

Unbalanced Fault Analysis Examples in a Radial System

Define units: MVA := MW MVAR := MW pu := 1

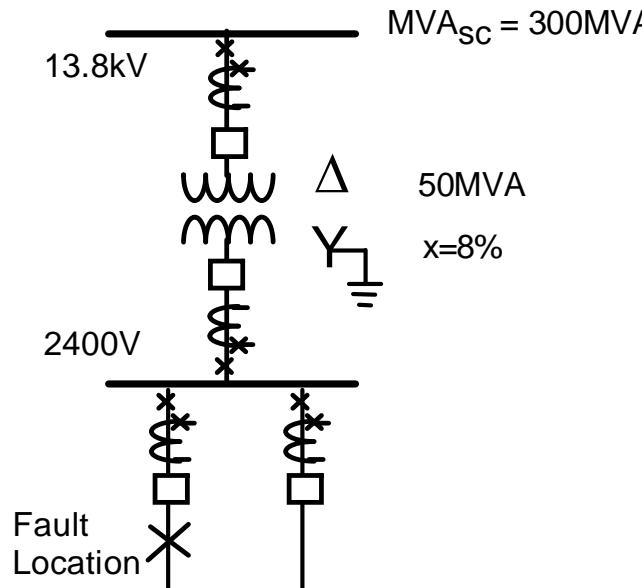
Define transformation

$$a := 1 \cdot e^{j \cdot 120\text{deg}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

- **Simple radial fault example**

Compare the per unit phase currents seen on the HV side of the transformer below with those seen by at the fault location for three phase, SLG and LL faults the left side feeder. Assume wye connected CT's.



Use transformer ratings as the per unit base.

$$V_{hi} := 13.8\text{kV}$$

$$V_{lo} := 2400\text{V}$$

$$X_{xfmr_pu} := 0.08\text{pu}$$

$$\text{MVArated} := 50\text{MVA}$$

$$S_B := 50\text{MVA}$$

$$\text{MVA}_{sc} := 300\text{MVA}$$

$$X_{src_pu} := \frac{(1.0\text{pu})^2}{\left(\frac{\text{MVA}_{sc}}{S_B}\right)}$$

$$X_{src_pu} = 0.167 \cdot \text{pu}$$

$$V_{src} := 1.0\text{pu}$$

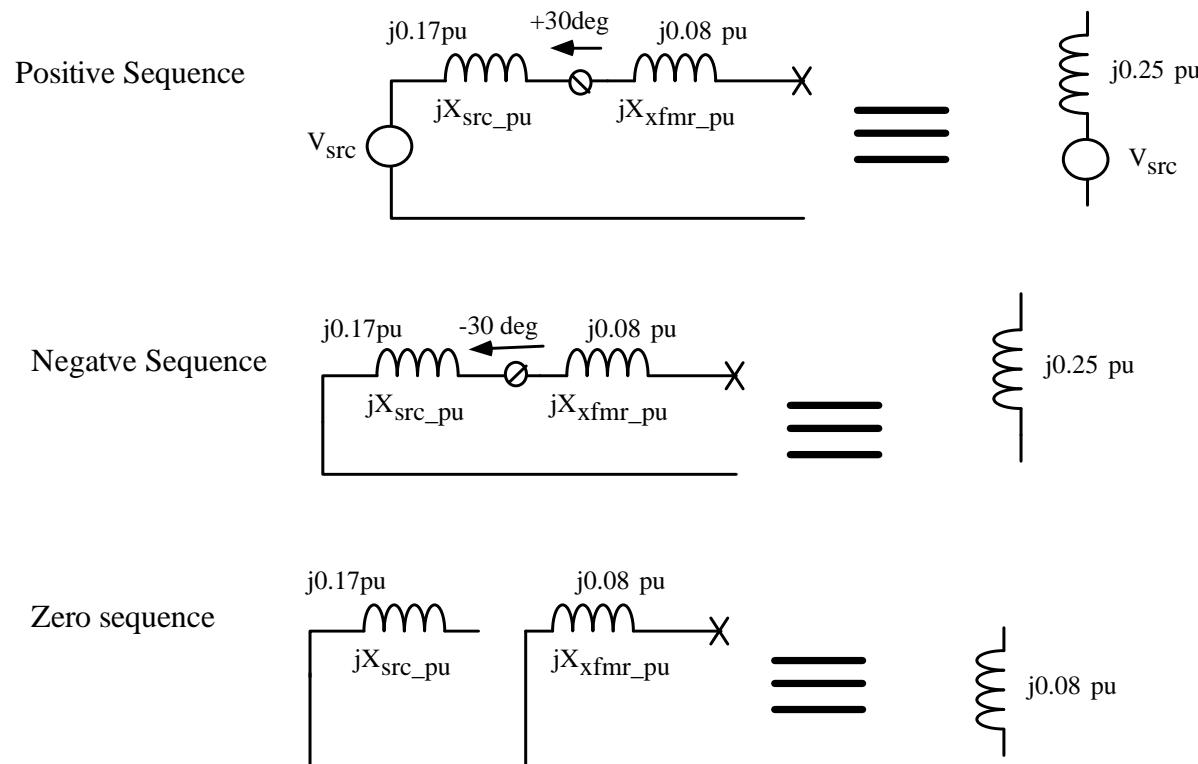
Sequence Impedances for Faults:

