Grounding Examples

\[ a := 1 \cdot e^{j120^\circ} \]

\[
A_{012} := \begin{pmatrix}
1 & 1 & 1 \\
1 & a & a \\
1 & a & a^2
\end{pmatrix}
\]

MVA := 1000kW \quad \text{pu} := 1

A 4160 V feeder is supplied by the WYE connected side of a 75 MVA transformer. The system \( \text{MVAsc} \) supplying the delta side of the transformer is 650 MVA. The transformer has a leakage reactance of 10%. A ground impedance will be connected in the neutral of 4.16kV side of the transformer to limit fault currents.

A Sketch the per unit diagram for the system

\[ \text{MVA} \_	ext{base} := 100 \text{MVA} \]

\[ \text{Srated} := 75 \text{MVA} \quad \text{V} \_	ext{LL} := 4.16 \text{kV} \]

\[ \text{V} \_	ext{ln} := \frac{\text{V} \_	ext{LL}}{\sqrt{3}} \quad \text{V} \_	ext{ln} = 2.402 \cdot \text{kV} \]

\[ \text{MVAsc} := 650 \text{MVA} \]

\[ \text{Xsrc} \_\text{pu} := \frac{1.0^2 \cdot \text{MVAsc}}{\text{MVA} \_\text{base}} \quad \text{Xsrc} \_\text{pu} = 0.154 \cdot \text{pu} \]

\[ \text{X} \_\text{xfmr} := 0.1 \cdot \left( \frac{4160 \text{V}}{4160 \text{V}} \right)^2 \left( \frac{\text{MVA} \_\text{base}}{\text{Srated}} \right) \]

\[ \text{X} \_\text{xfmr} = 0.133 \cdot \text{pu} \]

\[ \text{V} \_\text{src} \]

\[ \text{X} \_\text{src} \]

\[ \text{X} \_\text{xfmr} \]

\[ \text{j}0.154 \text{pu} \]

\[ \text{j}0.133 \text{pu} \]
B Determine sequence networks for the system

Positive Sequence:

\[
\begin{align*}
V_{\text{src}} & \quad \text{X}_{\text{src}} \quad \text{X}_{\text{fmr}} \\
j0.154\text{pu} & \quad j0.133\text{pu}
\end{align*}
\]

Negative Sequence:

\[
\begin{align*}
V_{\text{src}} & \quad \text{X}_{\text{src}} \quad \text{X}_{\text{fmr}} \\
j0.154\text{pu} & \quad j0.133\text{pu}
\end{align*}
\]

Zero Sequence:

\[
\begin{align*}
V_{\text{src}} & \quad \text{X}_{\text{src}} \quad \text{X}_{\text{fmr}} \\
j0.154\text{pu} & \quad j0.133\text{pu} \\
& \quad 3Z_{\text{gnd}}
\end{align*}
\]

Note that this is a Δ-Y grounded transformer. Also, assuming that zero sequence leakage impedance equal to positive and negative sequence values.

C Assume that the feeder is all overhead lines with negligible capacitance. Determine the ground reactance needed to limit the single line to ground fault current to 6000A.

\[
\begin{align*}
\text{If}_{\text{slgmax}} & := 6000\text{A} \\
\text{I}_{\text{base}} & := \frac{\text{MVA}_{\text{base}}}{\sqrt{3} \cdot V_{\text{LL}}} \\
\text{I}_{\text{base}} & = 13.88\cdot\text{kA} \\
\text{Z}_{\text{base}} & := \frac{V_{\text{LL}}^2}{\text{MVA}_{\text{base}}}
\end{align*}
\]

\[
\begin{align*}
\text{If}_{\text{pu}} & := \frac{\text{If}_{\text{slgmax}}}{\text{I}_{\text{base}}} \\
\text{If}_{\text{pu}} & = 0.432\cdot\text{pu}
\end{align*}
\]

