Grounding Examples

\[
a := 1 \cdot e^{j120\text{deg}}
\]

\[
A_{012} := \begin{pmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a & a^2
\end{pmatrix}
\]

\[
\text{MVA} := 1000\text{kW} \quad \text{pu} := 1
\]

A 4160 V feeder is supplied by the WYE connected side of a 75 MVA transformer. The system \( \text{MVA}_{sc} \) supplying the delta side of the transformer is 650 MVA. The transformer has a leakage reactance of 10%. A ground impedance will be connected in the neutral of 4.16kV side of the transformer to limit fault currents.

A Sketch the per unit diagram for the system

\[
\text{MVA}_{\text{base}} := 100\text{MVA}
\]

\[
\text{Srated} := 75\text{MVA} \quad V_{\text{LL}} := 4.16\text{kV} \quad V_{\text{ln}} := \frac{V_{\text{LL}}}{\sqrt{3}} \quad V_{\text{ln}} = 2.402\cdot\text{kV}
\]

\[
\text{MVAsc} := 650\text{MVA}
\]

\[
X_{\text{src\_pu}} := \frac{1.0^2}{\left(\frac{\text{MVAsc}}{\text{MVA}_{\text{base}}}\right)} \quad X_{\text{src\_pu}} = 0.154\cdot\text{pu}
\]

\[
X_{\text{xferm}} := 0.1 \cdot \left(\frac{4160\text{V}}{4160\text{V}}\right)^2 \left(\frac{\text{MVA}_{\text{base}}}{\text{Srated}}\right) \quad X_{\text{xferm}} = 0.133\cdot\text{pu}
\]
B Determine sequence networks for the system

Positive Sequence:

Negative Sequence:

Zero Sequence:

Note that this is a Δ-Y grounded transformer. Also, assuming that zero sequence leakage impedance equal to positive and negative sequence values.

C Assume that the feeder is all overhead lines with negligible capacitance. Determine the ground reactance needed to limit the single line to ground fault current to 6000A.

\[
\frac{I_{\text{slgmax}}}{I_{\text{base}}} = 6000A
\]

\[
I_{\text{base}} := \frac{\text{MVA}_{\text{base}}}{\sqrt{3} \cdot V_{\text{LL}}}
\]

\[
I_{\text{base}} = 13.88\cdot\text{kA}
\]

\[
Z_{\text{base}} := \frac{V_{\text{LL}}^2}{\text{MVA}_{\text{base}}}
\]

\[
I_{\text{ifpu}} := \frac{I_{\text{slgmax}}}{I_{\text{base}}}
\]

\[
I_{\text{ifpu}} = 0.432\cdot\text{pu}
\]

For a SLG fault we have (connect positive, negative and zero sequence circuits in series):

\[
I_0 = \frac{V_{\text{fault}}}{Z_1 + Z_2 + Z_0 + 3jX_{\text{gnd}}}
\]

where

\[
V_{\text{fault}} := 1.0 \cdot e^{-90\text{deg}}
\]

\[
Z_1 := jX_{\text{src\_pu}} + jX_{\text{xfmr}}
\]

\[
Z_2 := Z_1
\]

\[
Z_0 := jX_{\text{xfmr}}
\]

and we know for a SLG fault:

\[
I_0 := \frac{I_{\text{ifpu}}}{3}
\]

Solve for \(Z_{\text{gnd}}\)

\[
Z_{\text{gnd}} := \frac{1}{3} \left[ \frac{V_{\text{fault}}}{I_0} - (Z_1 + Z_2 + Z_0) \right]
\]

\[
Z_{\text{gnd}} = 2.0772i \quad \text{per unit}
\]
\[ X_{\text{gndpu}} := \text{Im}(Z_{\text{gnd}}) \quad \text{Xgndpu} = 2.077 \text{ per unit} \]

\[ X_{\text{gnd}} := \text{Xgndpu} \cdot Z_{\text{base}} \quad \text{Xgnd} = 0.359 \Omega \]

\[ L_{\text{gnd}} := \frac{X_{\text{gnd}}}{2 \cdot \pi \cdot 60 \text{Hz}} \quad \text{Lgnd} = 0.954 \cdot \text{mH at 60Hz} \]

**D** If the feeder is largely underground, the capacitance cannot be neglected. If the total per phase capacitance to ground is 1.5 \( \mu \text{F} \), determine the grounding resistance needed to limit the single line to ground fault current to 20 A.

\[ C_{\text{parasitic}} := 1.5 \mu \text{F} \quad X_c := \frac{1}{2 \cdot \pi \cdot 60 \text{Hz} \cdot C_{\text{parasitic}}} \quad X_c = 1.768 \cdot \text{k}\Omega \]

\[ X_{c\_pu} := \frac{X_c}{Z_{\text{base}}} \quad X_{c\_pu} = 10218.6 \cdot \text{pu} \]

\[ I_{\text{slg\_max}} := 20 \text{A} \quad I_{\text{slgpu}} := \frac{I_{\text{slg\_max}}}{I_{\text{base}}} \quad I_{\text{slgpu}} = 1.441 \times 10^{-3} \cdot \text{pu} \]