$$X_1 := X_{\text{src_pu}} + X_{\text{xfmr_pu}} \quad X_1 = 0.2467 \cdot \text{pu}$$

$$X_2 := X_1$$

$$X_0 := X_{\text{xfmr_pu}} \quad X_0 = 0.08 \cdot \text{pu}$$

Sequence Networks:



- Zbus matrices

$$Y_{B1} := \begin{pmatrix} \frac{1}{j \cdot X_{src_pu}} + \frac{1}{j \cdot X_{xfmr_pu}} & \frac{-1 \cdot e^{j \cdot 30\text{deg}}}{j \cdot X_{xfmr_pu}} \\ \frac{-1 \cdot e^{-j \cdot 30\text{deg}}}{j \cdot X_{xfmr_pu}} & \frac{1}{j \cdot X_{xfmr_pu}} \end{pmatrix} \quad Z_{B1} := Y_{B1}^{-1} \quad Z_{B1} = \begin{pmatrix} 0.1667i & -0.0833 + 0.1443i \\ 0.0833 + 0.1443i & 0.2467i \end{pmatrix} \cdot \text{pu}$$

$$Y_{B2} := \begin{pmatrix} \frac{1}{j \cdot X_{src_pu}} + \frac{1}{j \cdot X_{xfmr_pu}} & \frac{-1 \cdot e^{-j \cdot 30\text{deg}}}{j \cdot X_{xfmr_pu}} \\ \frac{-1 \cdot e^{j \cdot 30\text{deg}}}{j \cdot X_{xfmr_pu}} & \frac{1}{j \cdot X_{xfmr_pu}} \end{pmatrix} \quad Z_{B2} := Y_{B2}^{-1} \quad Z_{B2} = \begin{pmatrix} 0.1667i & 0.0833 + 0.1443i \\ -0.0833 + 0.1443i & 0.2467i \end{pmatrix} \cdot \text{pu}$$

$$Y_{B0} := \begin{pmatrix} \frac{1}{j \cdot X_{src_pu}} & 0 \\ 0 & \frac{1}{j \cdot X_{xfmr_pu}} \end{pmatrix} \quad Z_{B0} := Y_{B0}^{-1} \quad Z_{B0} = \begin{pmatrix} 0.1667i & 0 \\ 0 & 0.08i \end{pmatrix} \cdot \text{pu}$$

SLG fault on 2400V side of transformer:

$$I_{0_SLGf} := \frac{V_{src}}{j \cdot X_1 + j \cdot X_2 + j \cdot X_0} \quad I_{0_SLGf} = -1.7442i \cdot \text{pu}$$

$$I_{1_SLGf} := I_{0_SLGf} \quad I_{2_SLGf} := I_{0_SLGf}$$

$$I_{abc_SLGf} := A_{012} \cdot \begin{pmatrix} I_{0_SLGf} \\ I_{1_SLGf} \\ I_{2_SLGf} \end{pmatrix} \quad \overrightarrow{|I_{abc_SLGf}|} = \begin{pmatrix} 5.2326 \\ 0 \\ 0 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg(I_{abc_SLGf})} = \begin{pmatrix} -90 \\ 18.4349 \\ 18.4349 \end{pmatrix} \cdot deg$$

