Apparatus Models: Transformers

- Normally model as series impedance from winding resistance and leakage reactance
- Positive and negative impedances equal
- In a Y-Δ transformer that phase shift is in the opposite direction for negative sequence

Transformers (continued)

- Zero sequence impedance of the transformer depends on core construction
- \( X_0 = X_1 \) for single phase cores
- \( X_0 = X_1 \) for 5 leg or shell type core
  - Both have similar path for zero sequence current
### Three Leg Cores

- Three leg core $3\Omega$ transformers have $X_0 < X_1$
- Zero sequence flux forced out of core
- Excitation branch becomes significant
- This leakage flux travels through the oil and transformer tank.
- Oil and tank steel have a high reluctance and low inductance producing the low exciting branch inductance

### Transformer Connections

- Zero sequence circuit impacted $Y$ or $\Delta$
- Also impacted by grounding
- Tertiary also complicates connection
Transformer Connections

More Transformer Connections
Per Unit Equivalents

- Positive and negative sequence equivalent for Y-Y, Δ–Δ, (and Y-Δ, Δ-Y when phase shift isn't a concern):

\[ R + jX_l \]

\[ V_H \] \[ V_X \]

- Positive and Negative Sequence for Y-Δ, Δ-Y, ANSI phase shift

\[ R + jX_l \]

Pos Seq: \( e^{j30\text{deg}} \)
Neg Seq: \( e^{+j30\text{deg}} \)

\[ V_H \] \[ V_X \]
General Zero Sequence
Yg - Yg

\[ V_H - V_X \]

\[ R_0 + jX_0 \]

\[ 3*Z_{grH} \quad 3*Z_{grX} \]

Zero sequence Yg-Δ

\[ V_H - V_X \]

\[ R_0 + jX_0 \]

\[ 3*Z_{grH} \]
**Zero sequence Δ-Δ**

- \[ R_0 + jX_0 \]

- \[ V_H \]

- \[ V_X \]

**Transformers with Tertiaries**

- \( Z_1 = Z_2 \) - phase rotation doesn’t impact
- Low voltage winding often closed delta
- Zero sequence trap
- May float or ground corners through impedance
Transformers with Tertiaries

- See Fig A4.2.3 in Blackburn’s relaying book for more configurations

Per Unit Equivalents (solid grounding)
Calculating $Z_h$, $Z_m$ and $Z_l$

- Often given short circuit test data as $Z_{hm}$, $Z_{hl}$ and $Z_{ml}$
- Not all on same per unit base, so first do change of base

\[
X_h := \frac{1}{2} \left( X_{hl} + X_{hm} - X_{ml} \right)
\]
\[
X_m := \frac{1}{2} \left( X_{hm} + X_{ml} - X_{hl} \right)
\]
\[
X_l := \frac{1}{2} \left( X_{ml} + X_{hl} - X_{hm} \right)
\]

Component Modeling: Lines

- $Z_1=Z_2$ - phase rotation doesn’t impact
- Often approximated with per phase equivalent self impedance
- Zero sequence current flows through earth and $Z_0$ often 2-6 times $Z_1$
- Usually neglect capacitances unless transient case
  » Transient response matters for fast detection
Component Modeling: Lines

- Need to do detailed line constants calculation to get impedances accurately
- Form impedance matrix and then transform with Symmetrical Components Transformation matrix $Z_s = (A^{-1} \times Z \times A)$

Generators

- Positive sequence reactance broken into subtransient, transient and steady-state
- Direct vs Quadrature axis
  - Fault currents typically near zero power factor
  - D-axis only
- Subtransient or transient reactance based on time scale of interest
Generators: Negative Sequence

- MMF rotates backwards at twice the machines synchronous speed
- Double frequency currents in the rotor field and amortisseur windings
- Flux peak occurs along the d and q-axes
- $X_2 \approx 0.5(X_{d''} + X_{q''})$