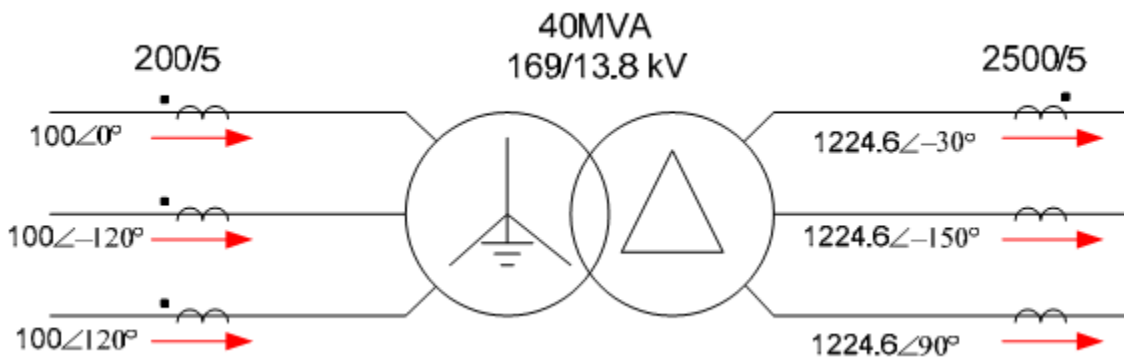


## Transformer Protection Example

- a) Consider the transformer shown. The transformer is protected by a digital current differential transformer relay. During routine maintenance a technician accidentally shorts out the C-phase CT on the HV side, determine whether the differential elements will operate and if so which “phase differential elements will operate.

Both set of CT are connected in wye (star). The matrix compensation is set so that correct phase correction is achieved. The TAP is set so that under full load conditions a one per unit “phase current” exists.



$$\text{MVA} := 1000\text{kW}$$

$$S_{\text{rated}} := 40\text{MVA}$$

$$V_{\text{HV}} := 169\text{kV}$$

$$V_{\text{LV}} := 13.8\text{kV}$$

$$\text{CTR}_{\text{HV}} := \frac{200}{5}$$

$$\text{CTR}_{\text{HV}} = 40$$

$$\text{CTR}_{\text{LV}} := \frac{2500}{5}$$

$$\text{CTR}_{\text{LV}} = 500$$

$$\text{Tap}_{\text{HV}} := \frac{S_{\text{rated}}}{\sqrt{3} \cdot V_{\text{HV}} \cdot \text{CTR}_{\text{HV}}}$$

$$\text{Tap}_{\text{HV}} = 3.4163 \text{ A}$$

$$\text{Tap}_{\text{LV}} := \frac{S_{\text{rated}}}{\sqrt{3} \cdot V_{\text{LV}} \cdot \text{CTR}_{\text{LV}}}$$

$$\text{Tap}_{\text{LV}} = 3.347 \text{ A}$$

$$\text{MAT}_{11} := \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\text{MAT}_0 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{MAT}_{12} := \frac{1}{3} \cdot \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$\text{MAT}_1 := \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

- Primary Currents

$$I_{A\_HV} := 100\text{A} \cdot e^{j \cdot 0\text{deg}}$$

$$I_{a\_LV} := 1224.6\text{A} \cdot e^{-j \cdot 30\text{deg}}$$

$$I_{B\_HV} := 100\text{A} \cdot e^{-j \cdot 120\text{deg}}$$

$$I_{b\_LV} := 1224.6\text{A} \cdot e^{-j \cdot 150\text{deg}}$$

$$I_{C\_HV} := 100\text{A} \cdot e^{j \cdot 120\text{deg}}$$

$$I_{c\_LV} := 1224.6\text{A} \cdot e^{j \cdot 90\text{deg}}$$

- Uncorrected CT Secondary Currents

$$\begin{pmatrix} I_{A_{HV\_sec}} \\ I_{B_{HV\_sec}} \\ I_{C_{HV\_sec}} \end{pmatrix} := \frac{1}{CTR_{HV}} \cdot \begin{pmatrix} I_{A\_HV} \\ I_{B\_HV} \\ I_{C\_HV} \end{pmatrix}$$

$$\begin{pmatrix} I_{a_{LV\_sec}} \\ I_{b_{LV\_sec}} \\ I_{c_{LV\_sec}} \end{pmatrix} := (-1) \cdot \left( \frac{1}{CTR_{LV}} \right) \cdot \begin{pmatrix} I_{a\_LV} \\ I_{b\_LV} \\ I_{c\_LV} \end{pmatrix}$$

