Digital Filter Examples

- Define sampling rate per cycle

  $RS := 16$

- Define length of sample data set, in cycles

  $CY := 8$

- Total number of samples:

  $M := CY \cdot RS$

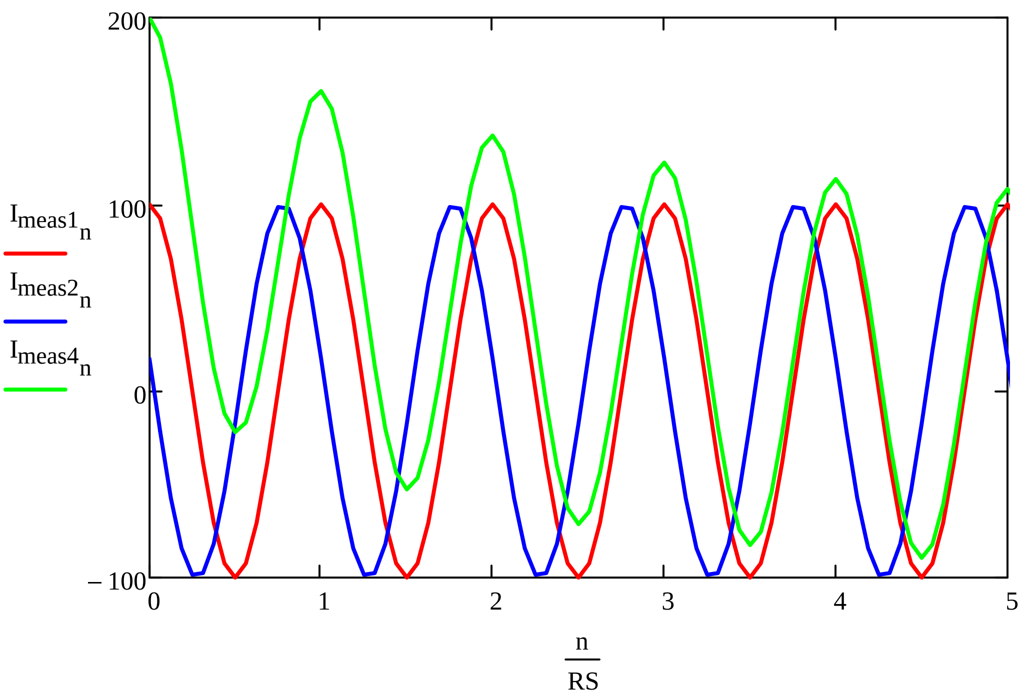
$n := 0, 1 .. M - 1$

$\Delta t := \frac{1}{RS \cdot 60Hz}$ $\Delta t = 1.042\,ms$ $t_n := 0, \Delta t .. n \cdot \Delta t$

- Create input data signal, sampled at RS per cycle

$I_{meas1_n} := 100 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{RS} + 0deg\right)$ $I_{meas2_n} := 100 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{RS} + 80deg\right)$

$I_{meas3_n} := 100 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{RS}\right) + 50$ $I_{meas4_n} := 100 \cdot \left(\cos\left(\frac{2 \cdot \pi \cdot n}{RS} + 0deg\right) + e^{\frac{-n}{2RS}}\right)$



- Lets look at the Cosine Filter Coefficients:

$k_4 := 0, 1 .. (4-1)$ $k_8 := 0, 1 .. (8-1)$ $k_{16} := 0, 1 .. (16-1)$ $k_{32} := 0, 1 .. (32-1)$

$$cos_{coef}(k, RS) := \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right)$$

$cos_{coef}(k_4, 4) =$

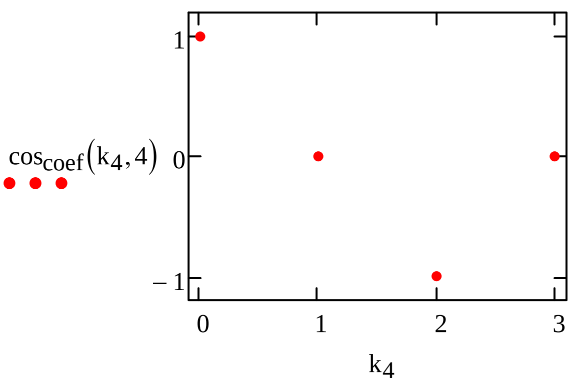
1
0
-1
0

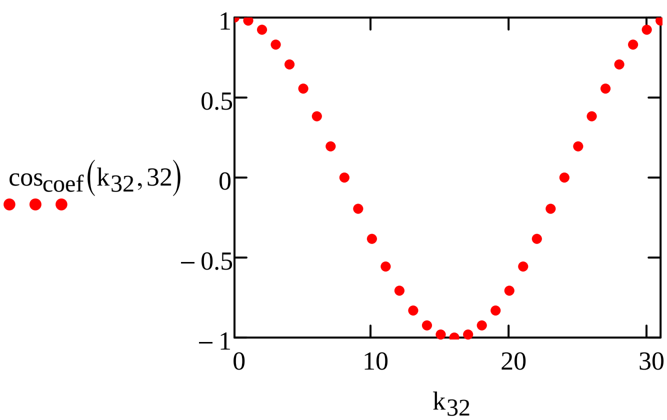
$cos_{coef}(k_8, 8) =$

1
0.707
0
-0.707
-1
-0.707
0
0.707

$cos_{coef}(k_{16}, 16) =$

1
0.924
0.707
0.383
0
-0.383
-0.707
-0.924
-1
-0.924
-0.707
-0.383
0
0.383
0.707
0.924





- Now lets look at the Sine Filter Coefficients:

$$\sin_{coef}(k, RS) := \sin\left(\frac{2\cdot\pi\cdot k}{RS}\right)$$

$\sin_{coef}(k_4, 4) =$

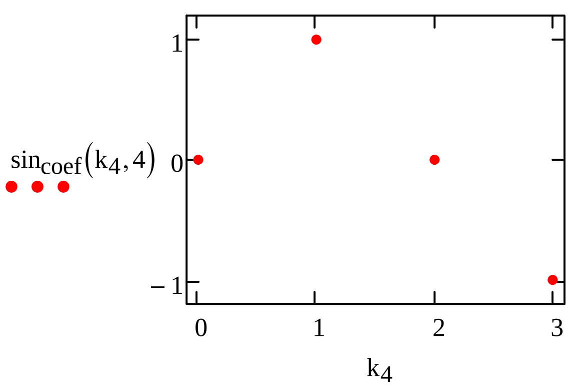
0
1
0
-1

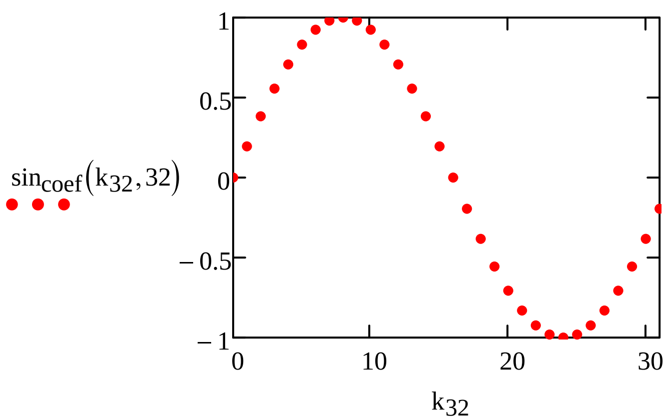
$\sin_{coef}(k_8, 8) =$

0
0.707
1
0.707
0
-0.707
-1
-0.707

$\sin_{coef}(k_{16}, 16) =$

0
0.383
0.707
0.924
1
0.924
0.707
0.383
0
-0.383
-0.707
-0.924
-1
-0.924
-0.707
-0.383





Now define Cosine and Sin filters

$$COSF(RS, A, q) := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[cos_{coef}(k, RS) \cdot A_{[q-(RS-1)]+k} \right]$$

$$SINF(RS, A, q) := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[sin_{coef}(k, RS) \cdot A_{[q-(RS-1)]+k} \right]$$

- Create a filter index, "i" (which includes RS samples of past history (so it starts at (RS - 1))

$$i := (RS - 1) .. M - 1$$

- Create a filter index, "v" (which includes RS/4 samples of past history for delaying cosine filter output put a quarter cycle (so it starts at (RS/4 - 1))

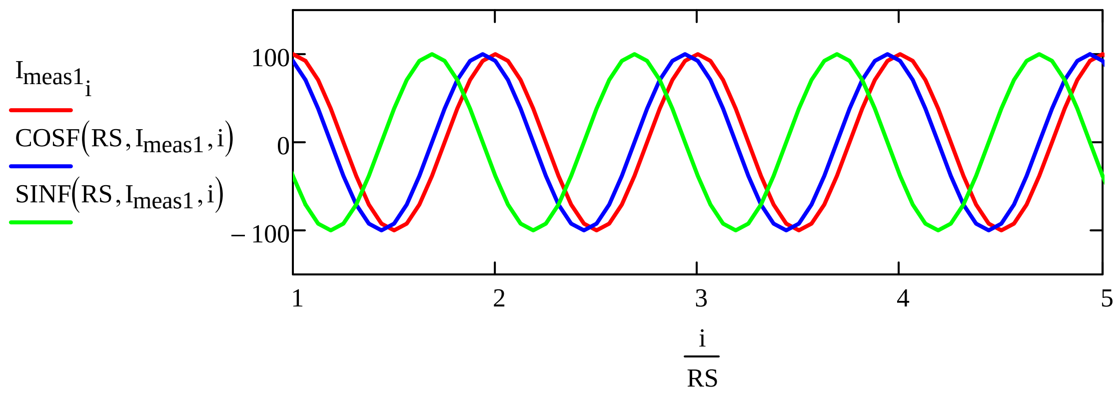
$$v := \left(RS + \frac{RS}{4} - 1 \right) .. M - 1$$

$COSF(RS, I_{meas1}, i) =$

100
92.388
70.711
38.268
0
-38.268
-70.711
-92.388
-100
-92.388
-70.711
-38.268
0
38.268
70.711
...