Now move the currents from the low side to the high side:

$$I_{1_SLGH} := I_{1_SLGf} \cdot e^{j \cdot 30deg} \quad I_{0_SLGH} := 0pu$$

$$I_{2_SLGH} := I_{2_SLGf} \cdot e^{-j \cdot 30deg}$$

$$I_{abc_SLGh} := A_{012} \cdot \begin{pmatrix} I_{0_SLGH} \\ I_{1_SLGH} \\ I_{2_SLGH} \end{pmatrix} \quad \overrightarrow{|I_{abc_SLGh}|} = \begin{pmatrix} 3.021 \\ 0 \\ 3.021 \end{pmatrix} \cdot pu$$

Currents are lower on the high side
and appear as a L-L fault not SLG

$$\overrightarrow{\arg(I_{abc_SLGh})} = \begin{pmatrix} -90 \\ 90 \\ 90 \end{pmatrix} \cdot deg$$

Voltage at the fault point:

$$V_{1_SLG} := V_{src} - j \cdot X_1 \cdot I_{1_SLGf} \quad |V_{1_SLG}| = 0.5698 \cdot pu \quad \arg(V_{1_SLG}) = 0 \cdot \text{deg}$$

$$V_{2_SLG} := 0 - j \cdot X_2 \cdot I_{2_SLGf} \quad |V_{2_SLG}| = 0.4302 \cdot pu \quad \arg(V_{2_SLG}) = 180 \cdot \text{deg}$$

$$V_{0_SLG} := 0 - j \cdot X_0 \cdot I_{0_SLGf} \quad |V_{0_SLG}| = 0.1395 \cdot pu \quad \arg(V_{0_SLG}) = 180 \cdot \text{deg}$$

$$V_{abc_SLG} := A_{012} \cdot \begin{pmatrix} V_{0_SLG} \\ V_{1_SLG} \\ V_{2_SLG} \end{pmatrix} \quad \overrightarrow{|V_{abc_SLG}|} = \begin{pmatrix} 0 \\ 0.891 \\ 0.891 \end{pmatrix} \cdot pu$$

$$\arg(V_{abc_SLG}_1) = -103.5868 \cdot \text{deg}$$

$$\arg(V_{abc_SLG}_2) = 103.5868 \cdot \text{deg}$$

- Now find the voltages on high side of transformer

$$V_{1_HV} := V_{src} \cdot e^{j \cdot 30\text{deg}} - j \cdot X_{src_pu} \cdot I_{1_SLGH} \quad |V_{1_HV}| = 0.7093 \cdot pu \quad \arg(V_{1_HV}) = 30 \cdot \text{deg}$$

$$V_{2_HV} := 0 - j \cdot X_{src_pu} \cdot I_{2_SLGH} \quad |V_{2_HV}| = 0.2907 \cdot pu \quad \arg(V_{2_HV}) = 150 \cdot \text{deg}$$

$$V_{0_HV} := 0$$

$$V_{ABC_HV} := A_{012} \cdot \begin{pmatrix} V_{0_HV} \\ V_{1_HV} \\ V_{2_HV} \end{pmatrix} \quad \overrightarrow{|V_{ABC_HV}|} = \begin{pmatrix} 0.6176 \\ 1 \\ 0.6176 \end{pmatrix} \cdot pu \quad \arg(V_{ABC_HV}) = \begin{pmatrix} 54.0562 \\ -90 \\ 125.9438 \end{pmatrix} \cdot deg$$

- Repeat using Zbus

$$I_{0_SLG} := \frac{V_{src}}{Z_{B1_{1,1}} + Z_{B2_{1,1}} + Z_{B0_{1,1}}} \quad |I_{0_SLG}| = 1.7442 \quad \arg(I_{0_SLG}) = -90 \cdot deg$$

$$I_{1_SLG} := I_{0_SLG} \quad I_{2_SLG} := I_{0_SLG}$$

$$I_{ABC_SLG} := A_{012} \cdot \begin{pmatrix} I_{0_SLG} \\ I_{1_SLG} \\ I_{2_SLG} \end{pmatrix} \quad \overrightarrow{|I_{ABC_SLG}|} = \begin{pmatrix} 5.2326 \\ 0 \\ 0 \end{pmatrix} \cdot pu \quad \overrightarrow{\arg(I_{ABC_SLG})} = \begin{pmatrix} -90 \\ 18.4349 \\ 18.4349 \end{pmatrix} \cdot deg$$

- Angles meaningless when magnitude is 0
- Now find voltages (again, in each sequence component)

$$\Delta V1 := Z_{B1} \cdot \begin{pmatrix} 0 \\ -I_{1_SLG} \end{pmatrix} \quad \overrightarrow{|\Delta V1|} = \begin{pmatrix} 0.2907 \\ 0.4302 \end{pmatrix} \quad \overrightarrow{\arg(\Delta V1)} = \begin{pmatrix} -150 \\ -180 \end{pmatrix} \cdot deg$$

