

## Directional Relay Model

The MathCAD sheet below implements some basic relay calculations. The file takes data read from a Comtrade file and postprocesses it.

The matrix "data" below is the data captured from a COMTRADE "\*.dat" file. To read in a data file remove the table currently at the top of the file. Then choose "Insert" ---> "Component".

- This will open a dialog box. **One option** is to choose "Input Table".
  - \* Then select the first cell in the table and right click your mouse and choose "Import".
  - \* Then browse to the "\*.dat" COMTRADE file and select. This will fill in the data in the table. Then name the variable as "data"
- Another option is to choose "File Read or Write".
  - \* This will open a dialog box, choose Text file
  - \* Browse for file with extension .txt or .csv.
  - \* Your assignments will tell you which files to open.
- The example below uses the File Read or Write option.

### Read Comtrade File Data

#### 1. Read Comtrade Configuration File:

config :=

..\FSLG83.cfc

Right click on the floppy disk icon and select "Properties", then "Browse" to open file browser. Choose the \*.cfg file from the contrade file (you will need to type the extension)

data :=

..\FSLG83.dat

Right click on the floppy disk icon and select "Properties", the "Browse" to open a file browser. Choose the \*.dat file from the contrade file (it should be an accepted file type)

The COMTRADE file is actually threefiles. One has an extension "\*.hdr". This file will be empty in this case. Another has the extension "\*.dat". This has the actual numerical data in columns of numbers. The third file is a configuration file and has the extension "\*.cfg" and this tells the program reading the numerical data what the columns represent. The configuration file provides scaling and offset information for each of the variables stored as vectors. Here is a typical entry:

```
1 , IN , , , A , 2 . 5162E-04 , 1 . 0045E00 , 0 . 0000E00 , -32765 , 32765 , 1 , 1 , P
```

Each data record starts entry number (1-7 here), the name for the measurements (for example "IN").

The number after the 4 commas (column 5 starting numbering with 0) is a scale factor. The next number (column 6) is an offset factor. If you don't change the scaling and offset factors, the waveforms you evaluate won't be correct. The MathCAD sheet has further instructions.

COMTRADE configuration file format:

1. The first row states how the file was created and the version of the standard
2. The second row gives the total number of inputs (7 for these cases), number of analog inputs (7 here) and number of digital inputs (0 here)
3. Rows 3 - 10 are the analog inputs, in the following order:
  - In (referred to as residual current below)
  - Ia
  - Ib
  - Ic
  - Van
  - Vbn
  - Vcn
4. Data sampled 16 times per cycle (960 Hz)

### Scaling and Offset Data from Comtrade CFG file

- Read individual cells (scale from column 5, offset from column 6)
- Remember row and column indices start at 0:

$I_{r\_scale} := \text{config}_{2,5}$	$I_{r\_scale} = 2.5162 \times 10^{-4}$	$I_{r\_offset} := \text{config}_{2,6}$	$I_{r\_offset} = 1.0045$
$I_{a\_scale} := \text{config}_{3,5}$	$I_{a\_scale} = 2.5216 \times 10^{-4}$	$I_{a\_offset} := \text{config}_{3,6}$	$I_{a\_offset} = -1.0116$
$I_{b\_scale} := \text{config}_{4,5}$	$I_{b\_scale} = 3.4249 \times 10^{-5}$	$I_{b\_offset} := \text{config}_{4,6}$	$I_{b\_offset} = 0.01$
$I_{c\_scale} := \text{config}_{5,5}$	$I_{c\_scale} = 3.4478 \times 10^{-5}$	$I_{c\_offset} := \text{config}_{5,6}$	$I_{c\_offset} = -2.2585 \times 10^{-3}$
$V_{a\_scale} := \text{config}_{6,5}$	$V_{a\_scale} = 5.4238 \times 10^{-3}$	$V_{a\_offset} := \text{config}_{6,6}$	$V_{a\_offset} = -7.407$
$V_{b\_scale} := \text{config}_{7,5}$	$V_{b\_scale} = 5.4189 \times 10^{-3}$	$V_{b\_offset} := \text{config}_{7,6}$	$V_{b\_offset} = 4.4371$
$V_{c\_scale} := \text{config}_{8,5}$	$V_{c\_scale} = 5.2139 \times 10^{-3}$	$V_{c\_offset} := \text{config}_{8,6}$	$V_{c\_offset} = -0.031$

Enter Constants. Note that RS is the sampling rate, and the value of 16 here is assuming that the COMTRADE file was sampled at that rate.

$$a := 1 \cdot e^{j \cdot 120 \text{deg}}$$

Phase A symmetrical  
components  
transform

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$RS := 16$$

Phase B symmetrical  
components  
transform

$$B_{012} := \begin{pmatrix} 1 & a & a^2 \\ 1 & 1 & 1 \\ 1 & a^2 & a \end{pmatrix}$$

Phase C symmetrical  
components  
transform

$$C_{012} := \begin{pmatrix} 1 & a^2 & a \\ 1 & a & a^2 \\ 1 & 1 & 1 \end{pmatrix}$$

Enter vector indices for filter and relay calculations (do not change these)

$$i := 0 \dots \text{rows}(\text{data}) - 1$$

$$v := \frac{RS}{4} \dots \text{rows}(\text{data}) - 1$$

Offset samples by 1/4 cycles for phasor calculation

$$d := 5 \cdot RS \dots \text{rows}(\text{data}) - 1$$

Offset samples by 5 cycles for trip calculation

$$If := RS - 1 \dots \text{rows}(\text{data}) - 1$$

### User Entered Parameters:

- I am entering typical values the current transformer ration (CTR) and voltage transformer ratio (PTR). You need to change these to match your calculations.