- Note the (-1), the diagram doesn't have the LV side current matching the CT polarity

- Corrected currents for relay

$$\begin{pmatrix} I_{A_{HV\_cor}} \\ I_{B_{HV\_cor}} \\ I_{C_{HV\_cor}} \end{pmatrix} := \frac{1}{Tap_{HV}} \cdot \text{MAT}_{11} \cdot \begin{pmatrix} I_{A_{HV\_sec}} \\ I_{B_{HV\_sec}} \\ I_{C_{HV\_sec}} \end{pmatrix}$$

$$\begin{pmatrix} I_{a_{LV\_cor}} \\ I_{b_{LV\_cor}} \\ I_{c_{LV\_cor}} \end{pmatrix} := \frac{1}{Tap_{LV}} \cdot \text{MAT}_0 \cdot \begin{pmatrix} I_{a_{LV\_sec}} \\ I_{b_{LV\_sec}} \\ I_{c_{LV\_sec}} \end{pmatrix}$$

- Operate and Restraint Current

Phase A:

$$IOP_A := |I_{A_{HV\_cor}} + I_{a_{LV\_cor}}|$$

$$IRTA_A := |I_{A_{HV\_cor}}| + |I_{a_{LV\_cor}}|$$

$$IOP_A = 2.252 \times 10^{-5}$$

$$IRTA_A = 1.464$$

Phase B:

$$IOP_B := |IB_{HV\_cor} + IB_{LV\_cor}|$$

$$IOP_B = 2.252 \times 10^{-5}$$

$$IRTA_B := |IB_{HV\_cor}| + |IB_{LV\_cor}|$$

$$IRTA_B = 1.464$$

Phase C:

$$IOP_C := |IC_{HV\_cor} + IC_{LV\_cor}|$$

$$IOP_C = 2.252 \times 10^{-5}$$

$$IRTA_C := |IC_{HV\_cor}| + |IC_{LV\_cor}|$$

$$IRTA_C = 1.464$$

- Alternate approach

$$\begin{pmatrix} IA_{HV\_cor} \\ IB_{HV\_cor} \\ IC_{HV\_cor} \end{pmatrix} := \frac{1}{Tap_{HV}} \cdot MAT_{12} \cdot MAT_0 \cdot \begin{pmatrix} IA_{HV\_sec} \\ IB_{HV\_sec} \\ IC_{HV\_sec} \end{pmatrix} \quad \begin{pmatrix} IA_{LV\_cor} \\ IB_{LV\_cor} \\ IC_{LV\_cor} \end{pmatrix} := \frac{1}{Tap_{LV}} \cdot MAT_1 \cdot \begin{pmatrix} Ia_{LV\_sec} \\ Ib_{LV\_sec} \\ Ic_{LV\_sec} \end{pmatrix}$$

- Operate and Restraint Current

Phase A:

$$IOP_A := |IA_{HV\_cor} + IA_{LV\_cor}|$$

$$IOP_A = 2.252 \times 10^{-5}$$

$$IRTA_A := |IA_{HV\_cor}| + |IA_{LV\_cor}|$$

$$IRTA_A = 1.464$$

Phase B:

$$IOP_B := |IB_{HV\_cor} + IB_{LV\_cor}|$$

$$IOP_B = 2.252 \times 10^{-5}$$

$$IRTA_B := |IB_{HV\_cor}| + |IB_{LV\_cor}|$$

$$IRTA_B = 1.464$$

Phase C:

$$IOP_C := |IC_{HV\_cor} + IC_{LV\_cor}|$$

$$IOP_C = 2.252 \times 10^{-5}$$

$$IRTA_C := |IC_{HV\_cor}| + |IC_{LV\_cor}|$$

$$IRTA_C = 1.464$$

**Repeat with the phase C CT shorted on the HV side:**

- Uncorrected CT Secondary Currents

$$\begin{pmatrix} I_{AHV\_sec} \\ I_{BHV\_sec} \\ I_{CHV\_sec} \end{pmatrix} := \frac{1}{CTR_{HV}} \cdot \begin{pmatrix} I_{A\_HV} \\ I_{B\_HV} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} I_{aLV\_sec} \\ I_{bLV\_sec} \\ I_{cLV\_sec} \end{pmatrix} := (-1) \cdot \left( \frac{1}{CTR_{LV}} \right) \cdot \begin{pmatrix} I_{a\_LV} \\ I_{b\_LV} \\ I_{c\_LV} \end{pmatrix}$$