$SINF(RS, I_{meas1}, i) =$

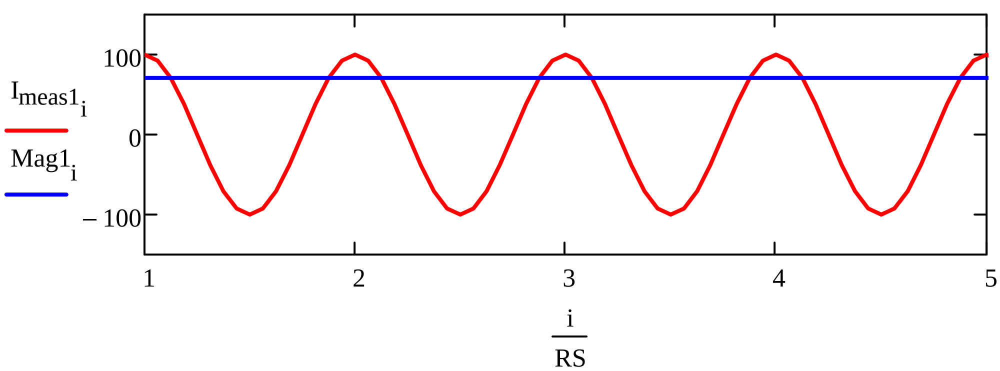
0
-38.268
-70.711
-92.388
-100
-92.388
-70.711
-38.268
0
38.268
70.711
92.388
100
92.388
70.711
...

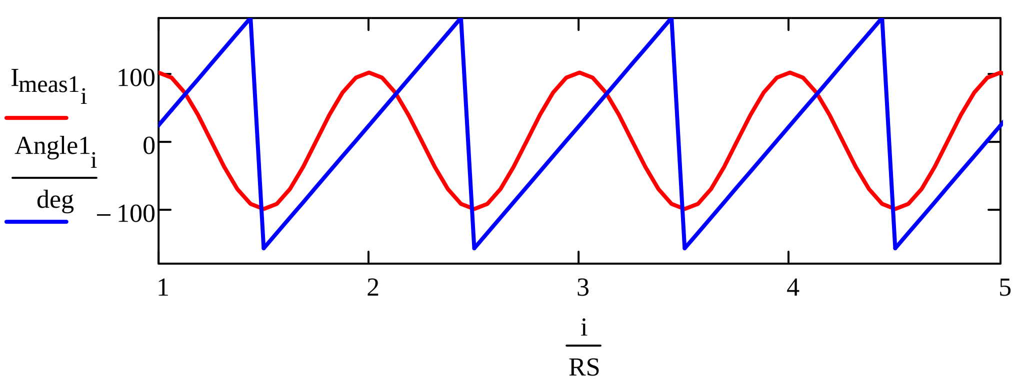


$$Phasor1_i := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas1}, i\right) - j \cdot SINF\left(RS, I_{meas1}, i\right)\right)$$

$$Mag1_i := \left|Phasor1_i\right|$$

$$Angle1_i := arg\left(Phasor1_i\right)$$

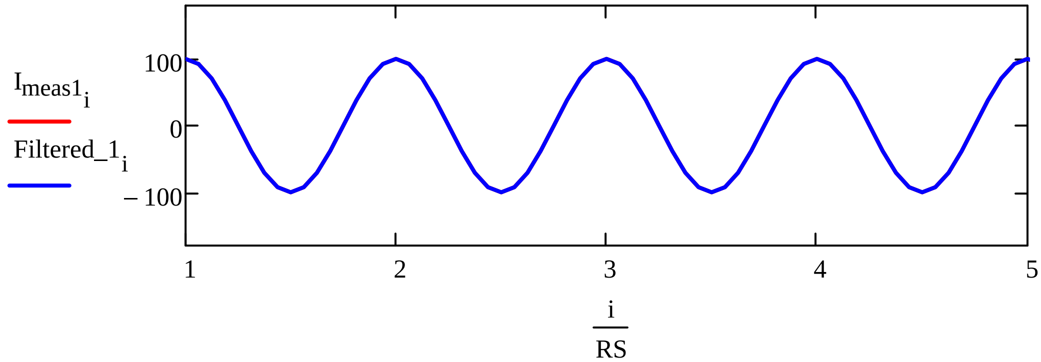




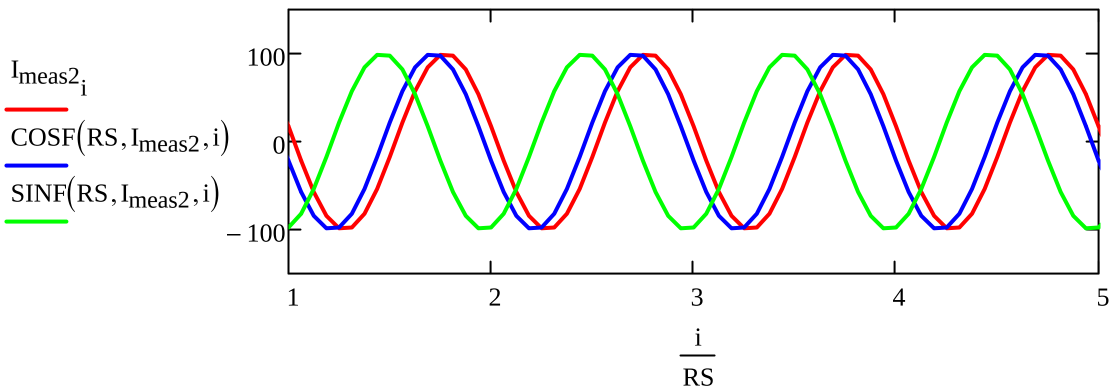
- So we need to compare this angle to a reference. In the case with only one measurement, we compare it to itself.

$$\theta1_i := Angle1_i - Angle1_i$$

$$Filtered\_1_i := \sqrt{2}\cdot Mag1_i\cdot \cos\left[\left(\frac{2\cdot\pi\cdot i}{RS}\right) + \theta1_i\right]$$



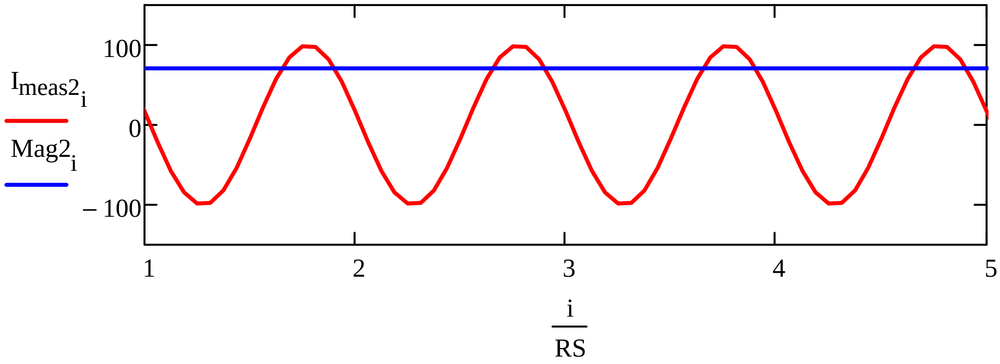
- Now repeat with the second signal, which is phase shifted