$$CTR := 1 \quad PTR := 1$$

- Relay Settings

*Moved later in the file close to the relay model and plots.*

### Read Data From Comtrade File:

- The data from the COMTRADE file is now read into vectors for MathCAD to use.
- Note that these vectors are assuming that IA, IB, etc are in certain columns in the table. The numbers below assume that the data is assigned as described in the other handout. Columns 0 and 1 of the COMTRADE file do not store data, so Column 2 is the first one of interest.
- Since the neutral current  $I_N$  was measured, that will be used for the residual current ( $IR = 3I_0$ ).

$$IR := \left( \frac{\text{data}^{(2)}}{CTR} \right) \cdot Ir_{scale} + Ir_{offset}$$

$$VA := \left( \frac{\text{data}^{(6)}}{PTR} \right) \cdot Va_{scale} + Va_{offset}$$

$$IA := \left( \frac{\text{data}^{(3)}}{CTR} \right) \cdot Ia_{scale} + Ia_{offset}$$

$$VB := \left( \frac{\text{data}^{(7)}}{PTR} \right) \cdot Vb_{scale} + Vb_{offset}$$

$$IB := \left( \frac{\text{data}^{(4)}}{CTR} \right) \cdot Ib_{scale} + Ib_{offset}$$

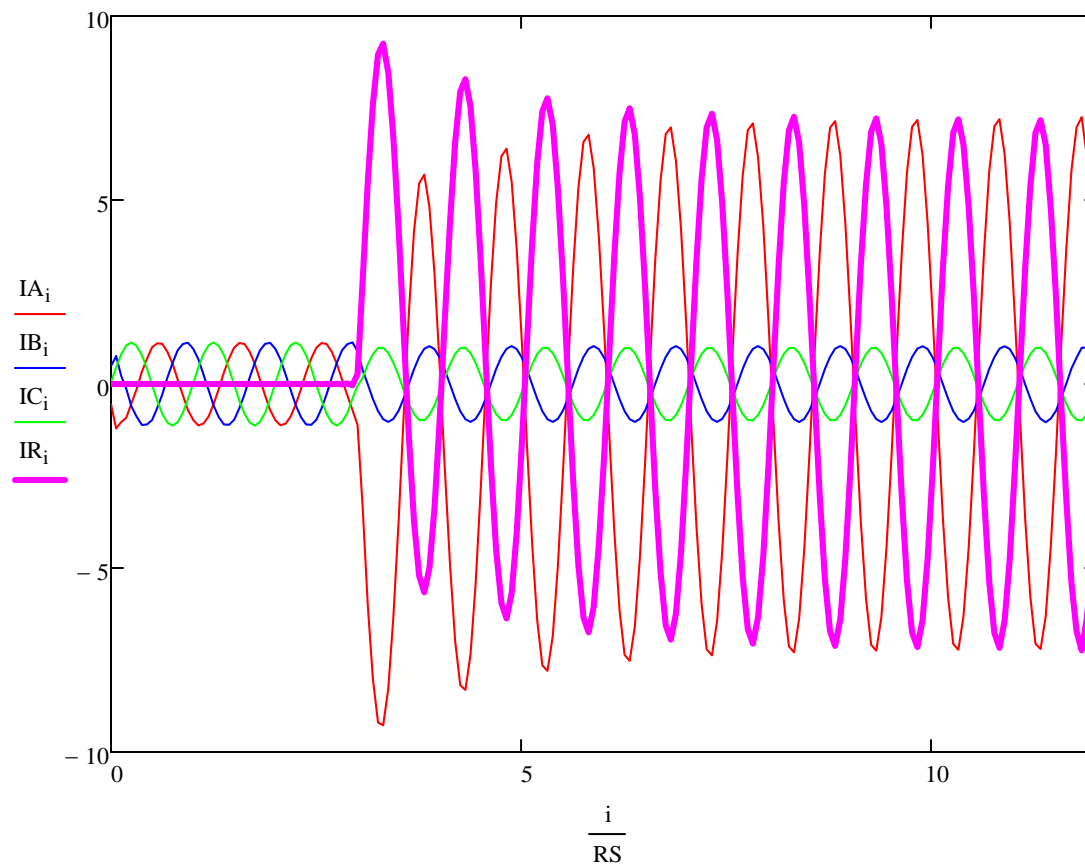
$$VC := \left( \frac{\text{data}^{(8)}}{PTR} \right) \cdot Vc_{scale} + Vc_{offset}$$

$$IC := \left( \frac{\text{data}^{(5)}}{CTR} \right) \cdot Ic_{scale} + Ic_{offset}$$