- Now when we find V1 we need to *include the transformer phase shift* in the prefault voltages

$$V1 := \begin{pmatrix} 1.0e^{j \cdot 30\text{deg}} \\ 1.0 \end{pmatrix} + \Delta V1 \quad \overrightarrow{|V1|} = \begin{pmatrix} 0.7093 \\ 0.5698 \end{pmatrix} \quad \overrightarrow{\arg(V1)} = \begin{pmatrix} 30 \\ 0 \end{pmatrix} \cdot \text{deg}$$

$$\Delta V2 := Z_{B2} \cdot \begin{pmatrix} 0 \\ -I_{2_SLG} \end{pmatrix} \quad \overrightarrow{|\Delta V2|} = \begin{pmatrix} 0.2907 \\ 0.4302 \end{pmatrix} \quad \overrightarrow{\arg(\Delta V2)} = \begin{pmatrix} 150 \\ 180 \end{pmatrix} \cdot \text{deg} \quad \text{Note the } -30 \text{ degree shift...}$$

$$V2 := \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \Delta V2 \quad \bullet \quad \text{No prefault voltage, so enter 0.}$$

$$\Delta V0 := Z_{B0} \cdot \begin{pmatrix} 0 \\ -I_{0_SLG} \end{pmatrix} \quad \Delta V0 = \begin{pmatrix} 0 \\ -0.1395 \end{pmatrix} \quad V0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \Delta V0 \quad \text{Again, no prefault voltage}$$

- ABC Voltages at 2.4kV Bus

$$V_{ABC_2400} := A_{012} \cdot \begin{pmatrix} V0_1 \\ V1_1 \\ V2_1 \end{pmatrix} \quad \overrightarrow{|V_{ABC_2400}|} = \begin{pmatrix} 0 \\ 0.891 \\ 0.891 \end{pmatrix} \cdot \text{pu} \quad \arg(V_{ABC_2400}) = -103.5868 \cdot \text{deg}$$

$$\arg(V_{ABC_2400}) = 103.5868 \cdot \text{deg}$$

- ABC Voltages at 13.8kV bus

$$V_{ABC_13.8kV} := A_{012} \cdot \begin{pmatrix} V0_0 \\ V1_0 \\ V2_0 \end{pmatrix} \quad \overrightarrow{|V_{ABC_13.8kV}|} = \begin{pmatrix} 0.6176 \\ 1 \\ 0.6176 \end{pmatrix} \cdot \text{pu} \quad \arg(V_{ABC_13.8kV}) = \begin{pmatrix} 54.0562 \\ -90 \\ 125.9438 \end{pmatrix} \cdot \text{deg}$$

- Branch currents

- HV side of transformer (note the phase shifts in the voltages):

$$I_{1_HV} := \frac{V_{10} - V_{11} e^{j \cdot 30\text{deg}}}{j \cdot X_{xfmr_pu}} \quad |I_{1_HV}| = 1.7442 \quad \arg(I_{1_HV}) = -60\text{-deg}$$

$$I_{2_HV} := \frac{V_{20} - V_{21} e^{-j \cdot 30\text{deg}}}{j \cdot X_{xfmr_pu}} \quad |I_{2_HV}| = 1.7442 \quad \arg(I_{2_HV}) = -120\text{-deg}$$

$$I_{0_HV} := 0$$

$$I_{ABC_HV} := A_{012} \cdot \begin{pmatrix} I_{0_HV} \\ I_{1_HV} \\ I_{2_HV} \end{pmatrix} \quad \overrightarrow{|I_{ABC_HV}|} = \begin{pmatrix} 3.021 \\ 0 \\ 3.021 \end{pmatrix} \cdot \text{pu} \quad \arg(I_{ABC_HV}) = \begin{pmatrix} -90 \\ 0 \\ 90 \end{pmatrix} \cdot \text{deg}$$