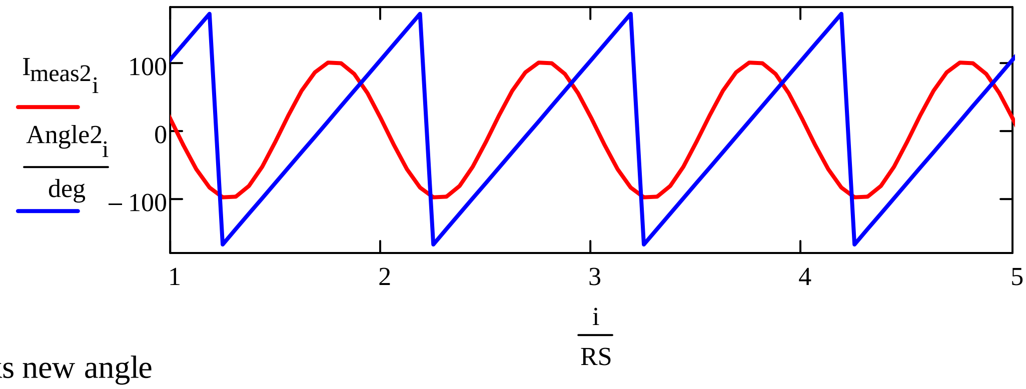


$$Phasor2_i := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas2}, i\right) - j\cdot SINF\left(RS, I_{meas2}, i\right)\right)$$

$$Mag2_i := \left|Phasor2_i\right|$$

$$Angle2_i := arg\left(Phasor2_i\right)$$



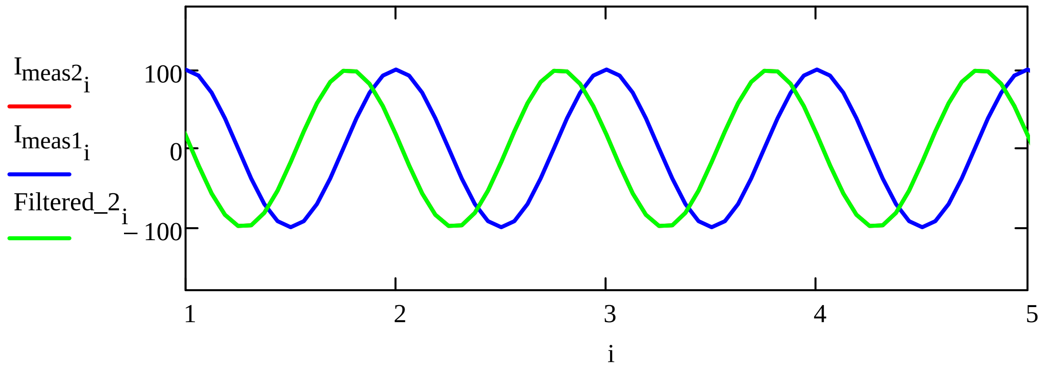


Tracks new angle

- So we need to compare this angle to a reference. In the case we'll use the first signal as a reference

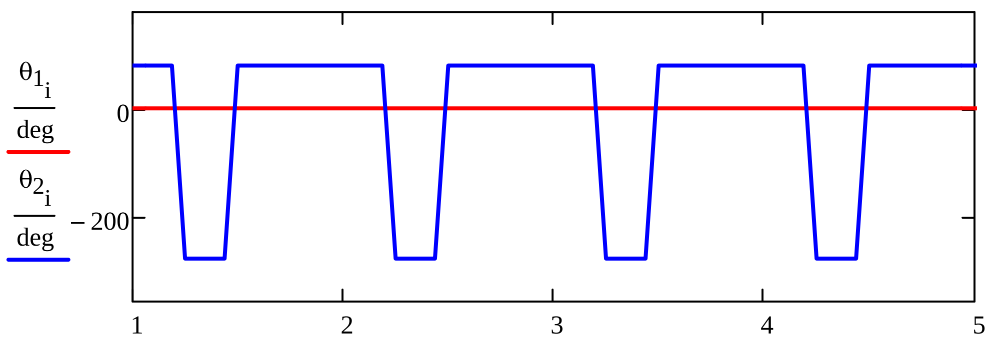
$$\theta_{2_i} := Angle2_i - Angle1_i$$

$$Filtered\_2_i := \sqrt{2} \cdot Mag2_i \cdot \cos\left[\left(\frac{2 \cdot \pi \cdot i}{RS}\right) + \theta_{2_i}\right]$$



Tracks the phase shift

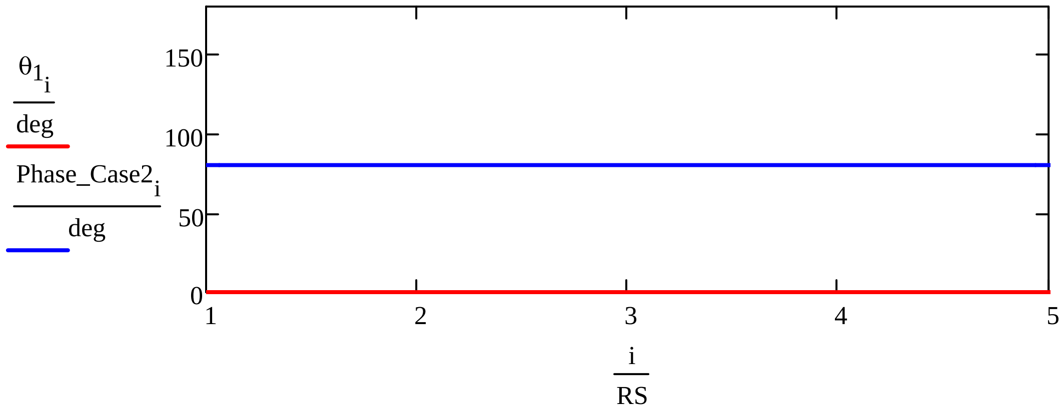
Now plot the angle



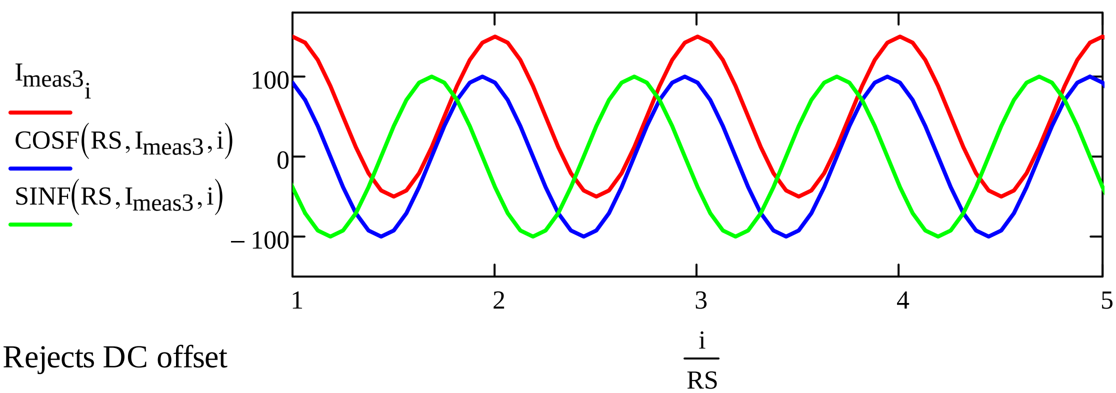
We have a problem in the angle
due to the difference in reset times

- Fix for the reset time issue:

$$Phase\_Case2_i := \begin{cases} Angle2_i - Angle1_i & \text{if } \left|Angle2_i - Angle1_i\right| < \pi \\ Angle2_i - Angle1_i - 2\cdot\pi & \text{if } \left(Angle2_i - Angle1_i\right) > \pi \\ Angle2_i - Angle1_i + 2\cdot\pi & \text{if } Angle2_i - Angle1_i < -(\pi) \end{cases}$$



- Now repeat with the third signal, which has a constant dc offset

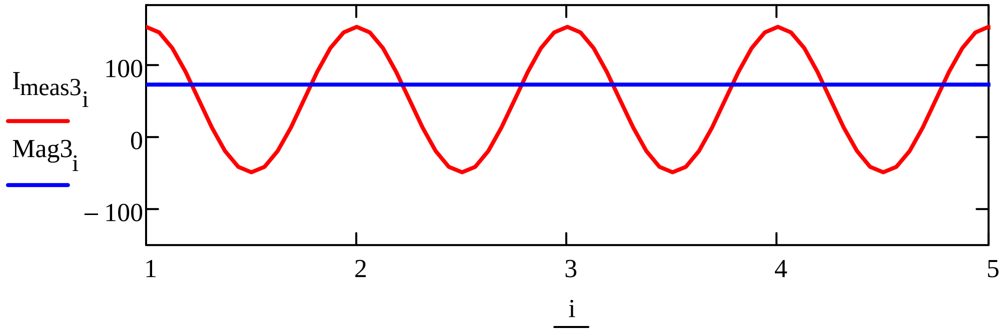


- Rejects DC offset

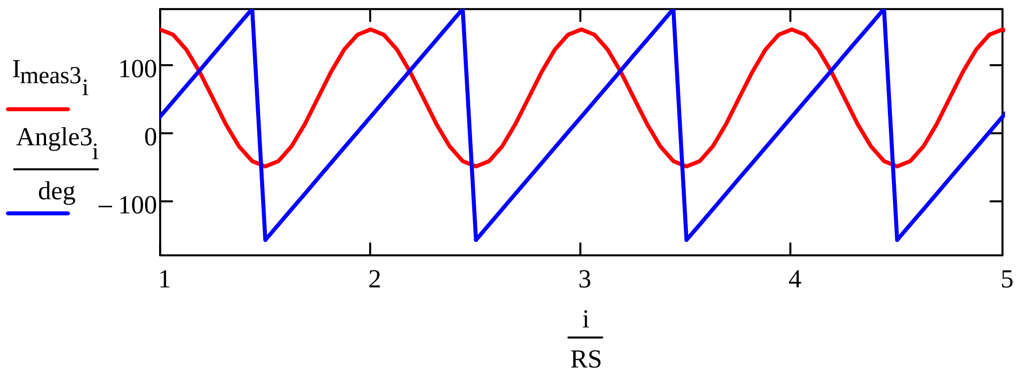
$$Phasor3_i := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas3}, i\right) - j \cdot SINF\left(RS, I_{meas3}, i\right)\right)$$