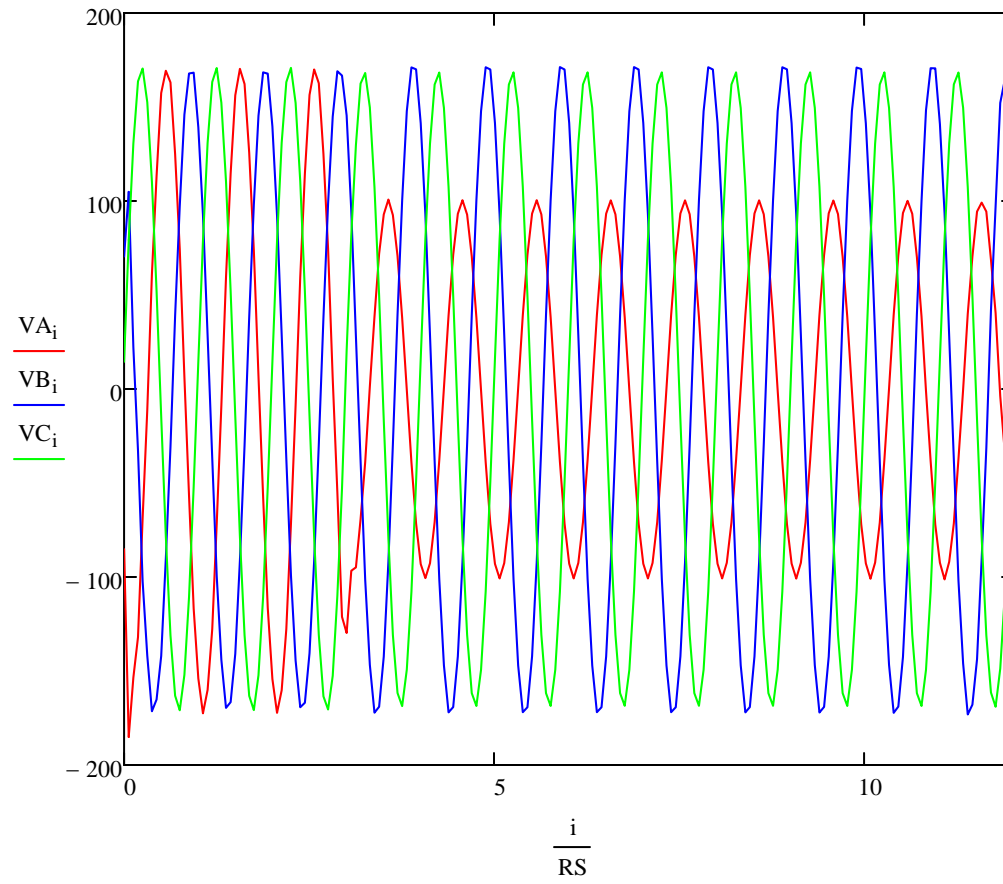
Now plot the currents and voltages. These should be sinusoidal. Note that the horizontal axis is in the number of cycles since the  $i/RS$  is sample number divided by sampling rate.

- As a check, the pre-fault data should be balanced three phase

Phase currents and residual current



Line to ground voltages:



### Digital Filter Stages:

- Now the data is run through a digital cosine filter as a first step in calculating magnitude and phase. Normally the first step would be to perform a low pass filtering operation on the data, but since ATP Analyzer already did this, we can skip that step.

$$I_{a_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot IA_{[If-(RS-1)]+k} \right]$$

$$V_{a_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot VA_{[If-(RS-1)]+k} \right]$$

$$I_{b_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot IB_{[If-(RS-1)]+k} \right]$$

$$V_{b_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot VB_{[If-(RS-1)]+k} \right]$$

$$I_{c_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot IC_{[If-(RS-1)]+k} \right]$$

$$V_{c_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot VC_{[If-(RS-1)]+k} \right]$$

$$I_{r_{If}} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[ \cos\left(\frac{2 \cdot \pi \cdot k}{RS}\right) \cdot IR_{[If-(RS-1)]+k} \right]$$



**Now create phasors.**

- To create a phasor we need a real and an imaginary part. If one uses Euler's identity, we need:  $\cos(A) + j \sin(A)$ . The cosine filter output gives us the cosine part. Rather than implementing a sine filter (which doesn't reject decaying DC offsets), create the sine term by delaying the cosine terms by 90 degrees (1/4 cycle-- or 4 samples when sampling at 16 samples per cycle). Also divide by  $\text{SQRT}(2)$  to get RMS phasors

$$IA_{cp}x_v := \frac{1}{\sqrt{2}} \left( Ia_v + j \cdot Ia_{v-\frac{RS}{4}} \right)$$

$$VA_{cp}x_v := \frac{1}{\sqrt{2}} \left( Va_v + j \cdot Va_{v-\frac{RS}{4}} \right)$$

$$IB_{cp}x_v := \frac{1}{\sqrt{2}} \left( Ib_v + j \cdot Ib_{v-\frac{RS}{4}} \right)$$

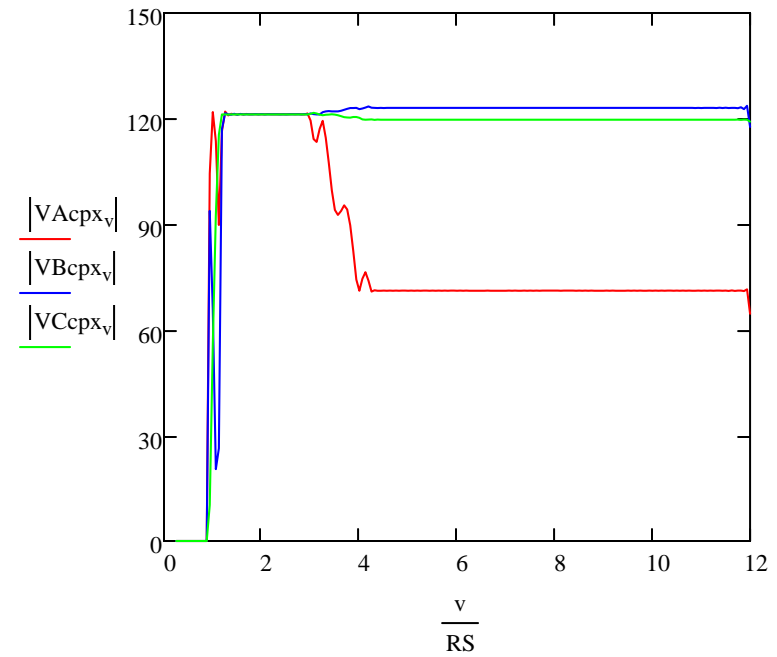
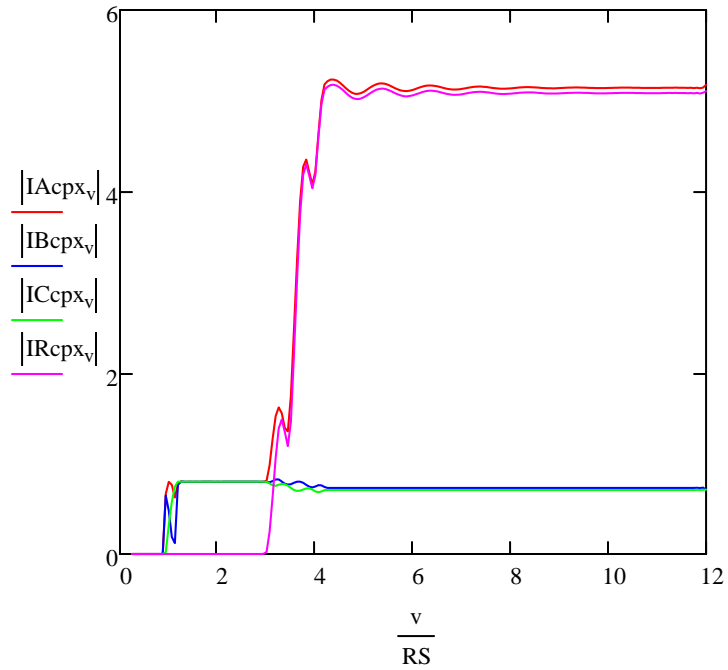
$$VB_{cp}x_v := \frac{1}{\sqrt{2}} \left( Vb_v + j \cdot Vb_{v-\frac{RS}{4}} \right)$$