- Note the (-1), the diagram doesn't have the LV side current matching the CT polarity

- Corrected currents for relay

$$\begin{pmatrix} I_{AHV\_cor} \\ I_{BHV\_cor} \\ I_{CHV\_cor} \end{pmatrix} := \frac{1}{Tap_{HV}} \cdot MAT_{11} \cdot \begin{pmatrix} I_{AHV\_sec} \\ I_{BHV\_sec} \\ I_{CHV\_sec} \end{pmatrix}$$

$$\begin{pmatrix} I_{aLV\_cor} \\ I_{bLV\_cor} \\ I_{cLV\_cor} \end{pmatrix} := \frac{1}{Tap_{LV}} \cdot MAT_{0} \cdot \begin{pmatrix} I_{aLV\_sec} \\ I_{bLV\_sec} \\ I_{cLV\_sec} \end{pmatrix}$$

- Operate and Restraint Current, with shorted CT

Phase A:

$$IOP_A := |I_{AHV\_cor} + I_{aLV\_cor}|$$

$$IOP_A = 0.422$$

$$IRTA_A := |I_{AHV\_cor}| + |I_{aLV\_cor}|$$

$$IRTA_A = 1.154$$

Phase B:

$$IOP_B := |I_{BHV\_cor} + I_{bLV\_cor}|$$

$$IOP_B = 2.252 \times 10^{-5}$$

$$IRTA_B := |I_{BHV\_cor}| + |I_{bLV\_cor}|$$

$$IRTA_B = 1.464$$

Phase C:

$$IOP_C := |I_{CHV\_cor} + I_{cLV\_cor}|$$

$$IOP_C = 0.422$$

$$IRTA_C := |I_{CHV\_cor}| + |I_{cLV\_cor}|$$

$$IRTA_C = 1.154$$

- Due to the delta winding (and correction matrix) two of the differential elements respond.

Check the ratio of IOP to IRTA:

$$\frac{IOP_A}{IRTA_A} = 36.602\%$$

So if the relay slope is set larger than 36.602% the relay will not operate for this condition. Otherwise it will operate.

- Alternate approach

$$\begin{pmatrix} I_{A_{HV\_cor}} \\ I_{B_{HV\_cor}} \\ I_{C_{HV\_cor}} \end{pmatrix} := \frac{1}{Tap_{HV}} \cdot MAT_{12} \cdot MAT_0 \cdot \begin{pmatrix} I_{A_{HV\_sec}} \\ I_{B_{HV\_sec}} \\ I_{C_{HV\_sec}} \end{pmatrix} \quad \begin{pmatrix} I_{A_{LV\_cor}} \\ I_{B_{LV\_cor}} \\ I_{C_{LV\_cor}} \end{pmatrix} := \frac{1}{Tap_{LV}} \cdot MAT_1 \cdot \begin{pmatrix} I_{a_{LV\_sec}} \\ I_{b_{LV\_sec}} \\ I_{c_{LV\_sec}} \end{pmatrix}$$

- Operate and Restraint Current, with shorted CT

Phase A:

$$IOP_A := |I_{A_{HV\_cor}} + I_{A_{LV\_cor}}|$$

$$IOP_A = 0.244$$

$$IRTA_A := |I_{A_{HV\_cor}}| + |I_{A_{LV\_cor}}|$$

$$IRTA_A = 1.377$$

Phase B:

$$IOP_B := |I_{B_{HV\_cor}} + I_{B_{LV\_cor}}|$$

$$IOP_B = 0.244$$

$$IRTA_B := |I_{B_{HV\_cor}}| + |I_{B_{LV\_cor}}|$$

$$IRTA_B = 1.377$$

Phase C:

$$IOP_C := |I_{C_{HV\_cor}} + I_{C_{LV\_cor}}|$$

$$IOP_C = 0.488$$

$$IRTA_C := |I_{C_{HV\_cor}}| + |I_{C_{LV\_cor}}|$$

$$IRTA_C = 0.976$$

- Due to the delta winding (and correction matrix) two of the differential elements respond.