$$Mag3_i := \left|Phasor3_i\right|$$

$$Angle3_i := \arg\left(Phasor3_i\right)$$



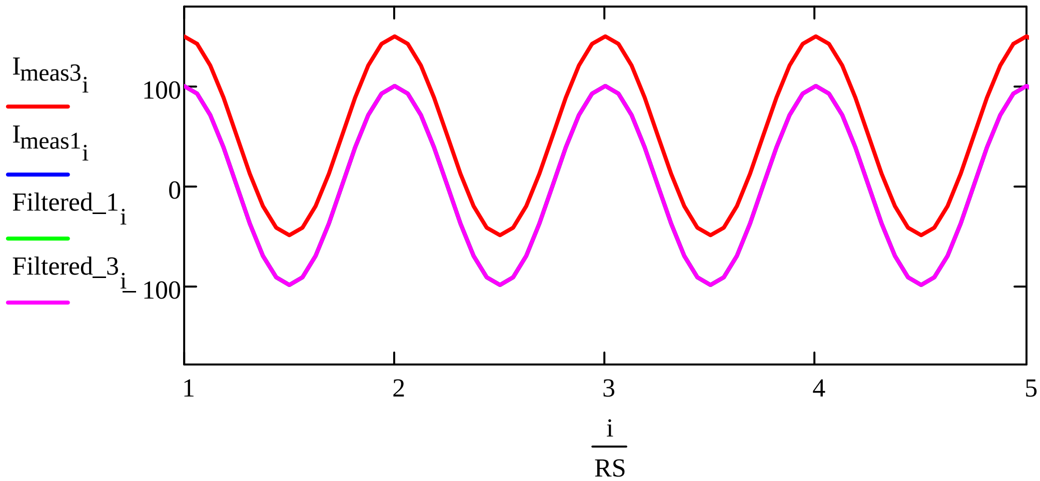
RMS magnitude of fundamental component



- Again need to compare this angle to a reference. In the case we'll use the first signal as a reference

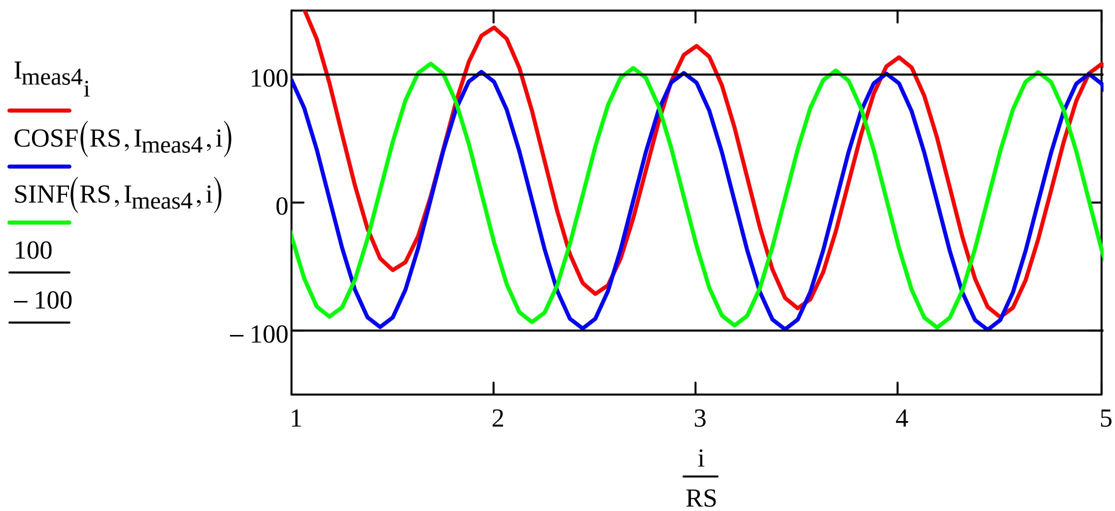
$$\theta_{3_i} := Angle3_i - Angle1_i$$

$$Filtered\_3_i := \sqrt{2} \cdot Mag3_i \cdot \cos\left[\left(\frac{2 \cdot \pi \cdot i}{RS}\right) + \theta_{3_i}\right]$$



Note the DC offset not in filtered results

- Now repeat with the fourth signal, which has a decaying DC offset.

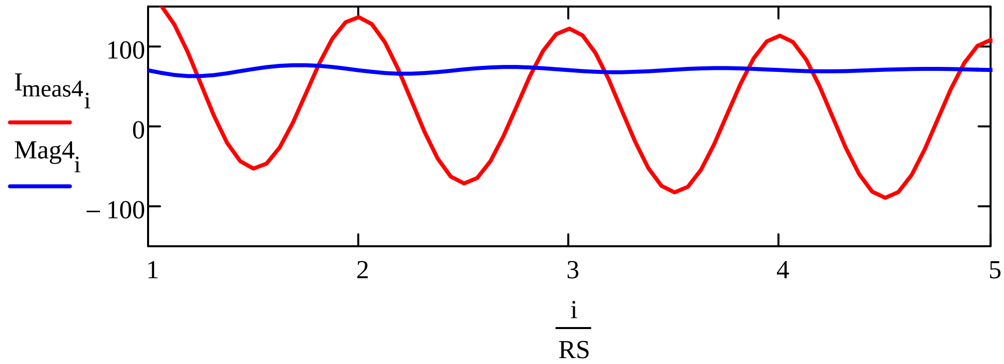


Sine filer passing some DC offset, but not cosine

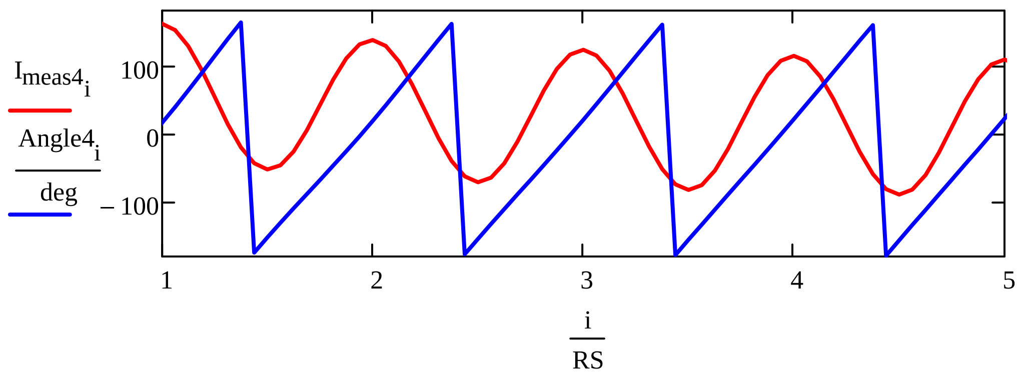
$$Phasor4_i := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas4}, i\right) - j \cdot SINF\left(RS, I_{meas4}, i\right)\right)$$

$$Mag4_i := \left|Phasor4_i\right|$$

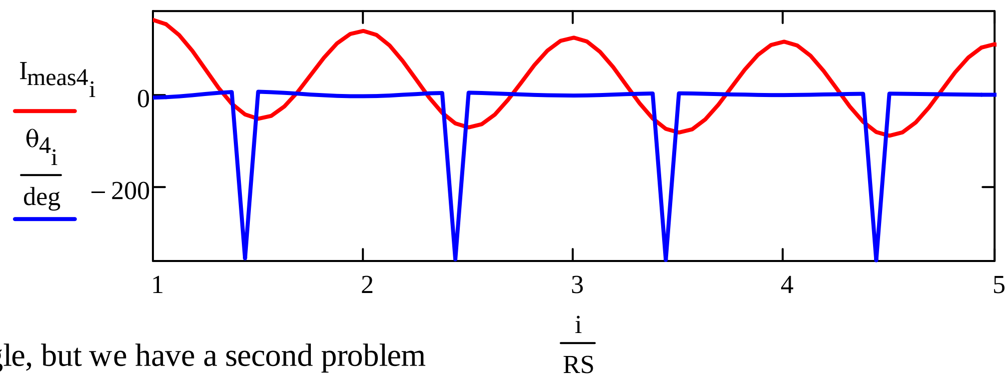
$$Angle4_i := \arg\left(Phasor4_i\right)$$