The sequence networks will now change with the addition of the capacitance as shown.
\[ I_0 = \frac{V_{\text{fault}}}{Z_1 + Z_2 + \frac{[(Z_0 + 3 \cdot R_g) \cdot (-jX_c)]}{Z_0 + 3R_g - j \cdot X_c}} \]

Note that \( Z_1 + Z_2 \) will be much much smaller than the parallel combination of \( 3R \) and \(-jX_c\), to that \( Z_1 \) and \( Z_2 \) can be neglected, as can \( Z_0 \).

We also, only care about the magnitude of the reduced current, not the angle.

So we are actually solving:

\[ |I_0| = \left| \frac{V_{\text{fault}}}{[(Z_0+3 \cdot R_g) \cdot (-jX_c)] / (Z_0+3R_g-j \cdot X_c)} \right| \]

which requires an iterative solution.

\[ I_0 := \frac{I_{\text{slgpu}}}{3} \]

Initial Guess:

\[ R_g := 1000 \]

MathCAD solve block:

Given

\[ I_0 - \left| \frac{V_{\text{fault}}}{(3 \cdot R_g + Z_0) \cdot (-jX_c \text{pu})} \right| = 0 \]

\[ R_{\text{gnd \ pu}} := \text{Find}(R_g) \]

\[ R_{\text{gnd \ pu}} = 708.786 \]

\[ R_{\text{gnd}} := R_{\text{gnd \ pu}} \cdot \text{Zbase} \]

\[ R_{\text{gnd}} = 122.66 \Omega \]
E Calculate the line to ground voltages on the unfaulted phases in parts C and D and calculate the zero sequence voltages and currents.

**Part C:**

\[
\begin{align*}
I_{0\text{-partC}} & := \frac{V_{\text{fault}}}{Z_0 + 3\cdot jX_{\text{gndpu}} + Z_1 + Z_2} \quad \Rightarrow \quad I_{1\text{-partC}} = 0.144 \\
& \quad I_{2\text{-partC}} := I_{0\text{-partC}}
\end{align*}
\]

as a check:

\[
\begin{align*}
I_{a\text{-partC}} & := 3\cdot I_{0\text{-partC}} \quad \Rightarrow \quad I_{a\text{-partC}} = 0.432 \quad \text{pu} \\
& \quad |I_{a\text{-partC}}| \cdot I_{\text{base}} = 6\cdot \text{kA}
\end{align*}
\]

\[
\begin{align*}
V_{1\_\text{partC}} & := V_{\text{fault}} - I_{1\text{-partC}} \cdot Z_1 \\
& \quad |V_{1\text{-partC}}| = 0.959 \text{i}
\end{align*}
\]

\[
\begin{align*}
V_{2\_\text{partC}} & := -I_{2\text{-partC}} \cdot Z_2 \\
& \quad |V_{2\text{-partC}}| = 0.041 \text{i}
\end{align*}
\]

\[
\begin{align*}
V_{0\_\text{partC}} & := -I_{0\text{-partC}} \cdot (Z_0 + 3\cdot jX_{\text{gndpu}}) \\
& \quad |V_{0\text{-partC}}| = 0.917 \text{i}
\end{align*}
\]

\[
\begin{align*}
V_{abc\_\text{partC}} & := \begin{bmatrix} V_{0\text{-partC}} \\ V_{1\text{-partC}} \\ V_{2\text{-partC}} \end{bmatrix} \\
& \quad |V_{abc\_\text{partC}}| = \begin{bmatrix} 0 \\ 0.866 - 1.376 \text{i} \\ -0.866 - 1.376 \text{i} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\text{arg} (V_{abc\_\text{partC}}) & \approx \begin{bmatrix} 90 \\ -57.812 \text{ deg} \\ -122.188 \text{ deg} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
V_{\text{ln}} \cdot |V_{abc\_\text{partC}}| & = \begin{bmatrix} 0 \\ 3.905 \text{i} \\ 3.905 \text{i} \end{bmatrix} \text{kV}
\end{align*}
\]