$$IC_{cp}x_v := \frac{1}{\sqrt{2}} \left( Ic_v + j \cdot Ic_{v-\frac{RS}{4}} \right)$$

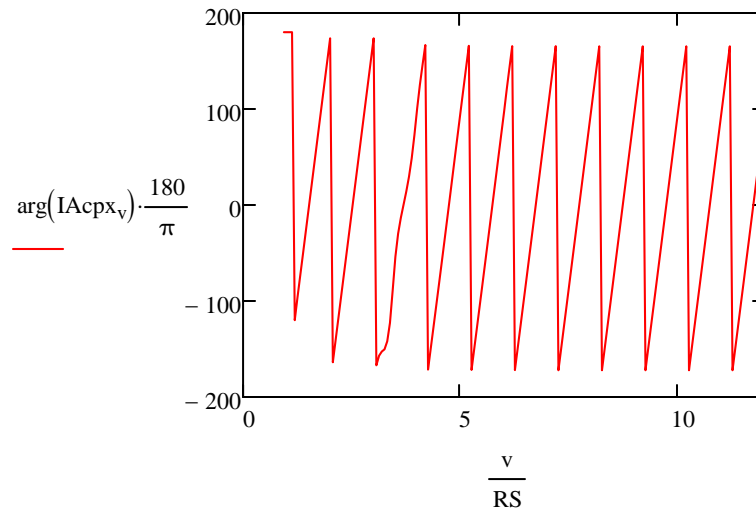
$$VC_{cp}x_v := \frac{1}{\sqrt{2}} \left( Vc_v + j \cdot Vc_{v-\frac{RS}{4}} \right)$$

$$IR_{cp}x_v := \frac{1}{\sqrt{2}} \left( Ir_v + j \cdot Ir_{v-\frac{RS}{4}} \right)$$

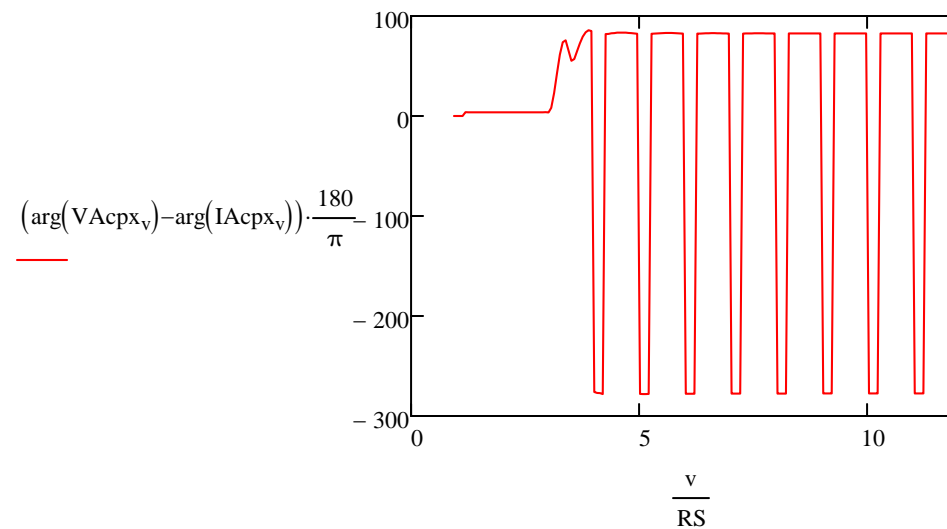
- Each of these terms is a phasor with magnitude and phase (we are only uses magnitude for now).