For a SLG fault we have (connect positive, negative and zero sequence circuits in series):

\[
\begin{align*}
I_0 & = \frac{V_{\text{fault}}}{Z_1 + Z_2 + Z_0 + 3jX_{\text{gnd}}} \\
\text{where} & \\
V_{\text{fault}} & := 1.0 \cdot e^{\text{j} - 90\text{deg}} \\
Z_1 & := jX_{\text{src\_pu}} + jX_{\text{xfmr}} \\
Z_2 & := Z_1 \\
Z_0 & := jX_{\text{xfmr}}
\end{align*}
\]

and we know for a SLG fault: 

\[
I_0 := \frac{\text{If}_{\text{pu}}}{3}
\]

Solve for \(Z_{\text{gnd}}\)

\[
Z_{\text{gnd}} := \frac{1}{3} \left[ \frac{V_{\text{fault}}}{I_0} - (Z_1 + Z_2 + Z_0) \right]
\]

\[
Z_{\text{gnd}} = 2.0772i \quad \text{per unit}
\]
\[ X_{\text{gndpu}} := \text{Im}(Z_{\text{gnd}}) \quad X_{\text{gndpu}} = 2.077 \text{ per unit} \]

\[ X_{\text{gnd}} := X_{\text{gndpu}} \cdot Z_{\text{base}} \]

\[ X_{\text{gnd}} = 0.359 \Omega \]

\[ L_{\text{gnd}} := \frac{X_{\text{gnd}}}{2 \cdot \pi \cdot 60 \text{Hz}} \quad L_{\text{gnd}} = 0.954 \cdot \text{mH at 60Hz} \]

**D** If the feeder is largely underground, the capacitance cannot be neglected. If the total per phase capacitance to ground is 1.5 \( \mu \text{F} \), determine the grounding resistance needed to limit the single line to ground fault current to 20 A.

\[ C_{\text{parasitic}} := 1.5 \mu \text{F} \]

\[ X_c := \frac{1}{2 \cdot \pi \cdot 60 \text{Hz} \cdot C_{\text{parasitic}}} \quad X_c = 1.768 \cdot \text{k\Omega} \]

\[ X_{c\_pu} := \frac{X_c}{Z_{\text{base}}} \quad X_{c\_pu} = 10218.6 \cdot \text{pu} \]

\[ I_{\text{slg\_max}} := 20 \text{A} \quad I_{\text{slgpu}} := \frac{I_{\text{slg\_max}}}{I_{\text{base}}} \quad I_{\text{slgpu}} = 1.441 \times 10^{-3} \cdot \text{pu} \]

The sequence networks will now change with the addition of the capacitance as shown.
\[ I_0 = \frac{V_{\text{fault}}}{Z_1 + Z_2 + \frac{[(Z_0 + 3R_g)(-jX_c)]}{Z_0 + 3R_g - jX_c}} \]

Note that \( Z_1 + Z_2 \) will be much much smaller than the parallel combination of \( 3R \) and \(-jX_c\), to that \( Z_1 \) and \( Z_2 \) can be neglected, as can \( Z_0 \).

We also, only care about the magnitude of the reduced current, not the angle.

So we are actually solving:

\[ |I_0| = \left| \frac{V_{\text{fault}}}{\frac{[(Z_0 + 3R_g)(-jX_c)]}{Z_0 + 3R_g - jX_c}} \right| \]

which requires an iterative solution.

\[ I_0 := \frac{I_{\text{slgppu}}}{3} \]

Initial Guess:

\[ R_g := 1000 \]

MathCAD solve block:

Given

\[ I_0 - \left| \frac{V_{\text{fault}}}{\frac{(3\cdot R_g + Z_0)(-j\cdot X_c_{\text{pu}})}{(3\cdot R_g + Z_0) - j\cdot X_c_{\text{pu}}}} \right| = 0 \]

\[ R_{\text{gndpu}} := \text{Find}(R_g) \]

\[ R_{\text{gndpu}} = 708.786 \]

\[ R_{\text{gnd}} := R_{\text{gndpu}} \cdot Z_{\text{base}} \quad \text{Rgnd} = 122.66 \Omega \]
E Calculate the line to ground voltages on the unfaulted phases in parts C and D and calculate the zero sequence voltages and currents.