Magnitude has error with decaying offset



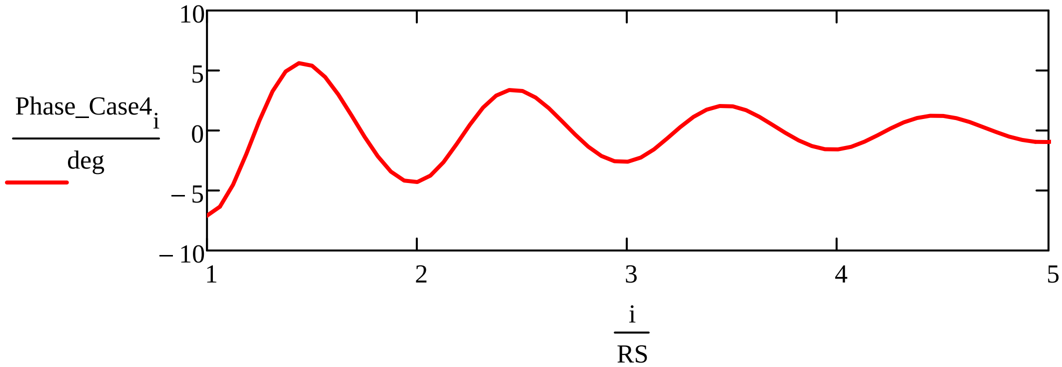
$\theta_{4_i} := Angle4_i - Angle1_i$



As does angle, but we have a second problem due to reset times

- Fix for the reset time issue:

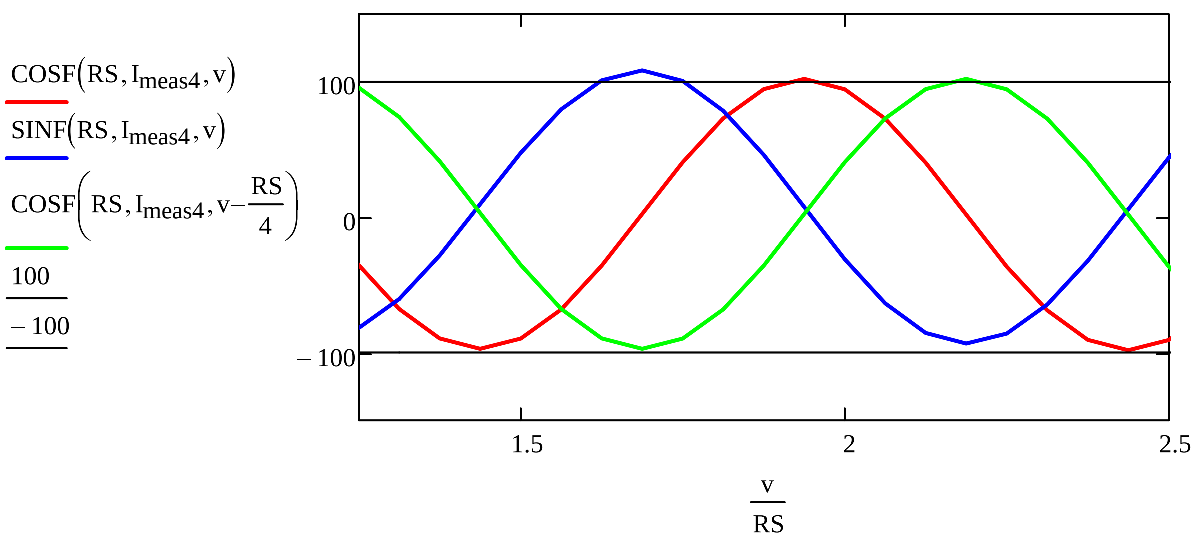
$$Phase\_Case4_i := \begin{cases} Angle4_i - Angle1_i & \text{if } \left|Angle4_i - Angle1_i\right| < \pi \\ Angle4_i - Angle1_i - 2\cdot\pi & \text{if } \left(Angle4_i - Angle1_i\right) > \pi \\ Angle4_i - Angle1_i + 2\cdot\pi & \text{if } Angle4_i - Angle1_i < -(\pi) \end{cases}$$



So still see decaying dc offset problem in angle calculation.

Alternative to using Sine Filter:

Note that delaying a cosine by 90 degrees (1/4 cycle) give a sine function

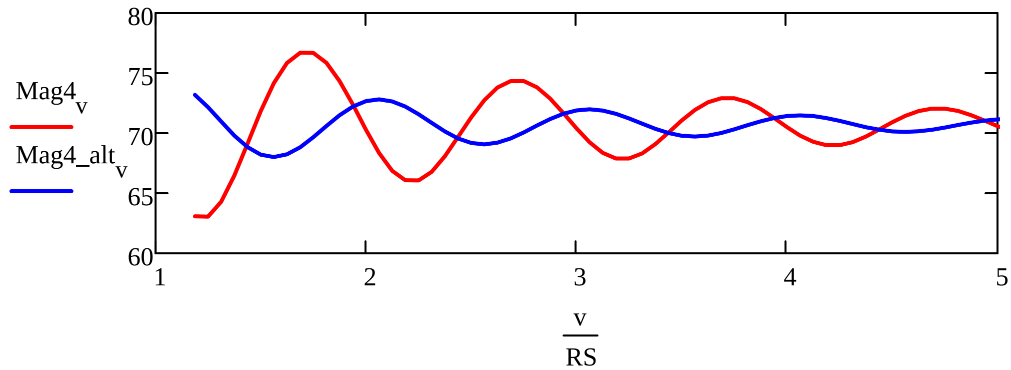


Note I'm changing index to "v" instead of "i" due to different starting point

$$Phasor4\_alt_v := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas4}, v\right) + j \cdot COSF\left(RS, I_{meas4}, v - \frac{RS}{4}\right)\right)$$

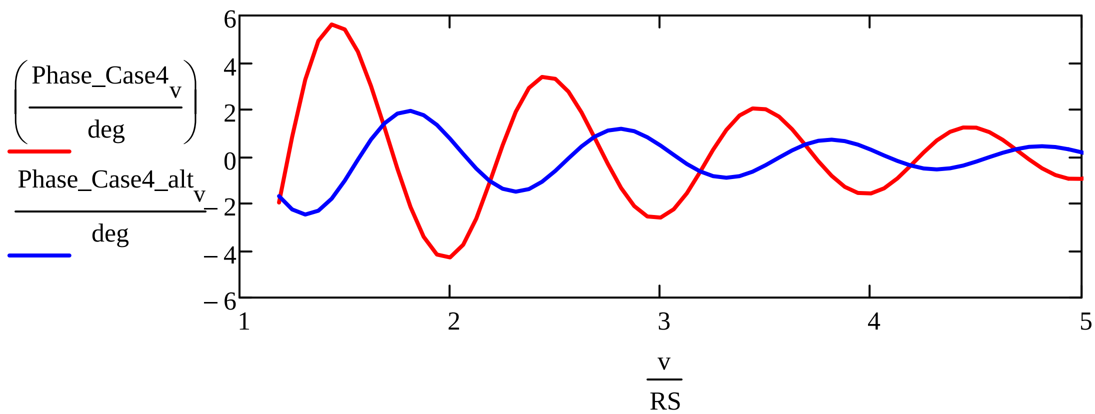
$$Mag4\_alt_v := \left|Phasor4\_alt_v\right|$$

$$Angle4\_alt_v := \arg\left(Phasor4\_alt_v\right)$$



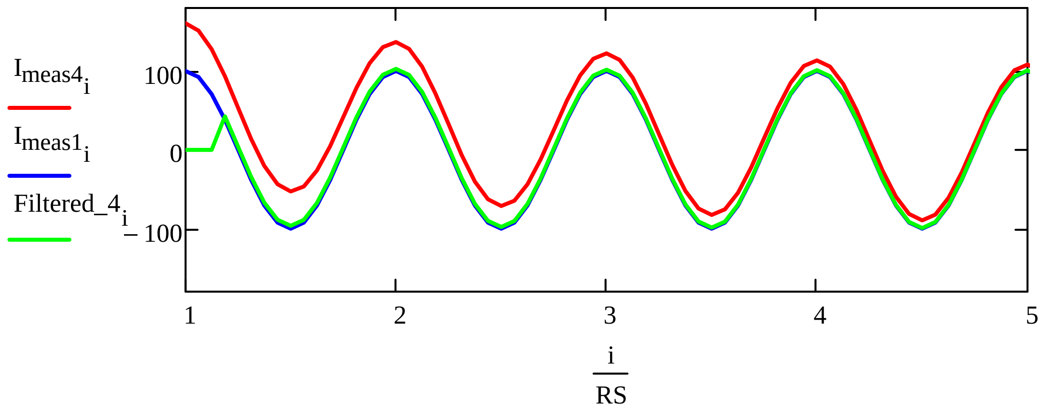
Magnitude has less error due to DC offset, but
cosine isn't perfect rejection either

$$Phase\_Case4\_alt_v := \begin{cases} Angle4\_alt_v - Angle1_v & \text{if } \left|Angle4\_alt_v - Angle1_v\right| < \pi \\ Angle4\_alt_v - Angle1_v - 2\cdot\pi & \text{if } \left(Angle4\_alt_v - Angle1_v\right) > \pi \\ Angle4\_alt_v - Angle1_v + 2\cdot\pi & \text{if } Angle4\_alt_v - Angle1_v < -(\pi) \end{cases}$$



Again, much better, but not perfect.