Since  $\omega t$  is continually growing as time passes, the angle is not constant:



If we try to calculate an angle difference we have a problem due to the different reset times



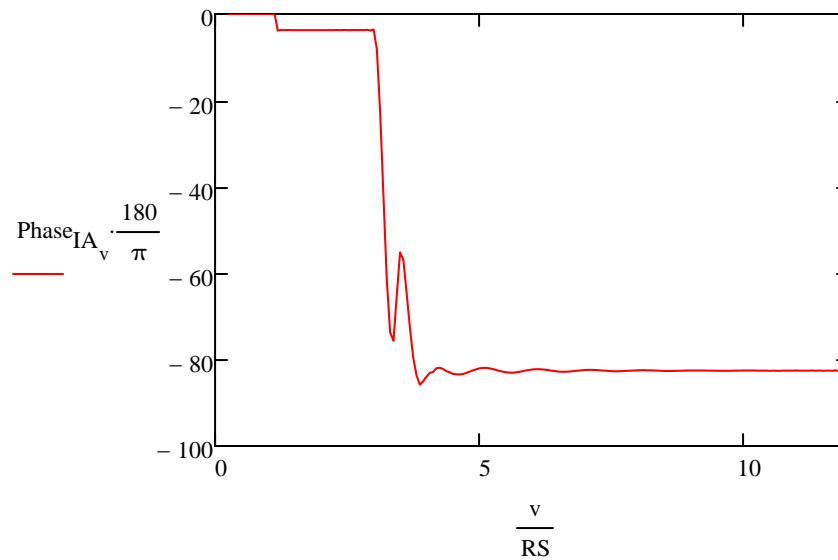
We don't get a nice smooth angle, because the two waveforms don't "reset" to -180 deg at the same time, so first we need to define phasors with a reference angle. For example, using Van as a reference:

$$\theta_{VA_v} := \arg(VA_{cp}x_v + 0.000001)$$

$$\theta_{IA_v} := \arg[(IA_{cp}x)_v + 0.000001]$$

add a small number to avoid numerical problems

$$\text{Phase}_{IA_v} := \begin{cases} \theta_{IA_v} - \theta_{VA_v} & \text{if } |\theta_{IA_v} - \theta_{VA_v}| < \pi \\ (\theta_{IA_v} - \theta_{VA_v} - 2 \cdot \pi) & \text{if } (\theta_{IA_v} - \theta_{VA_v}) > \pi \\ (\theta_{IA_v} - \theta_{VA_v} + 2 \cdot \pi) & \text{if } \theta_{IA_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$



Notes:

1. Angle of zero until filter catches up
2. Then slightly lagging
3. Then nearly -90 deg due to fault.

$$\theta_{IB_v} := \arg[(IB_{cpx})_v + 0.000001]$$

$$\text{Phase}_{IB_v} := \begin{cases} \theta_{IB_v} - \theta_{VA_v} & \text{if } |\theta_{IB_v} - \theta_{VA_v}| < \pi \\ (\theta_{IB_v} - \theta_{VA_v} - 2\pi) & \text{if } (\theta_{IB_v} - \theta_{VA_v}) > \pi \\ (\theta_{IB_v} - \theta_{VA_v} + 2\pi) & \text{if } \theta_{IB_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$

$$\theta_{IC_v} := \arg[(IC_{cpx})_v + 0.000001]$$

$$\text{Phase}_{IC_v} := \begin{cases} \theta_{IC_v} - \theta_{VA_v} & \text{if } |\theta_{IC_v} - \theta_{VA_v}| < \pi \\ (\theta_{IC_v} - \theta_{VA_v} - 2\pi) & \text{if } (\theta_{IC_v} - \theta_{VA_v}) > \pi \\ (\theta_{IC_v} - \theta_{VA_v} + 2\pi) & \text{if } \theta_{IC_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$

$$\theta_{IR_v} := \arg[(IR_{cpx})_v + 0.000001]$$

$$\text{Phase}_{IR_v} := \begin{cases} \theta_{IR_v} - \theta_{VA_v} & \text{if } |\theta_{IR_v} - \theta_{VA_v}| < \pi \\ (\theta_{IR_v} - \theta_{VA_v} - 2\pi) & \text{if } (\theta_{IR_v} - \theta_{VA_v}) > \pi \\ (\theta_{IR_v} - \theta_{VA_v} + 2\pi) & \text{if } \theta_{IR_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$