**Part C:**

\[
I_{0\text{-partC}} := \frac{V_{\text{fault}}}{Z_0 + 3 \cdot jX_{\text{gndpu}} + Z_1 + Z_2}
\]

**Part C:**

\[
I_{0\text{-partC}} = 0.144 \quad I_{1\text{-partC}} := 10 \cdot I_{0\text{-partC}}
\]

\[
I_{2\text{-partC}} := 10 \cdot I_{0\text{-partC}}
\]

as a check: \[
I_{a\text{-partC}} := 3 \cdot I_{0\text{-partC}} \quad I_{a\text{-partC}} = 0.432 \text{ pu}
\]

\[
V_{1\text{-partC}} := V_{\text{fault}} - I_{1\text{-partC}} \cdot Z_1 \quad V_{1\text{-partC}} = 0.959i
\]

\[
V_{2\text{-partC}} := -I_{2\text{-partC}} \cdot Z_2 \quad V_{2\text{-partC}} = -0.041i
\]

\[
V_{0\text{-partC}} := -I_{0\text{-partC}} \cdot (Z_0 + 3 \cdot jX_{\text{gndpu}}) \quad V_{0\text{-partC}} = -0.917i
\]

\[
V_{\text{abc-partC}} = A_{012} \cdot \begin{pmatrix} V_{0\text{-partC}} \\ V_{1\text{-partC}} \\ V_{2\text{-partC}} \end{pmatrix}
\]

\[
V_{\text{abc-partC}} = \begin{pmatrix} 0 \\ 0.866 - 1.376i \\ -0.866 - 1.376i \end{pmatrix}
\]

\[
|V_{\text{abc-partC}}| = \begin{pmatrix} 0 \\ 1.626 \\ 1.626 \end{pmatrix}
\]

\[
\text{arg}(V_{\text{abc-partC}}) = \begin{pmatrix} 90 \text{ deg} \\ -57.812 \text{ deg} \\ -122.188 \text{ deg} \end{pmatrix}
\]

\[
V_{\text{ln}} \cdot |V_{\text{abc-partC}}| = \begin{pmatrix} 0 \\ 3.905 \text{ kV} \\ 3.905 \text{ kV} \end{pmatrix}
\]

**Part D:**

\[
Z_{\text{gndD}} := \frac{(3 \cdot R_{\text{gnd-pu}} + Z_0) \cdot (-jX_{\text{c-pu}})}{(3 \cdot R_{\text{gnd-pu}} + Z_0) - j \cdot X_{\text{c-pu}}}
\]

\[
I_{0\text{-partD}} := \frac{V_{\text{fault}}}{Z_{\text{gndD}} + Z_1 + Z_2} \quad I_{0\text{-partD}} = -9.771 \times 10^{-5} + 4.703i \times 10^{-4}
\]

\[
I_{1\text{-partD}} := I_{0\text{-partD}} \quad I_{2\text{-partD}} := I_{0\text{-partD}} \quad |I_{0\text{-partD}}| = 4.804 \times 10^{-4}
\]
as a check: \[ I_{a\_partD} := 3 \cdot I_{0\_partD} \quad |I_{a\_partD}| = 1.441 \times 10^{-3} \text{ pu} \]

\[ |I_{a\_partD}| \cdot I_{\text{base}} = 20.001 \cdot \text{A} \]

\[ V_{1\_partD} := V_{\text{fault}} - I_{1\_partD} \cdot Z_1 \quad |V_{1\_partD}| = 1 \]