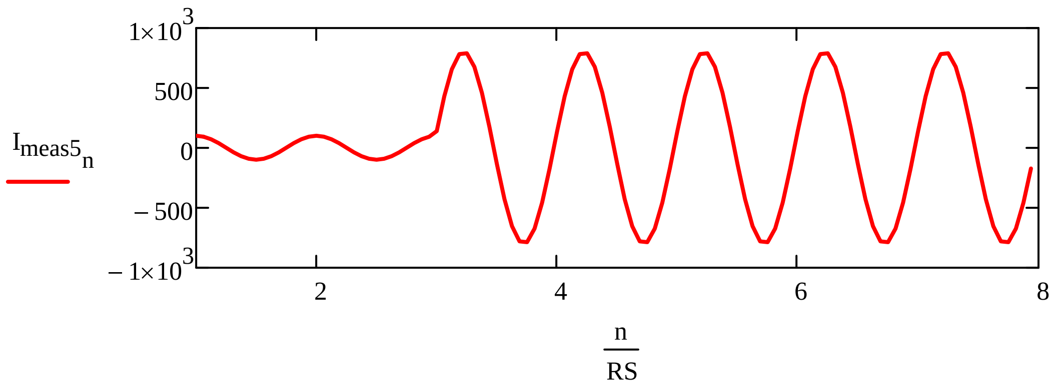
$$Filtered\_4_v := \sqrt{2}\cdot Mag4\_alt_v\cdot \cos\left[\left(\frac{2\cdot\pi\cdot v}{RS}\right) + Phase\_Case4\_alt_v\right]$$



DC offset largely removed, but not entirely

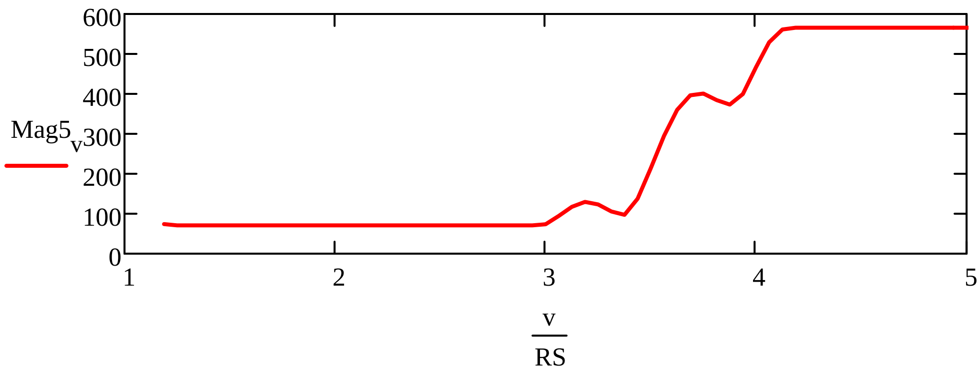
A few more cases:

$$I_{meas5_n} := \begin{cases} 100\cdot\cos\left(\frac{2\cdot\pi\cdot n}{RS} + 0deg\right) & \text{if } 0 < n < 3\cdot RS \\ 800\cdot\cos\left[\frac{2\cdot\pi\cdot(n + 2\cdot RS)}{RS} - 80deg\right] & \text{otherwise} \end{cases}$$

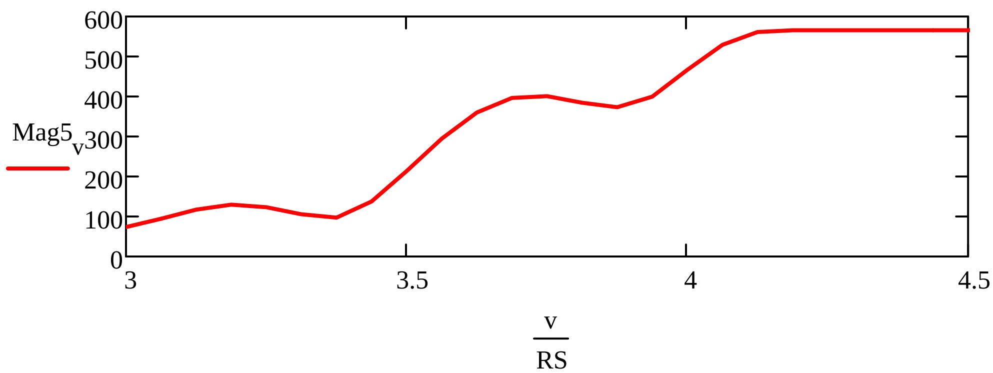


$$Phasor5_v := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas5}, v\right) + j\cdot COSF\left(RS, I_{meas5}, v - \frac{RS}{4}\right)\right)$$

$$Mag5_v := \left|Phasor5_v\right| \qquad Angle5_v := \arg\left(Phasor5_v\right)$$

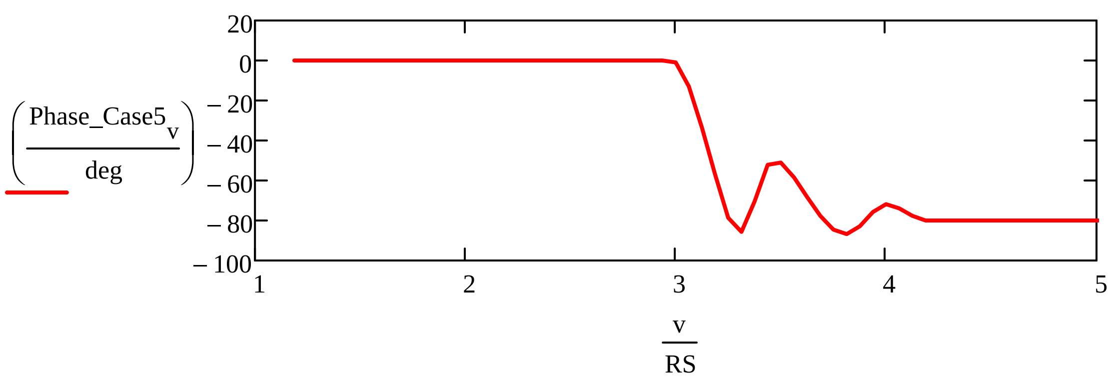


- Notice the filter response time

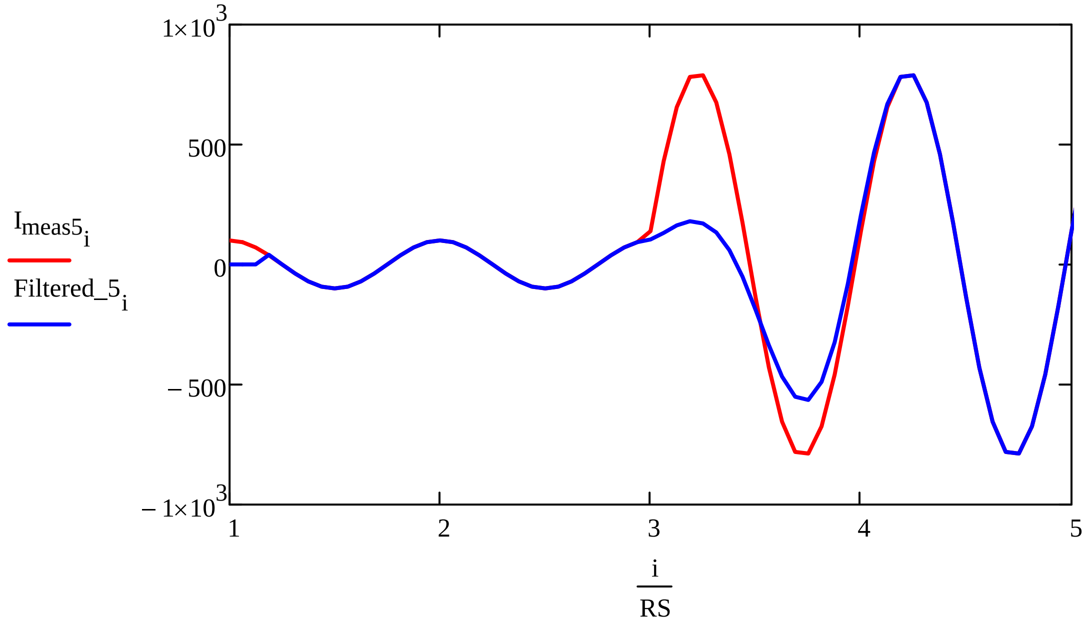


Takes a little over a cycle

$$\text{Phase\_Case5}_v := \begin{cases} \text{Angle5}_v - \text{Angle1}_v & \text{if } \left|\text{Angle5}_v - \text{Angle1}_v\right| < \pi \\ \text{Angle5}_v - \text{Angle1}_v - 2\cdot\pi & \text{if } \left(\text{Angle5}_v - \text{Angle1}_v\right) > \pi \\ \text{Angle5}_v - \text{Angle1}_v + 2\cdot\pi & \text{if } \text{Angle5}_v - \text{Angle1}_v < -(\pi) \end{cases}$$



$$\text{Filtered\_5}_v := \sqrt{2}\cdot \text{Mag5}_v\cdot \cos\left[\left(\frac{2\cdot\pi\cdot v}{RS}\right) + \text{Phase\_Case5}_v\right]$$