Check the ratio of IOPC to IRTC and IOPA to IRTA:

$$\frac{IOP_A}{IRTA_A} = 17.712\%$$

$$\frac{IOP_C}{IRTA_C} = 49.999\%$$

Note that the error is more significant in this case because the shorted CT was on HV side, and we are no longer scaling the current down as much.

So if these correction matrices are used, the relay slope need to be set larger than 50% the relay so the relay not operate for this condition. Otherwise it will operate.

*Now repeat if the shorted CT is on the delta side:*

- Uncorrected CT Secondary Currents

$$\begin{pmatrix} I_{AHV\_sec} \\ I_{BHV\_sec} \\ I_{CHV\_sec} \end{pmatrix} := \frac{1}{CTR_{HV}} \cdot \begin{pmatrix} I_{A\_HV} \\ I_{B\_HV} \\ I_{C\_HV} \end{pmatrix}$$

$$\begin{pmatrix} I_{aLV\_sec} \\ I_{bLV\_sec} \\ I_{cLV\_sec} \end{pmatrix} := (-1) \cdot \left( \frac{1}{CTR_{LV}} \right) \cdot \begin{pmatrix} I_{a\_LV} \\ I_{b\_LV} \\ 0A \end{pmatrix}$$

- Note the (-1), the diagram doesn't have the LV side current matching the CT polarity

- Corrected currents for relay

$$\begin{pmatrix} I_{AHV\_cor} \\ I_{BHV\_cor} \\ I_{CHV\_cor} \end{pmatrix} := \frac{1}{Tap_{HV}} \cdot MAT_{11} \cdot \begin{pmatrix} I_{AHV\_sec} \\ I_{BHV\_sec} \\ I_{CHV\_sec} \end{pmatrix}$$

$$\begin{pmatrix} I_{aLV\_cor} \\ I_{bLV\_cor} \\ I_{cLV\_cor} \end{pmatrix} := \frac{1}{Tap_{LV}} \cdot MAT_0 \cdot \begin{pmatrix} I_{aLV\_sec} \\ I_{bLV\_sec} \\ I_{cLV\_sec} \end{pmatrix}$$

- Operate and Restraint Current, with shorted CT

Phase A:

$$IOP_A := |I_{AHV\_cor} + I_{aLV\_cor}|$$

$$IRTA_A := |I_{AHV\_cor}| + |I_{aLV\_cor}|$$

$$IOP_A = 2.252 \times 10^{-5}$$

$$IRTA_A = 1.464$$

Phase B:

$$IOP_B := |I_{BHV\_cor} + I_{bLV\_cor}|$$

$$IRTA_B := |I_{BHV\_cor}| + |I_{bLV\_cor}|$$

$$IOP_B = 2.252 \times 10^{-5}$$

$$IRTA_B = 1.464$$

Phase C:

$$IOP_C := |I_{CHV\_cor} + I_{cLV\_cor}|$$

$$IRTA_C := |I_{CHV\_cor}| + |I_{cLV\_cor}|$$

$$IOP_C = 0.732$$

$$IRTA_C = 0.732$$

- Due to the delta winding (and correction matrix) two of the differential elements respond.

Check the ratio of IOP to IRTA:

$$\frac{IOP_C}{IRTA_C} = 100\%$$

So now the relay slope needs to be set larger than 100% so the relay will not operate for this condition. Otherwise it will operate.

- Alternate approach

$$\begin{pmatrix} I_{AHV\_cor} \\ I_{BHV\_cor} \\ I_{CHV\_cor} \end{pmatrix} := \frac{1}{Tap_{HV}} \cdot MAT_{12} \cdot MAT_0 \cdot \begin{pmatrix} I_{AHV\_sec} \\ I_{BHV\_sec} \\ I_{CHV\_sec} \end{pmatrix} \quad \begin{pmatrix} I_{ALV\_cor} \\ I_{BLV\_cor} \\ I_{CLV\_cor} \end{pmatrix} := \frac{1}{Tap_{LV}} \cdot MAT_1 \cdot \begin{pmatrix} I_{aLV\_sec} \\ I_{bLV\_sec} \\ I_{cLV\_sec} \end{pmatrix}$$