$$\theta_{VA_v} := \arg[(VA_{cpx})_v + 0.000001]$$

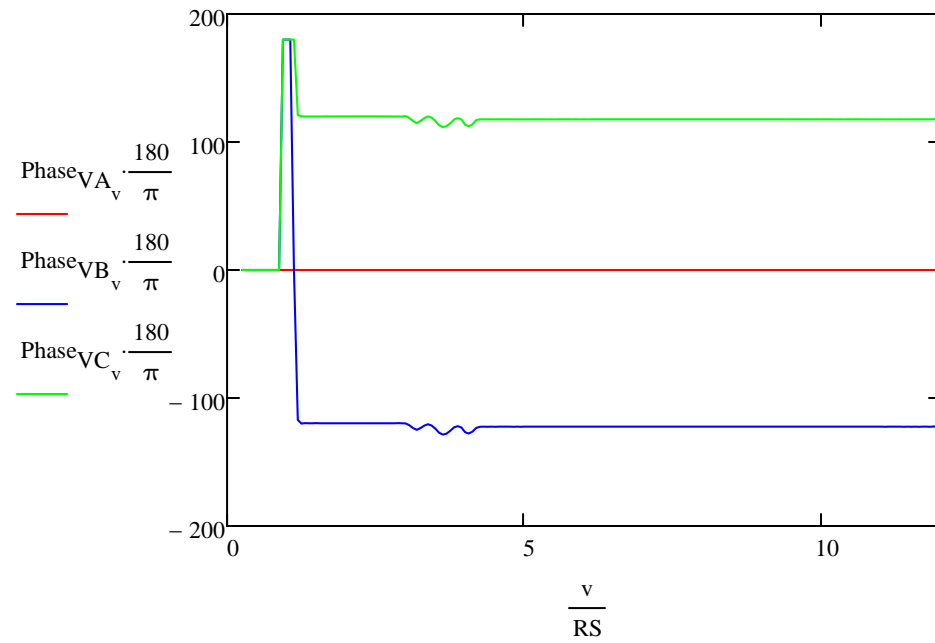
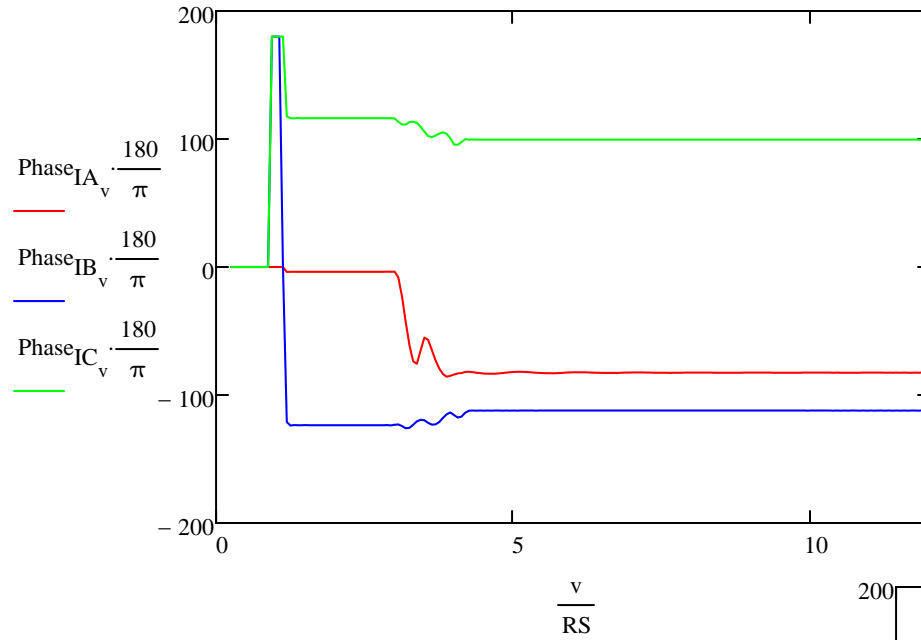
$$\text{Phase}_{VA_v} := \begin{cases} \theta_{VA_v} - \theta_{VA_v} & \text{if } |\theta_{VA_v} - \theta_{VA_v}| < \pi \\ (\theta_{VA_v} - \theta_{VA_v} - 2\pi) & \text{if } (\theta_{VA_v} - \theta_{VA_v}) > \pi \\ (\theta_{VA_v} - \theta_{VA_v} + 2\pi) & \text{if } \theta_{VA_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$

$$\theta_{VB_v} := \arg[(VB_{cpx})_v + 0.000001]$$

$$\text{Phase}_{VB_v} := \begin{cases} \theta_{VB_v} - \theta_{VA_v} & \text{if } |\theta_{VB_v} - \theta_{VA_v}| < \pi \\ (\theta_{VB_v} - \theta_{VA_v} - 2\pi) & \text{if } (\theta_{VB_v} - \theta_{VA_v}) > \pi \\ (\theta_{VB_v} - \theta_{VA_v} + 2\pi) & \text{if } \theta_{VB_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$

$$\theta_{VC_v} := \arg[(VC_{cpx})_v + 0.000001]$$

$$\text{Phase}_{VC_v} := \begin{cases} \theta_{VC_v} - \theta_{VA_v} & \text{if } |\theta_{VC_v} - \theta_{VA_v}| < \pi \\ (\theta_{VC_v} - \theta_{VA_v} - 2\pi) & \text{if } (\theta_{VC_v} - \theta_{VA_v}) > \pi \\ (\theta_{VC_v} - \theta_{VA_v} + 2\pi) & \text{if } \theta_{VC_v} - \theta_{VA_v} < -(\pi) \\ \pi & \text{otherwise} \end{cases}$$



$$Z1 := 1\text{ohm} + j \cdot 10\text{ohm} \quad Z1\text{MAG} := |Z1| \quad Z1\text{MAG} = 10.0499 \Omega$$

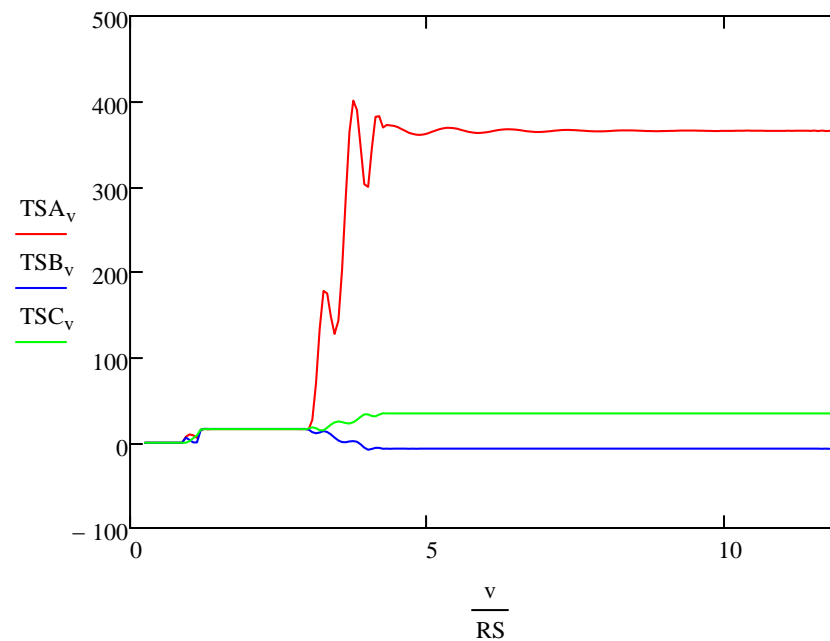
$$Z0 := 3 \cdot Z1 \quad Z1\text{ANG} := \arg(Z1) \quad Z1\text{ANG} = 84.2894 \cdot \text{deg}$$

$$\text{MTA} := Z1\text{ANG}$$

$$\text{TSA}_v := |V_{\text{Acp}x_v}| \cdot |I_{\text{Acp}x_v}| \cdot \cos[\theta_{V_{\text{A}_v}} - (\theta_{I_{\text{A}_v}} + \text{MTA})]$$

$$\text{TSB}_v := |V_{\text{Bcp}x_v}| \cdot |I_{\text{Bcp}x_v}| \cdot \cos[\theta_{V_{\text{B}_v}} - (\theta_{I_{\text{B}_v}} + \text{MTA})]$$

$$\text{TSC}_v := |V_{\text{Ccp}x_v}| \cdot |I_{\text{Ccp}x_v}| \cdot \cos[\theta_{V_{\text{C}_v}} - (\theta_{I_{\text{C}_v}} + \text{MTA})]$$

