Very High Frequency Transformer Model

Problem 1: A grounded shunt reactor with a single layer winding and a grounded neutral has:
\[ \alpha = 6 \]
A step voltage surge of 750 kV is applied.

A. Find the maximum voltage seen at any point on the winding
B. What point on the winding sees the maximum voltage?

Note: Use the analytical equations described in class

\[ X := 1, 0.95, 0 \]

The plot shows three traces. The middle trace is a plot of the even steady-state voltage distribution from the top of the winding to the neutral. The lower plot shows the initial voltage distribution, where more of the voltage is dropped along the top of the winding initially. This lower curve will then “oscillate” an equal distance above the even distribution line, shown in the upper curve.

We can find the maximum value and the point of the maximum value from the curve above. However, we can also derive an expression for the maximum value by taking the derivative of the top curve with respect to \( x \), and setting it equal to zero. The equation for the top curve is:

\[
\frac{d}{dx} \left[ 750kV \cdot \left( \frac{2x - \sinh(\alpha \cdot x)}{\sinh(\alpha)} \right) \right] = 750kV \cdot \left( \frac{\alpha \cdot \cosh \left( \frac{\alpha \cdot x}{\text{Length}} \right)}{\text{Length} \cdot \sinh(\alpha)} \right) = 0
\]
Since this is a nonlinear function an iterative solver is needed. I'm using a MathCAD solve block.

Set constant for normalized length of the winding:

\[ \text{Length} := 1 \]

Set initial value of unknown position \( x := 1 \)

Given

\[
750kV \cdot \left( \frac{2}{\text{Length}} - \frac{\alpha \cdot \text{cosh} \left( \frac{\alpha \cdot x}{\text{Length}} \right)}{\text{Length} \cdot \text{sinh} \left( \alpha \right)} \right) = 0
\]

\[ X_{\text{max}} := \text{Find}(x) \quad X_{\text{max}} = 0.817 \quad X_{\text{max}} = 81.69\% \]

This tells us that the maximum voltage occurs 81.7% of the way from the neutral to the top of the winding. This also matches the plot above. Now we can solve for the maximum voltage:

\[
V_{\text{Max}} := 750kV \cdot \left( \frac{2X_{\text{max}}}{\text{Length}} - \frac{\text{sinh} \left( \frac{\alpha \cdot X_{\text{max}}}{\text{Length}} \right)}{\text{sinh} \left( \alpha \right)} \right)
\]

\[ V_{\text{Max}} = 975.36\cdot\text{kV} \]

**Problem 2:** A single layer coil has 50 turns, with \( C_g = 2.4 \, \text{pF/turn} \) and \( C_s = 370 \, \text{pF/turn} \). The coil has a floating neutral, and experiences a step voltage of 400 kV.

A. Determine the initial voltage on the neutral
B. What is the maximum voltage on the neutral from the resulting oscillation?

- Define unit "turn" \( \text{turn} := 1 \)
- Define capacitance per turn values
  \[ C_g' := 2.4 \, \frac{\text{pF}}{\text{turn}} \quad C_s' := 370 \, \frac{\text{pF}}{\text{turn}} \]
- Define number of turns
  \[ N_{\text{turns}} := 50 \, \text{turn} \]
- Find total shunt capacitance (all are parallel, so multiply by number of turns):

\[ C_g := N_{\text{turns}} \cdot C_g' \quad C_g = 120\,\text{pF} \]

- Find total series capacitance (all are equal, so divide by number of turn to turn capacitance in series (note, since this isn't a toroid, there one less turn to turn capacitance than the number of turns).

\[ C_s := \frac{C_s'}{N_{\text{turns}} - 1} \quad C_s = 7.55\,\text{pF} \]

- Then we can find \( \alpha \):

\[ \alpha := \sqrt{\frac{C_g}{C_s}} \quad \alpha = 3.99 \]

- Now we can plot the voltage distributions as shown in the figure below.

\[ X := 1, 0.95 .. 0 \quad \text{Len} := 1 \]
The middle trace shows the steady-state voltage distribution, \( X=1 \) is the top of the winding and \( X=0 \) is the neutral. The lower trace shows the initial voltage distribution, we want to find the initial voltage at \( X=0 \). The upper trace shows the envelope that the voltage will ring up to. We want to find the voltage for \( X=0 \) on that curve as well.

To find the initial voltage at \( X=0 \), we plug \( X=0 \) into the equation for the lower curve. And to find the maximum value, we use the expression for the upper curve.

\[
x := 0
\]

\[
V_{\min 0} := 400 \text{kV} \cdot \frac{\cosh\left(\frac{\alpha \cdot x}{\text{Len}}\right)}{\cosh(\alpha)}
\]

\[
V_{\min 0} = 14.85 \text{kV}
\]

\[
V_{\max 0} := 400 \text{kV} \cdot \left(2 - \frac{\cosh\left(\frac{\alpha \cdot x}{\text{Len}}\right)}{\cosh(\alpha)}\right)
\]

\[
V_{\max 0} = 785.15 \text{kV}
\]