- LV side of transformer (note the phase shifts in the voltages):

$$I_{1_LV} := \frac{V_{10} e^{-j \cdot 30\text{deg}} - V_{11}}{j \cdot X_{xfmr_pu}} \quad |I_{1_LV}| = 1.7442 \quad \arg(I_{1_LV}) = -90\text{-deg}$$

$$I_{2_LV} := \frac{V_{20} e^{j \cdot 30\text{deg}} - V_{21}}{j \cdot X_{xfmr_pu}} \quad |I_{2_LV}| = 1.7442 \quad \arg(I_{2_LV}) = -90\text{-deg}$$

$$I_{0_LV} := \frac{0 - V_{01}}{(j \cdot X_{xfmr_pu})} \quad |I_{0_LV}| = 1.7442 \quad \arg(I_{0_LV}) = -90\text{-deg}$$

$$I_{ABC_LV} := A_{012} \cdot \begin{pmatrix} I_{0_LV} \\ I_{1_LV} \\ I_{2_LV} \end{pmatrix} \quad \overrightarrow{|I_{ABC_LV}|} = \begin{pmatrix} 5.2326 \\ 0 \\ 0 \end{pmatrix} \cdot pu \quad \arg(I_{ABC_LV}) = \begin{pmatrix} -90 \\ -51.5793 \\ 144.2147 \end{pmatrix} \cdot deg$$

Line to Line fault:

$$I_{0_LLf} := 0 \cdot pu$$

$$I_{1_LLf} := \frac{V_{src}}{j \cdot X_1 + j \cdot X_2} \quad I_{1_LLf} = -2.027i \cdot pu$$

$$I_{2_LLf} := -I_{1_LLf}$$

$$I_{abc_LLf} := A_{012} \cdot \begin{pmatrix} I_{0_LLf} \\ I_{1_LLf} \\ I_{2_LLf} \end{pmatrix} \quad \overrightarrow{|I_{abc_LLf}|} = \begin{pmatrix} 0 \\ 3.5109 \\ 3.5109 \end{pmatrix} \cdot pu$$

$$\arg(I_{abc_LLf}_1) = 180 \cdot deg$$

$$\arg(I_{abc_LLf}_2) = 0 \cdot deg$$

Now move the currents from the low side to the high side (note, you must transform the sequence currents):

$$I_{1_LLH} := I_{1_LLf} \cdot e^{j \cdot 30\text{deg}} \quad I_{0_LLH} := 0 \text{pu}$$

$$I_{2_LLH} := I_{2_LLf} \cdot e^{-j \cdot 30\text{deg}}$$

$$I_{abc_LLh} := A_{012} \cdot \begin{pmatrix} I_{0_LLH} \\ I_{1_LLH} \\ I_{2_LLH} \end{pmatrix} \quad \overrightarrow{|I_{abc_LLh}|} = \begin{pmatrix} 2.027 \\ 4.0541 \\ 2.027 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(I_{abc_LLh})} = \begin{pmatrix} 0 \\ 180 \\ 0 \end{pmatrix} \cdot \text{deg}$$

Phase B per unit current is larger on the high side of the transformer than on the low side, use caution with coordination

Voltage at the fault point:

$$V_{1_LL} := V_{src} - j \cdot X_1 \cdot I_{1_LLf} \quad |V_{1_LL}| = 0.5 \cdot \text{pu} \quad \arg(V_{1_LL}) = 0 \cdot \text{deg}$$

$$V_{2_LL} := 0 - j \cdot X_2 \cdot I_{2_LLf} \quad |V_{2_LL}| = 0.5 \cdot \text{pu} \quad \arg(V_{2_LL}) = 0 \cdot \text{deg}$$

$$V_{0_LL} := 0 - j \cdot X_0 \cdot I_{0_LLf} \quad |V_{0_LL}| = 0 \cdot \text{pu}$$

$$V_{abc_LL} := A_{012} \cdot \begin{pmatrix} V_{0_LL} \\ V_{1_LL} \\ V_{2_LL} \end{pmatrix} \quad \overrightarrow{|V_{abc_LL}|} = \begin{pmatrix} 1 \\ 0.5 \\ 0.5 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(V_{abc_LL})} = \begin{pmatrix} 0 \\ 180 \\ 180 \end{pmatrix} \cdot \text{deg}$$