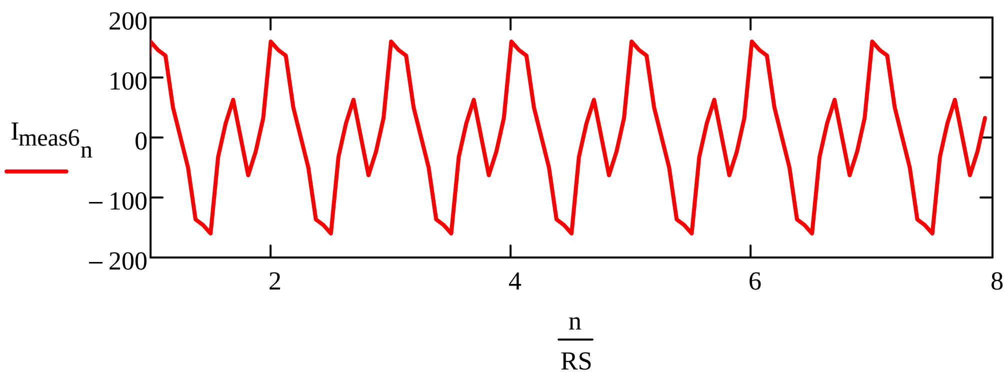


Notice delay in filter response

- Now add some harmonics.

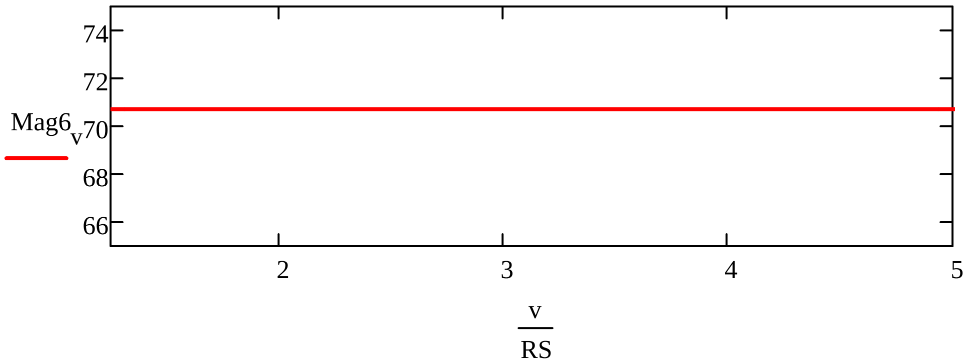
First, integer harmonics

$$I_{meas6_n} := 100\cdot\cos\left(\frac{2\cdot\pi\cdot n}{RS}\right) + 80\cdot\sin\left[2\left(\frac{2\cdot\pi\cdot n}{RS}\right)\right] + 40\cdot\cos\left[3\left(\frac{2\cdot\pi\cdot n}{RS}\right)\right] + 20\cdot\cos\left[7\left(\frac{2\cdot\pi\cdot n}{RS}\right)\right]$$

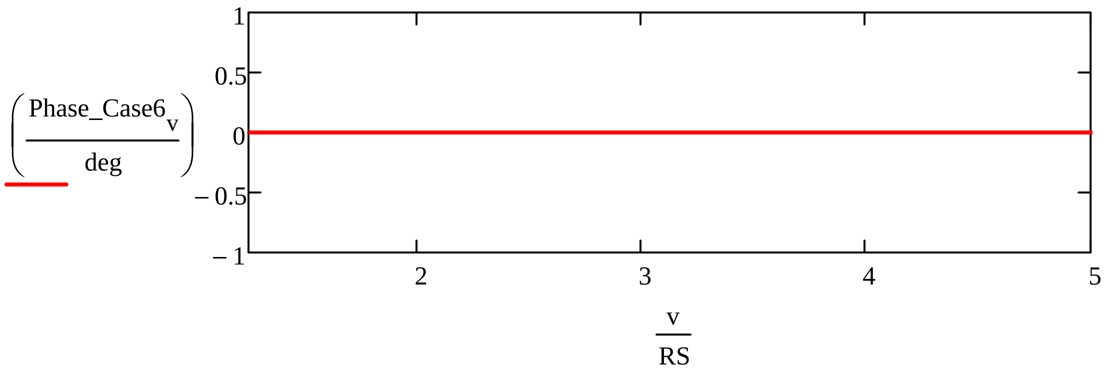


$$Phasor6_v := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas6}, v\right) + j\cdot COSF\left(RS, I_{meas6}, v - \frac{RS}{4}\right)\right)$$

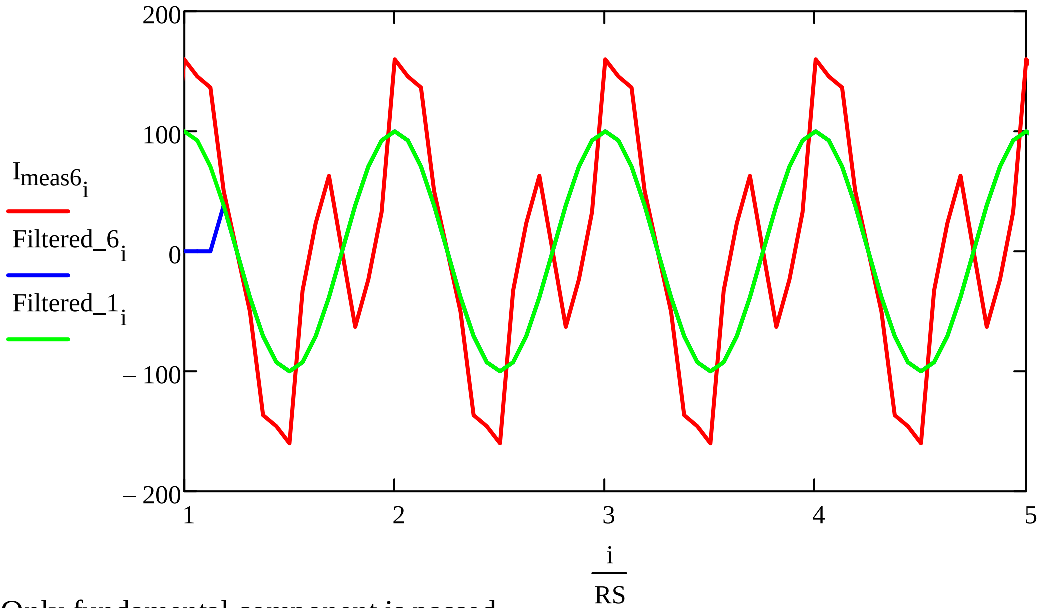
$$Mag6_v := \left|Phasor6_v\right| \qquad Angle6_v := \arg\left(Phasor6_v\right)$$



$$Phase\_Case6_v := \begin{cases} Angle6_v - Angle1_v & \text{if } \left|Angle6_v - Angle1_v\right| < \pi \\ Angle6_v - Angle1_v - 2\cdot\pi & \text{if } \left(Angle6_v - Angle1_v\right) > \pi \\ Angle6_v - Angle1_v + 2\cdot\pi & \text{if } Angle6_v - Angle1_v < -(\pi) \end{cases}$$



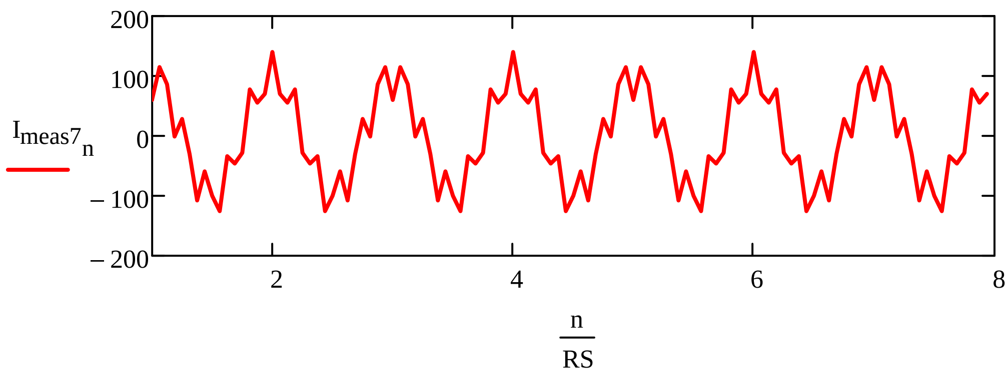
$$Filtered\_6_v := \sqrt{2}\cdot Mag6_v\cdot \cos\left[\left(\frac{2\cdot\pi\cdot v}{RS}\right) + Phase\_Case6_v\right]$$



Only fundamental component is passed.