**Part D:**

\[
\begin{align*}
Z_{\text{gndD}} & := \frac{(3 \cdot \text{Rgnd\_pu} + Z_0) \cdot (-j \cdot X_{\text{c\_pu}})}{(3 \cdot \text{Rgnd\_pu} + Z_0) - j \cdot X_{\text{c\_pu}}} \\
& \quad I_{0\text{-partD}} := \frac{V_{\text{fault}}}{Z_{\text{gndD}} + Z_1 + Z_2} \quad \Rightarrow \quad I_{0\text{-partD}} = -9.771 \times 10^{-5} + 4.703 \times 10^{-4} \text{i}
\end{align*}
\]

\[
\begin{align*}
I_{1\text{-partD}} & := I_{0\text{-partD}} \\
I_{2\text{-partD}} & := I_{0\text{-partD}} \quad \Rightarrow \quad |I_{0\text{-partD}}| = 4.804 \times 10^{-4}
\end{align*}
\]
as a check: \[ I_{a\text{, partD}} := 3 \cdot I_{0\text{, partD}} \]
\[ |I_{a\text{, partD}}| = 1.441 \times 10^{-3} \text{ pu} \]
\[ |I_{a\text{, partD}}| \cdot I_{\text{base}} = 20.001 \cdot \text{A} \]

\[ V_{1\text{, partD}} := V_{\text{fault}} - I_{1\text{, partD}} \cdot Z_{1} \]
\[ |V_{1\text{, partD}}| = 1 \]
\[ V_{2\text{, partD}} := -I_{2\text{, partD}} \cdot Z_{2} \]
\[ |V_{2\text{, partD}}| = 1.38 \times 10^{-4} \]
\[ V_{0\text{, partD}} := -I_{0\text{, partD}} \cdot (Z_{\text{gndD}}) \]
\[ |V_{0\text{, partD}}| = 1 \]

\[ V_{\text{abc\_partD}} := A_{012} \cdot \begin{pmatrix} V_{0\text{, partD}} \\ V_{1\text{, partD}} \\ V_{2\text{, partD}} \end{pmatrix} \]
\[ V_{\text{abc\_partD}} = \begin{pmatrix} 0 \\ 0.866 - 1.5i \\ -0.866 - 1.5i \end{pmatrix} \]

\[ |V_{\text{abc\_partD}}| = \begin{pmatrix} 0 \\ 1.732 \\ 1.732 \end{pmatrix} \]
\[ \text{arg}(V_{\text{abc\_partD}}) = \begin{pmatrix} 107.593 \\ -60.013 \\ -120.01 \end{pmatrix} \text{ deg} \]
\[ V_{\text{ln\_partD}} \cdot |V_{\text{abc\_partD}}| = \begin{pmatrix} 0 \\ 4.16 \\ 4.161 \end{pmatrix} \text{ kV} \]

**Compute the single line to ground fault current and the voltage on the unfaulted phases if the transformer is solidly grounded. Calculate the zero sequence voltages and currents.**

\[ I_{0\text{, gnd}} := \frac{V_{\text{fault}}}{Z_{0} + Z_{1} + Z_{2}} \]
\[ I_{0\text{, gnd}} = 1.413 \]
\[ I_{1\text{, gnd}} := I_{0\text{, gnd}} \]
\[ I_{2\text{, gnd}} := I_{0\text{, gnd}} \]
\[ I_{a\text{, gnd}} := 3 \cdot I_{0\text{, gnd}} \]
\[ I_{a\text{, gnd}} = 4.239 \text{ per unit} \]
\[ I_{a\text{, gnd}} \cdot I_{\text{base}} = 58.833 \cdot \text{kA} \]

\[ V_{1\text{, gnd}} := V_{\text{fault}} - I_{1\text{, gnd}} \cdot Z_{1} \]
\[ V_{1\text{, gnd}} = 0.594i \]
\[ V_{2\text{, gnd}} := -I_{2\text{, gnd}} \cdot Z_{2} \]
\[ V_{2\text{, gnd}} = -0.406i \]
\[ V_{0\text{, gnd}} := -I_{0\text{, gnd}} \cdot Z_{0} \]
\[ V_{0\text{, gnd}} = -0.18841i \]
\[
\begin{pmatrix}
0 \\
0.866 - 0.283i \\
-0.866 - 0.283i
\end{pmatrix}
\]

Note that \(V_b\) and \(V_c\) are nearly 1.0 per unit, and are slightly depressed. If \(Z_0 = Z_1 = Z_2\) then they would be 1.0 and offset from each other by 120 degrees.

For the different grounded cases described above, discuss the available quantities to measure for ground fault protection and suggest a scheme to consider (based on voltage, current, etc).

For the high resistance grounded case, there isn't enough current available for doing ground fault protection, but \(V_0\) is 1pu, so there is enough voltage available to use that to identify the presence of a ground fault.

For the case with the low inductance grounded, there is sufficient \(I_0\) for detecting a fault, although any fault impedance (resistance) may make this too difficult. There is also probably sufficient \(V_0\) to use that to detect the fault.

For the solidly grounded case, \(V_0\) is pretty small, and it might be hard to discriminate sufficiently to identify a fault. On the other hand, \(I_0\) is large.