Double Line to Ground fault:

$$I_{1_DLGf} := \frac{V_{src}}{j \cdot X_1 + \left(\frac{1}{j \cdot X_2} + \frac{1}{j \cdot X_0} \right)^{-1}} \quad I_{1_DLGf} = -3.2565i \cdot \text{pu}$$

$$I_{2_DLGf} := -I_{1_DLGf} \cdot \left(\frac{j \cdot X_0}{j \cdot X_2 + j \cdot X_0} \right) \quad I_{2_DLGf} = 0.7975i \cdot \text{pu}$$

$$I_{0_DLGf} := -I_{1_DLGf} \cdot \left(\frac{j \cdot X_2}{j \cdot X_2 + j \cdot X_0} \right) \quad I_{0_DLGf} = 2.459i \cdot \text{pu}$$

$$I_{abc_DLGf} := A_{012} \cdot \begin{pmatrix} I_{0_DLGf} \\ I_{1_DLGf} \\ I_{2_DLGf} \end{pmatrix} \quad \overrightarrow{|I_{abc_DLGf}|} = \begin{pmatrix} 0 \\ 5.0923 \\ 5.0923 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(I_{abc_DLGf})} = \begin{pmatrix} -90 \\ 133.5868 \\ 46.4132 \end{pmatrix} \cdot \text{deg}$$

Now move the currents from the low side to the high side:

$$I_{1_DLGH} := I_{1_DLGf} \cdot e^{j \cdot 30\text{deg}} \quad I_{0_DLGH} := 0 \text{pu}$$

$$I_{2_DLGH} := I_{2_DLGf} \cdot e^{-j \cdot 30\text{deg}}$$

$$I_{abc_DLGh} := A_{012} \cdot \begin{pmatrix} I_{0_DLGH} \\ I_{1_DLGH} \\ I_{2_DLGH} \end{pmatrix} \xrightarrow{\quad} = \begin{pmatrix} 2.9401 \\ 4.0541 \\ 2.9401 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(I_{abc_DLGh})} = \begin{pmatrix} -46.4132 \\ 180 \\ 46.4132 \end{pmatrix} \cdot \text{deg}$$

Voltage at the fault point:

$$V_{1_DLG} := V_{src} - j \cdot X_1 \cdot I_{1_DLGf} \quad |V_{1_DLG}| = 0.1967 \cdot \text{pu} \quad \arg(V_{1_DLG}) = 0 \cdot \text{deg}$$

$$V_{2_DLG} := 0 - j \cdot X_2 \cdot I_{2_DLGf} \quad |V_{2_DLG}| = 0.1967 \cdot \text{pu} \quad \arg(V_{2_DLG}) = 0 \cdot \text{deg}$$

$$V_{0_DLG} := 0 - j \cdot X_0 \cdot I_{0_DLGf} \quad |V_{0_DLG}| = 0.1967 \cdot \text{pu} \quad \arg(V_{0_DLG}) = 0 \cdot \text{deg}$$

$$V_{abc_DLG} := A_{012} \cdot \begin{pmatrix} V_{0_DLG} \\ V_{1_DLG} \\ V_{2_DLG} \end{pmatrix} \xrightarrow{\quad} = \begin{pmatrix} 0.5902 \\ 0 \\ 0 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(V_{abc_DLG})} = \begin{pmatrix} 0 \\ -165.9638 \\ 123.6901 \end{pmatrix} \cdot \text{deg}$$

- Add fault resistance - both R_f and R_g

$$I_{1_DLG}(R_f, R_g) := \frac{V_{src}}{(R_f + j \cdot X_1) + \left[\frac{1}{(R_f + j \cdot X_2)} + \frac{1}{(R_f + 3 \cdot R_g + j \cdot X_0)} \right]^{-1}}$$

$$I_{1_DLG}(0,0) = -3.2565i \cdot \text{pu}$$

$$I_{2_DLG}(R_f, R_g) := -I_{1_DLG}(R_f, R_g) \cdot \left[\frac{R_f + 3R_g + j \cdot X_0}{(R_f + j \cdot X_2) + (R_f + 3 \cdot R_g + j \cdot X_0)} \right]$$

$$I_{2_DLG}(0,0) = 0.7975i \cdot \text{pu}$$

$$I_{0_DLG}(R_f, R_g) := -I_{1_DLG}(R_f, R_g) \cdot \left[\frac{R_f + j \cdot X_2}{(R_f + j \cdot X_2) + (R_f + 3 \cdot R_g + j \cdot X_0)} \right]$$

$$I_{0_DLG}(0,0) = 2.459i \cdot \text{pu}$$

Now move the currents from the low side to the high side:

$$I_{1_DLGh}(R_f, R_g) := I_{1_DLG}(R_f, R_g) \cdot e^{j \cdot 30\text{deg}}$$

$$I_{2_DLGh}(R_f, R_g) := I_{2_DLG}(R_f, R_g) \cdot e^{-j \cdot 30\text{deg}}$$

$$I_{0_DLGh} := 0 \text{pu}$$

Voltage at the fault point:

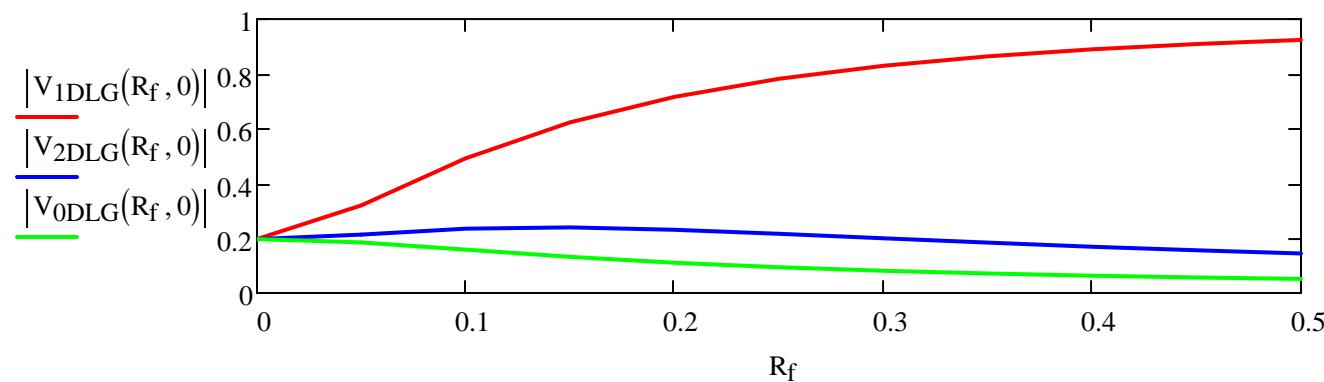
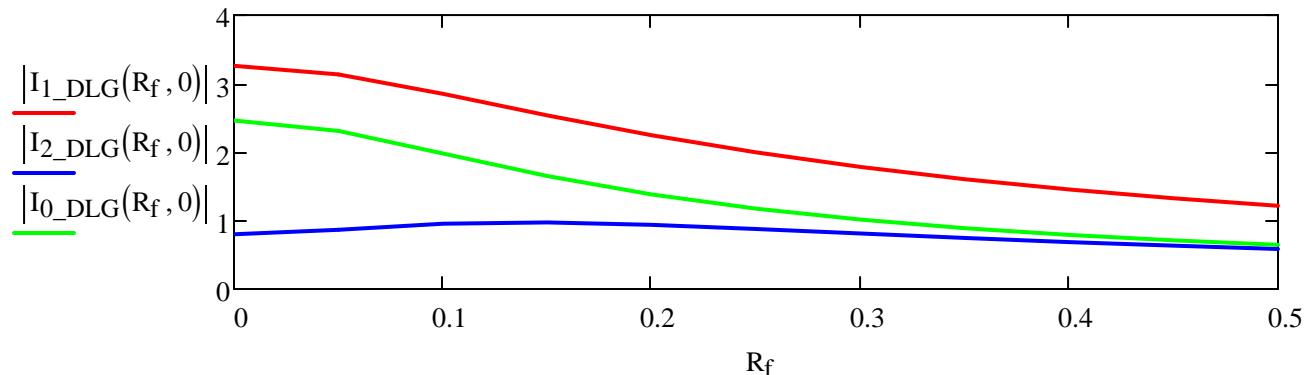
$$V_{1DLG}(R_f, R_g) := V_{src} - j \cdot X_1 \cdot I_{1_DLG}(R_f, R_g)$$