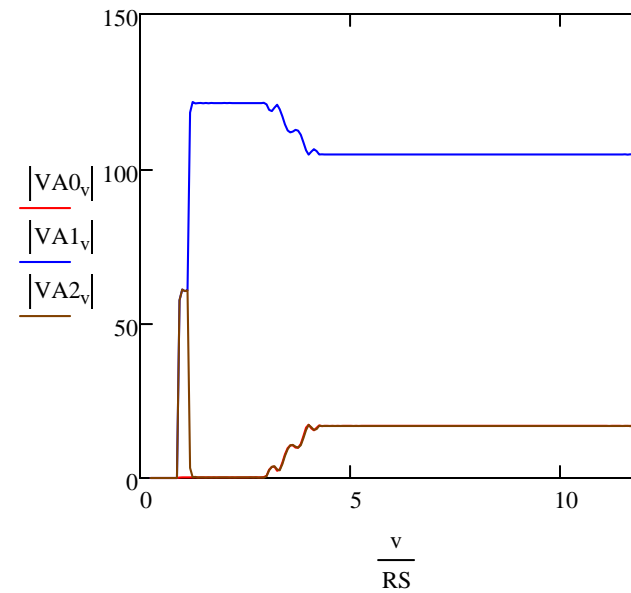
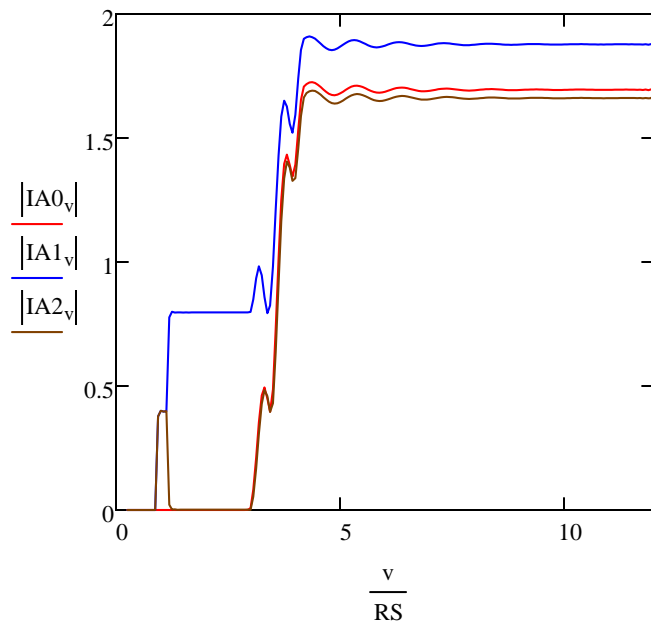


**Create symmetrical components:**

Phase A Components:

$$\begin{pmatrix} IA0_v \\ IA1_v \\ IA2_v \end{pmatrix} := A_{012}^{-1} \cdot \begin{pmatrix} IA_{cpx}_v \\ IB_{cpx}_v \\ IC_{cpx}_v \end{pmatrix}$$

$$\begin{pmatrix} VA0_v \\ VA1_v \\ VA2_v \end{pmatrix} := A_{012}^{-1} \cdot \begin{pmatrix} VA_{cpx}_v \\ VB_{cpx}_v \\ VC_{cpx}_v \end{pmatrix}$$





Phase B Components

$$\begin{pmatrix} IB0_v \\ IB1_v \\ IB2_v \end{pmatrix} := B_{012}^{-1} \cdot \begin{pmatrix} IAcp_{x_v} \\ IBcp_{x_v} \\ ICcp_{x_v} \end{pmatrix}$$

$$\begin{pmatrix} VB0_v \\ VB1_v \\ VB2_v \end{pmatrix} := B_{012}^{-1} \cdot \begin{pmatrix} VAcp_{x_v} \\ VBcp_{x_v} \\ VCcp_{x_v} \end{pmatrix}$$

Phase C Components:

$$\begin{pmatrix} IC0_v \\ IC1_v \\ IC2_v \end{pmatrix} := C_{012}^{-1} \cdot \begin{pmatrix} IAcp_{x_v} \\ IBcp_{x_v} \\ ICcp_{x_v} \end{pmatrix}$$

$$\begin{pmatrix} VC0_v \\ VC1_v \\ VC2_v \end{pmatrix} := C_{012}^{-1} \cdot \begin{pmatrix} VAcp_{x_v} \\ VBcp_{x_v} \\ VCcp_{x_v} \end{pmatrix}$$

The magnitudes for the phase A, B and C symmetrical components will be the same, but the angles will differ. So this will matter more later.

