{ij}$. Note, that this will also result if a Fourier Transformation is applied to the equations.

Equations (3) and (4) can be re-written in the same manner used for the single phase case to give:

\[
\begin{align*}
\frac{\partial^2 E}{\partial x^2} &= [Z'][Y']E \\
\frac{\partial^2 I}{\partial x^2} &= [Y'][Z']I
\end{align*}
\] (5) (6)

A modal transformation can be used to decouple these matrix equations. We can define two modal matrices, $T_e$ and $T_i$, where $T_e$ is the eigenvector matrix for $[Z'][Y']$ and $T_i$ is the eigenvector matrix $[Y'][Z']$. The most effective way to calculate these is to use a dedicated eigenvalue routine, such as the EIG function in Matlab.

These are constant matrices, independent of time and position ($x$). We can then define vectors $E_m$ and $I_m$ to represent the variables transformed into the modal domain, such that:

\[
\begin{align*}
E &= T_e E_m \\
I &= T_i I_m
\end{align*}
\] (7) (8)

Next substitute $E_m$ and $I_m$ into equations (5) and (6) resulting in:

\[
\begin{align*}
-\frac{T_e}{\partial x^2} E_m &= [Z'][Y'][T_e]E_m \\
-\frac{T_i}{\partial x^2} I_m &= [Y'][Z'][T_i]I_m
\end{align*}
\] (9) (10)
Which can be re-arranged as:

\[-\frac{\partial^2 E_m}{\partial x^2} = [T_e]^{-1}[Z'][Y'][T_e]E_m = \Lambda E_m \tag{11}\]
\[-\frac{\partial^2 I_m}{\partial x^2} = [T_i]^{-1}[Y'][Z'][T_i]I_m = \Lambda I_m \tag{12}\]

Notice that both \([Z'][Y']\) and \([Y'][Z']\) have the same eigenvalues. The theorem predicting this result is explained in many graduate level linear systems texts. The matrix \(\Lambda\) is a diagonal matrix of eigenvalues, resulting in \(m\) uncoupled equations, where \(m\) is the number of modes. Both voltage and current will have the same modes of propagation, as is demonstrated by having the same eigenvalues.

The transformation matrix \(T_e\) is not unique. Any matrix of the form \([D][T_e]\) is also a modal matrix. Where \([D] = d[I]\) where \(I\) is the identity matrix. In addition, \(T_e\) and \(T_i\) are related, as would be expected, since they lead to the same eigenvalues. We know that:

\[\Lambda = [T_e]^{-1}[Z'][Y'][T_e] \tag{13}\]

Then we can take the transpose

\[\Lambda^t = [[T_e]^{-1}[Z'][Y'][T_e]]^t \tag{14}\]

We know \(\Lambda = \Lambda^t\) since \(\Lambda\) is diagonal. We can simplify this expression to be (recall, that \([Y']\) and \([Z']\) are symmetric):

\[\Lambda = [T_e^*][Y'][Z'][T_e^*]^{-1} \tag{15}\]

So \([T_e^*]^{-1}\) is also a modal matrix for \([Y'][Z']\), which also has a modal matrix \(T_i\) defined by:

\[\Lambda = [T_i]^{-1}[Y'][Z'][T_i] \tag{16}\]

We can therefore conclude that:

\[[T_e][T_i^*] = [D] \tag{17}\]

where \([D] = d[I]\), and \(d\) could possibly equal one.

We can now diagonalize equations (3) and (4) as well.
from $Z_m' = \begin{bmatrix} z_0' & 0 & 0 \\ 0 & z_1' & 0 \\ 0 & 0 & z_2' \end{bmatrix}$

$R_0' + jwL_0'$

\[
\begin{bmatrix}
R_0' & 0 & 0 & 0 \\
0 & R_1' & 0 & 0 \\
0 & 0 & R_2' & 0 \\
0 & 0 & 0 & R_3'
\end{bmatrix}
\]

\[
\begin{bmatrix}
L_0' & 0 & 0 & 0 \\
0 & L_1' & 0 & 0 \\
0 & 0 & L_2' & 0 \\
0 & 0 & 0 & L_3'
\end{bmatrix}
\]