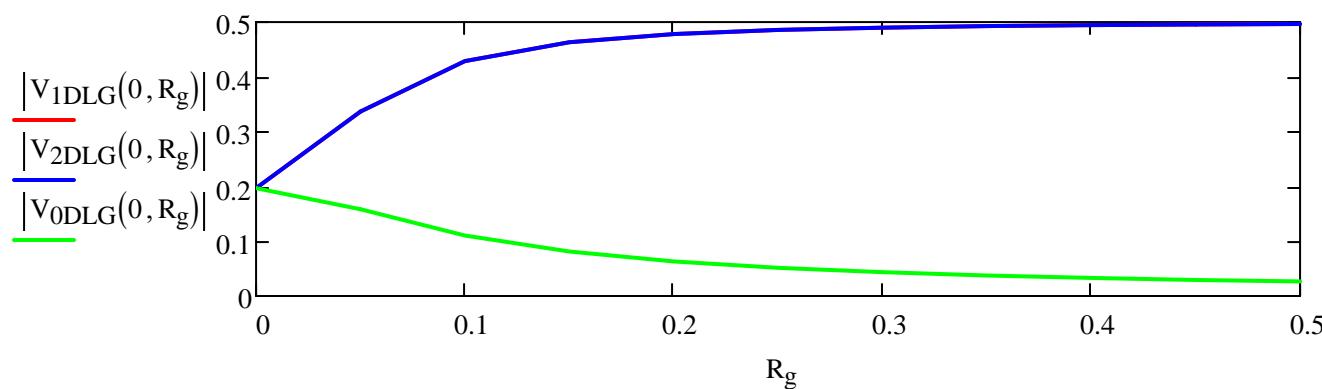
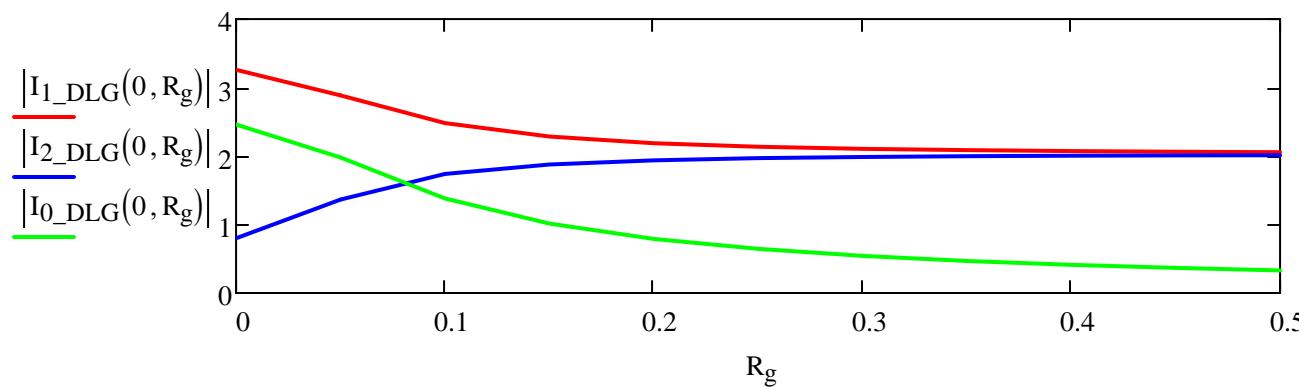
$$V_{2DLG}(R_f, R_g) := 0 - j \cdot X_2 \cdot I_{2_DLG}(R_f, R_g)$$

$$V_{0DLG}(R_f, R_g) := 0 - j \cdot X_0 \cdot I_{0_DLG}(R_f, R_g)$$

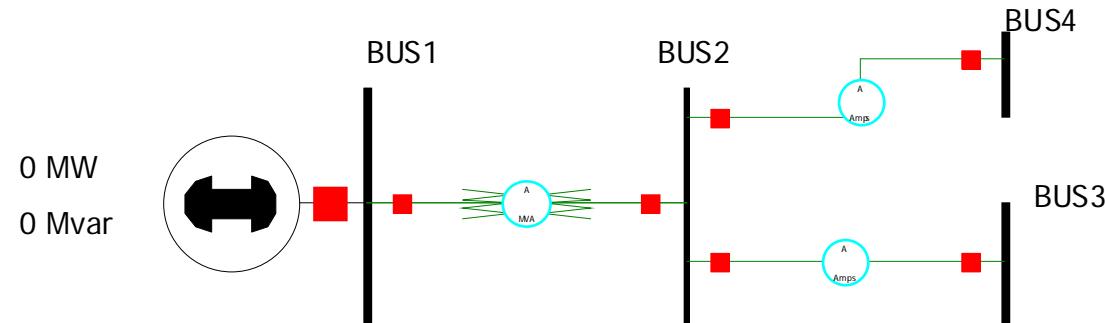
$$R_f := 0, 0.05 .. 0.5$$

$$R_g := 0, 0.05 .. 0.5$$





- Fault program results-- using Powerworld for example



Notes:

1. Slack bus angle set to +30 degree so BUS2 is at 0 degrees prefault
2. System MVA base set to 50MVA instead of 100MVA
3. No load on the system
4. Place the faults at 0% of the way on the line from BUS2 to BUS 3
5. Powerworld is set up to display phase currents, not the sequence currents