Calculate negative sequence impedance and zero sequence impedance

$$Z_{2A_v} := \frac{\text{Re} \left[ VA_{2_v} \cdot \left[ IA_{2_v} \cdot (1 \cdot e^{j \cdot Z1 \text{ANG}}) \right] \right]}{\left( |IA_{2_v}| \right)^2 + .00001}$$

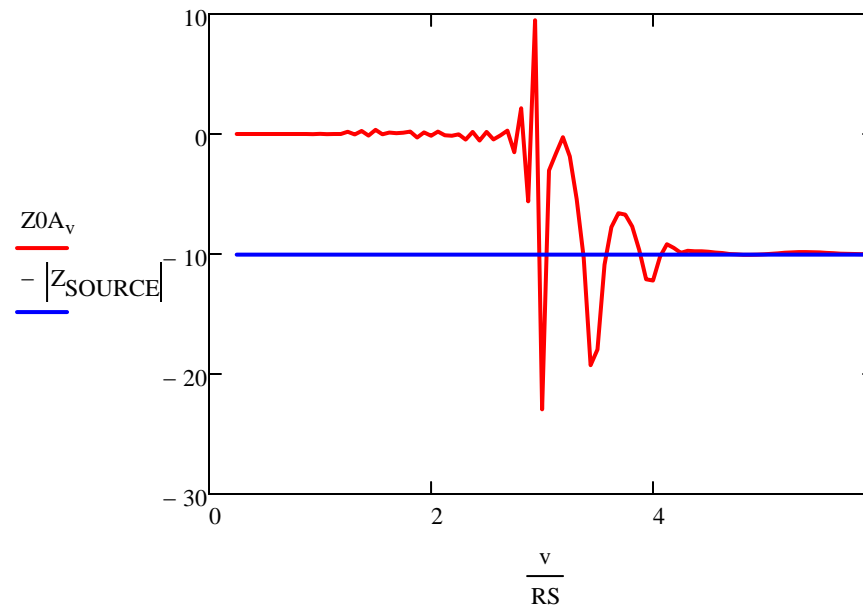
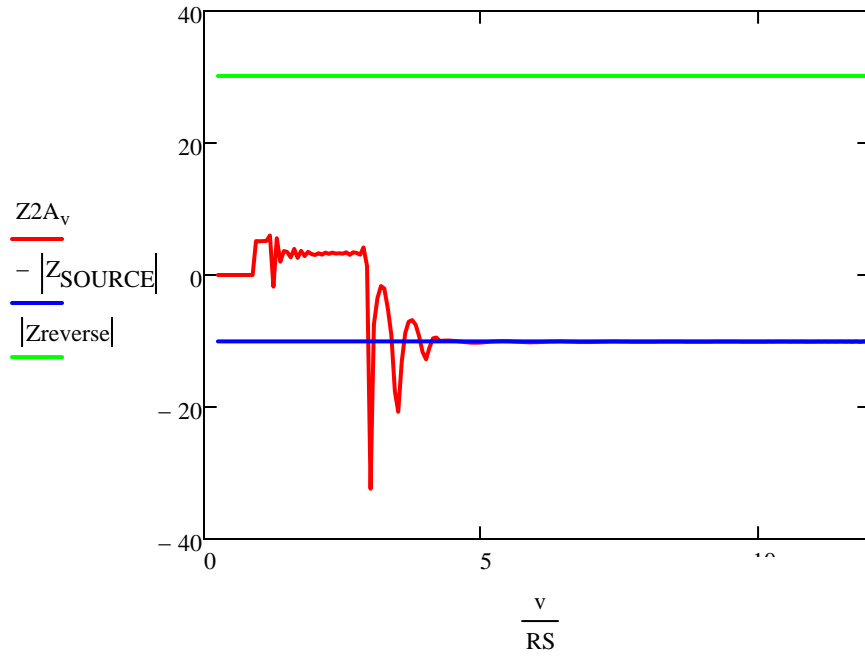
Avoid divide by 0

$$Z_{0A_v} := \frac{\text{Re} \left[ VA_{0_v} \cdot \left[ IA_{0_v} \cdot (1 \cdot e^{j \cdot Z1 \text{ANG}}) \right] \right]}{\left( |IA_{0_v}| \right)^2 + .00001}$$

Avoid divide by 0

$$Z_{\text{SOURCE}} := (1 + j \cdot 10) \text{ohm}$$

$$Z_{\text{reverse}} := Z_{\text{SOURCE}} + 2 \cdot Z1$$



### Alternate Approach for Three Phase Faults

- An alternate approach instead of the torque equation is to compare the angle single phase impedance calculated from measurements to thresholds

$$-90\text{deg} + Z1\text{ANG} < \theta_{Z1} < 90\text{deg} + Z1\text{ANG}$$

$$\theta_{V1_v} := \arg(VA1_v + 0.000001) \quad \theta_{I1_v} := \arg[(IA1)_v + 0.000001]$$

$$\text{Phase}_{I1_v} := \begin{cases} \theta_{I1_v} - \theta_{V1_v} & \text{if } |\theta_{I1_v} - \theta_{V1_v}| < \pi \\ (\theta_{I1_v} - \theta_{V1_v} - 2 \cdot \pi) & \text{if } (\theta_{I1_v} - \theta_{V1_v}) > \pi \\ (\theta_{I1_v} - \theta_{V1_v} + 2 \cdot \pi) & \text{if } \theta_{I1_v} - \theta_{V1_v} < -(\pi) \end{cases}$$

$$\text{Phase}_{V1_v} := \begin{cases} \theta_{V1_v} - \theta_{V1_v} & \text{if } |\theta_{V1_v} - \theta_{V1_v}| < \pi \\ (\theta_{V1_v} - \theta_{V1_v} - 2 \cdot \pi) & \text{if } (\theta_{V1_v} - \theta_{V1_v}) > \pi \\ (\theta_{V1_v} - \theta_{V1_v} + 2 \cdot \pi) & \text{if } \theta_{V1_v} - \theta_{V1_v} < -(\pi) \end{cases}$$

$$Z1A_v := \frac{|VA1_v| \cdot e^{j \cdot \text{Phase}_{V1_v}}}{|IA1_v| \cdot e^{j \cdot \text{Phase}_{I1_v} + 0.000001}}$$