\[ V_{2\_partD} := -I_{2\_partD} \cdot Z_2 \quad |V_{2\_partD}| = 1.38 \times 10^{-4} \]

\[ V_{0\_partD} := -I_{0\_partD} \cdot (Z_{\text{gndD}}) \quad |V_{0\_partD}| = 1 \]

\[ \begin{bmatrix} V_{0\_partD} \\ V_{1\_partD} \\ V_{2\_partD} \end{bmatrix} = A_{012} \cdot V_{abc\_partD} \]

\[ \begin{bmatrix} V_{abc\_partD} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.866 - 1.5i \\ -0.866 - 1.5i \end{bmatrix} \]

\[ |V_{abc\_partD}| = \begin{bmatrix} 0 \\ 1.732 \\ 1.732 \end{bmatrix} \]

\[ \text{arg}(V_{abc\_partD}) = \begin{bmatrix} 107.593 \\ -60.013 \text{ deg} \\ -120.01 \end{bmatrix} \]

\[ V_{\ln|V_{abc\_partD}|} = \begin{bmatrix} 0 \\ 4.16 \text{ kV} \\ 4.161 \text{ kV} \end{bmatrix} \]

F Compute the single line to ground fault current and the voltage on the unfaulted phases if the transformer is solidly grounded. Calculate the zero sequence voltages and currents.

\[ I_{0\_gnd} := \frac{V_{\text{fault}}}{Z_0 + Z_1 + Z_2} \quad I_{1\_gnd} := I_{0\_gnd} \quad I_{2\_gnd} := I_{0\_gnd} \]

\[ I_{a\_gnd} := 3 \cdot I_{0\_gnd} \quad |I_{a\_gnd}| = 4.239 \text{ per unit} \]

\[ I_{a\_gnd} \cdot I_{\text{base}} = 58.833 \cdot \text{kA} \]

\[ V_{1\_gnd} := V_{\text{fault}} - I_{1\_gnd} \cdot Z_1 \quad V_{1\_gnd} = 0.594i \]

\[ V_{2\_gnd} := -I_{2\_gnd} \cdot Z_2 \quad V_{2\_gnd} = -0.406i \]

\[ V_{0\_gnd} := -I_{0\_gnd} \cdot Z_0 \quad V_{0\_gnd} = -0.18841i \]
**G** For the different grounded cases described above, discuss the available quantities to measure for ground fault protection and suggest a scheme to consider (based on voltage, current, etc).

For the high resistance grounded case, there isn't enough current available for doing ground fault protection, but $V_0$ is 1pu, so there is enough voltage available to use that to identify the presence of a ground fault.

For the case with the low inductance grounded, there is sufficient $I_0$ for detecting a fault, although any fault impedance (resistance) may make this too difficult. There is also probably sufficient $V_0$ to use that to detect the fault.

For the solidly grounded case, $V_0$ is pretty small, and it might be hard to discriminate sufficiently to identify a fault. On the other hand, $I_0$ is large.

\[
\begin{align*}
V_{\text{abc}} := A_{012} & \begin{pmatrix}
V_{0\_\text{gnd}} + 10^{-16} \\
V_{1\_\text{gnd}} \\
V_{2\_\text{gnd}}
\end{pmatrix} \\
V_{\text{abc}} &= \begin{pmatrix}
0 \\
0.866 - 0.283i \\
-0.866 - 0.283i
\end{pmatrix} \\
|V_{\text{abc}}| &= \begin{pmatrix}
0 \\
0.911 \\
0.911
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\text{arg}(V_{\text{abc}}) &= \begin{pmatrix}
0 \\
-18.073 \\
-161.927
\end{pmatrix} \cdot \text{deg}
\end{align*}
\]

Note that $V_b$ and $V_c$ are nearly 1.0 per unit, and are slightly depressed. If $Z_0=Z_1=Z_2$ then they would be 1.0 and offset from each other by 120 degrees.