Traveling wave relationships for mode 0, mode 1, mode 2

3 independent Transmission Line models
Uniformly Transposed Lines

The above analysis can be simplified if the transmission line is uniformly transposed. Then the matrices for \([L']\) and \([C']\) are simplified, with all of the diagonal elements equal and all of the off-diagonal elements equal, as shown below for a transposed 3 phase line with 3 conductors:

\[
[L'] = \begin{bmatrix}
L'_s & L'_{mu} & L'_{mu} \\
L'_{mu} & L'_s & L'_{mu} \\
L'_{mu} & L'_{mu} & L'_s
\end{bmatrix}
\]

and

\[
[C'] = \begin{bmatrix}
C'_s & C'_{mu} & C'_{mu} \\
C'_{mu} & C'_s & C'_{mu} \\
C'_{mu} & C'_{mu} & C'_s
\end{bmatrix}
\]

Then we’ll find that \(T_e = [Z'][Y'] = T_i = [Y'][Z']\) when we repeat the analysis presented in equations (3) through (8).

One possible transformation matrix that will accomplish this is the “Karrenbauer” transformation:

\[
[T_i] = \begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2
\end{bmatrix}
\]

and

\[
[T_i]^{-1} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 0 \\
1 & 0 & -1
\end{bmatrix}
\]

However, the Clarke transformation (also called the \(\alpha - \beta\) transform) is more commonly used in EMTP-like programs, where

\[
[T_i] = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & \sqrt{2} & 0 \\
1 & -\frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{2} \\
1 & -\frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2}
\end{bmatrix}
\]

and

\[
[T_i]^{-1} = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
\sqrt{2} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix}
\]
Both of these transformations result in the same modal impedances and admittances as would result from applying the symmetrical components transformation in the 60 Hz phase domain. The advantage of the Clarke Transform is that $T_i^{-1} = T_i^t$, making the calculation of $T_e$ or modal quantities easier. In this case we see that:

\[
[Z'_m] = \begin{bmatrix}
Z'_s + 2Z'_{mu} & 0 & 0 \\
0 & Z'_a - Z'_{mu} & 0 \\
0 & 0 & Z'_i - Z'_{mu}
\end{bmatrix}
\]

and

\[
[Y'_m] = \begin{bmatrix}
Y'_s + 2Y'_{mu} & 0 & 0 \\
0 & Y'_a - Y'_{mu} & 0 \\
0 & 0 & Y'_i - Y'_{mu}
\end{bmatrix}
\]

Then solving

\[
-\frac{\partial E_m}{\partial x} = [Z'_m]I_m
\]

(27)

\[
-\frac{\partial I_m}{\partial x} = [Y'_m]E_m
\]

(28)

where $E_m = [E_0, E_1, E_2]^t$ and $I_m = [I_0, I_1, I_2]^t$. shows that each set of three differential equations only has two unique equations. Modes 1 and 2 will have identical solutions. This is only true in the transposed case.

We can solve the differential equations for each modal voltage and current separately, and repeat the analysis used for single phase lines, only in this case, each single phase line represents a different mode. It is again expedient to use lossless line models, and lump the resistances at the ends of the lines.

In this case we’ll propagation velocities and characteristic impedances for each of the modal voltages and currents, where:

\[
\nu_0 = \frac{1}{\sqrt{L_0C_0}} \quad \text{and} \quad Z_0 = \sqrt{\frac{L_0}{C_0}}
\]

(29)

\[
\nu_1 = \frac{1}{\sqrt{L_1C_1}} \quad \text{and} \quad Z_1 = \sqrt{\frac{L_1}{C_1}}
\]

(30)

\[
\nu_2 = \nu_1 \quad \text{and} \quad Z_1 = Z_2
\]

(31)

In this case the characteristic impedances are the same for the voltage and current equations if the Karrenbauer transformation is used.
Three Phase Travelling Wave Examples

Applied voltage: \( V_m := 107.8 \text{kV} \)

Balanced three phase set with close at peak of phase:

\[
V_{abc0} := \begin{bmatrix}
\frac{V_m}{2} \\
-\frac{V_m}{2} \\
-\frac{V_m}{2}
\end{bmatrix}
\]

Clarke Transform:

\[
T_i := \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & \sqrt{2} & 0 \\
1 & -1 & \sqrt{3} \\
1 & -1 & -\sqrt{3}
\end{bmatrix}
\]

\[
T_e := T_i
\]

\[
T_e T_i^T = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[ V_{\text{modal0}} := T_e^{-1} \cdot V_{\text{abc0}} \]

\[ V_{\text{modal0}} = \begin{pmatrix} 0 \\ 132.03 -kV \\ 0 \end{pmatrix} \]

\[ \text{Vref} = \begin{pmatrix} 0 \\ 2.132.05 -kV \\ 0 \end{pmatrix} \]