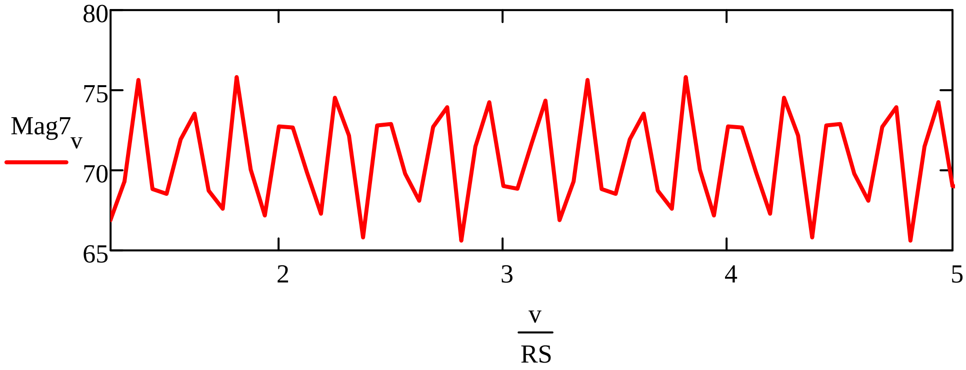
- Now, how about a non-interger harmonic

$$I_{meas7_n} := 100 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{RS}\right) + 40 \cdot \cos\left[5.5\left(\frac{2 \cdot \pi \cdot n}{RS}\right)\right]$$



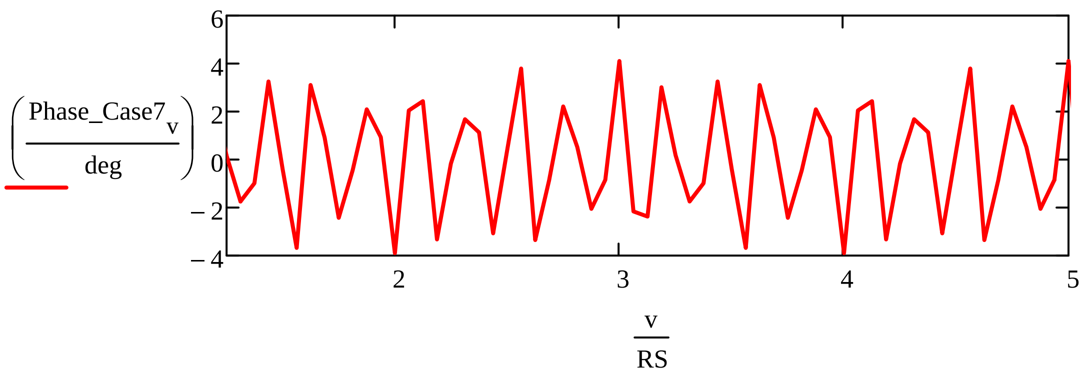
$$Phasor7_v := \frac{1}{\sqrt{2}}\left(COSF\left(RS, I_{meas7}, v\right) + j \cdot COSF\left(RS, I_{meas7}, v - \frac{RS}{4}\right)\right)$$

$Mag7_v := \left|Phasor7_v\right| \qquad Angle7_v := \arg\left(Phasor7_v\right)$

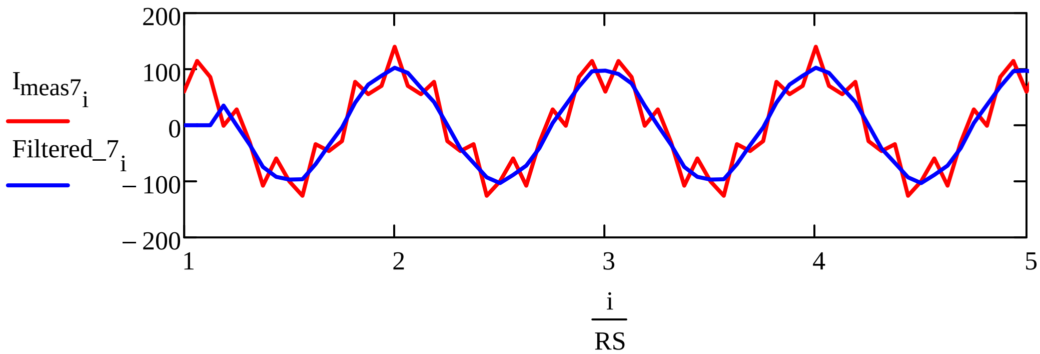


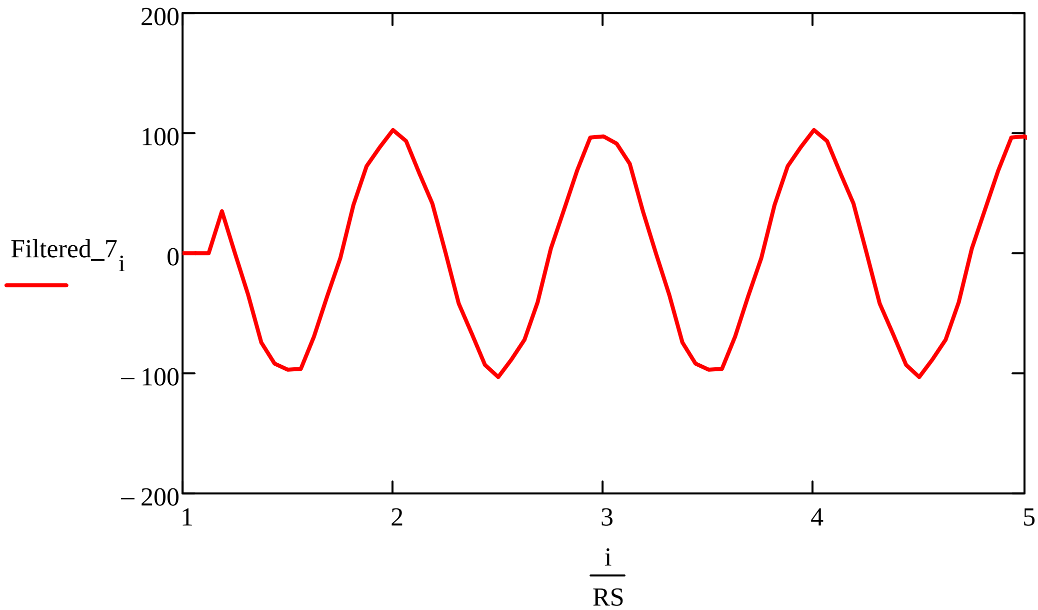
- Not so good. Filter doesn't have a gain of 0 at non-integer harmonics

$$Phase\_Case7_v := \begin{cases} Angle7_v - Angle1_v & \text{if } \left|Angle7_v - Angle1_v\right| < \pi \\ Angle7_v - Angle1_v - 2\cdot\pi & \text{if } \left(Angle7_v - Angle1_v\right) > \pi \\ Angle7_v - Angle1_v + 2\cdot\pi & \text{if } Angle7_v - Angle1_v < -(\pi) \end{cases}$$



$$Filtered\_7_v := \sqrt{2}\cdot Mag7_v\cdot\cos\left[\left(\frac{2\cdot\pi\cdot v}{RS}\right) + Phase\_Case7_v\right]$$





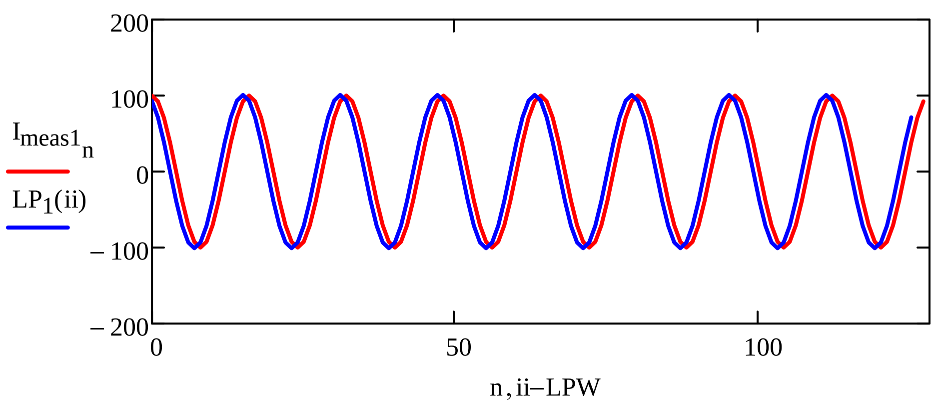
Distortion definitely attenuated, but not eliminated.

- A low pass filter is needed
- USe a simple filter for the moment

$$LPW := \mathrm{floor}\left(\frac{3}{60Hz\cdot\Delta t\cdot RS}\right) \qquad LPW = 3$$

$$ii := LPW\,..\,\mathrm{ceil}\left(\frac{CY}{60Hz\cdot\Delta t}\right)$$

$$LP_1(ii) := \left(1+\frac{1}{RS}\right)\cdot\sum_{k=0}^{LPW-1}\frac{I_{meas1_{ii-LPW+k}}}{LPW}$$



$$LP_7(ii) := \left(1 + \frac{1}{RS}\right) \cdot \sum_{k=0}^{LPW-1} \frac{I_{meas7_{ii-LPW+k}}}{LPW}$$