- Close into open circuit, we expect receiving end voltage to double for each mode, and the convert back to ABC

Voltages when close all three poles:

- Note that each individual phase doubles the initial applied voltage
- Suggests only one mode excited, as implied above

Initial modal current

\[ \frac{V_{\text{modal0}}}{Z_c1} = 489.33 \, \text{A} \]
Modal Transformations for transposed line:

Clark Transformation:

\[ T_i := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} & 0 \\ 1 & -\frac{1}{\sqrt{2}} & \sqrt{3} \\ 1 & -\frac{1}{\sqrt{2}} & -\sqrt{3} \end{pmatrix} \]

\[ T_i = \begin{pmatrix} 0.577 & 0.816 & 0 \\ 0.577 & -0.408 & 0.707 \\ 0.577 & -0.408 & -0.707 \end{pmatrix} \]

\[ (T_i^{-1}) = \begin{pmatrix} 0.577 & 0.577 & 0.577 \\ 0.816 & -0.408 & -0.408 \\ 0 & 0.707 & -0.707 \end{pmatrix} \]

\[ T_e := T_i \]

\[ L_{modal} := T_e^{-1} \cdot L_{net} \cdot T_i \]

As a check:

\[ L_{net_{0,0}} + 2 \cdot L_{net_{0,1}} = 5.8831 \cdot \frac{mH}{mi} \]

\[ L_{net_{0,0}} - L_{net_{0,1}} = 1.3386 \cdot \frac{mH}{mi} \]

\[ R_{modal} := T_e^{-1} \cdot R_{net} \cdot T_i \]

\[ R_{modal} = \begin{pmatrix} 0.314 & 0 & 0 \\ 0 & 0.029 & 0 \\ 0 & 0 & 0.029 \end{pmatrix} \cdot \frac{ohm}{mi} \]
Single phase close to open circuited line:

- Voltage

\[ V_{\text{modal\_single0}} := T_e \begin{pmatrix} 107.8 \text{kV} \\ 0 \text{kV} \\ 0 \text{kV} \end{pmatrix}, \quad V_{\text{modal\_single0}} = \begin{pmatrix} 62.24 \text{kV} \\ 88.02 \text{kV} \\ 0 \end{pmatrix} \text{ kV} \]

Note the cross-coupling, the first term is a ground mode term.

Receiving end: \[ V_{\text{recmodal}} := 2 \cdot V_{\text{modal\_single0}} \]

\[ V_{\text{abc\_rec}} := T_c \cdot V_{\text{recmodal}} \]

\[ V_{\text{abc\_rec}} = \begin{pmatrix} 215.6 \text{kV} \\ 0 \\ 0 \end{pmatrix} \text{ kV} \]
As a check:
\[
R_{ac} = \frac{0.029}{3} \text{ ohm/mi}
\]
\[
R_{\text{net},0} - R_{\text{net},1} = \frac{0.029}{3} \text{ ohm/mi}
\]
and
\[
R_{\text{net},0} + 2R_{\text{net},1} = \frac{0.314}{3} \text{ ohm/mi}
\]

\[
\left(\frac{R_{ac}}{3}\right) - \left(\frac{R_{\text{net},0} - R_{\text{net},1}}{3}\right) = 8.2849 \times 10^{-8} \text{ m-kg/A}^2 \cdot \text{s}^3
\]

\[
C_{\text{modal}} := \frac{1}{T_i} C_{\text{net},0} \cdot T_e
\]

As a check:
\[
C_{\text{net},0} + 2C_{\text{net},1} = 0.0102 \text{ \mu F/mi}
\]
\[
C_{\text{net},0} - C_{\text{net},1} = 0.0225 \text{ \mu F/mi}
\]

Length := 230mi

Characteristic Impedances:
\[
Z_0 := \sqrt{\frac{L_{\text{modal},0,0}}{C_{\text{modal},0,0}}}
\]
\[
Z_0 = 758.5565 \Omega
\]
\[
Z_1 := \sqrt{\frac{L_{\text{modal},1,1}}{C_{\text{modal},1,1}}}
\]
\[
Z_1 = 243.9525 \Omega
\]
\[
Z_2 := Z_1
\]