- Operate and Restraint Current, with shorted CT

Phase A:

$$IOP_A := |I_{AHV\_cor} + I_{ALV\_cor}|$$

$$IRTA_A := |I_{AHV\_cor}| + |I_{ALV\_cor}|$$

$$IOP_A = 2.252 \times 10^{-5}$$

$$IRTA_A = 1.464$$

Phase B:

$$IOP_B := |I_{BHV\_cor} + I_{BLV\_cor}|$$

$$IRTA_B := |I_{BHV\_cor}| + |I_{BLV\_cor}|$$

$$IOP_B = 0.423$$

$$IRTA_B = 1.154$$

Phase C:

$$IOP_C := |I_{CHV\_cor} + I_{CLV\_cor}|$$

$$IRTA_C := |I_{CHV\_cor}| + |I_{CLV\_cor}|$$

$$IOP_C = 0.423$$

$$IRTA_C = 1.154$$

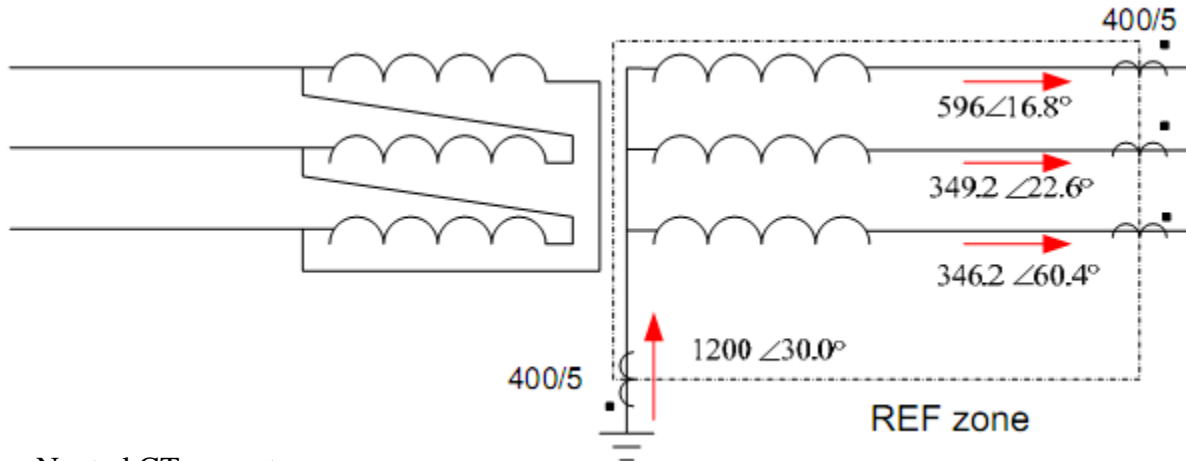
- Due to the delta winding (and correction matrix) two of the differential elements respond.

Check the ratio of  $IOP_C$  to  $IRTC$  and  $IOPA$  to  $IRTA$ :

$$\frac{IOP_C}{IRTA_C} = 36.604\%$$

So now we back to the case that if the relay slope is set larger than 36.602% the relay will not operate for this condition. Otherwise it will operate.

b) Consider the restricted earth fault protection scheme shown below. Determine whether the earth fault is inside or outside the protected zone, give reasons for your answer.



Neutral CT current:

$$I_N := 1200 \cdot e^{j \cdot 30 \text{deg}}$$

$$CTR_{\text{Neutral}} := \frac{400}{5}$$

LV CT currents (note negative sign due to polarity of CT compared to labelled current)

$$I_{A\_LV} := -596 \cdot e^{j \cdot 16.8 \text{deg}}$$

$$CTR_{\text{phase}} := \frac{400}{5}$$

$$I_{B\_LV} := -349.2 \cdot e^{j \cdot 22.6 \cdot \text{deg}}$$

$$I_{C\_LV} := -346.2 \cdot e^{j \cdot 60.4 \text{deg}}$$

$$I_R := I_{A\_LV} + I_{B\_LV} + I_{C\_LV}$$

$$|I_R| = 1225.161 \quad \arg(I_R) = -150.275 \cdot \text{deg}$$

$$I_{N\_sec} := \frac{I_N}{CTR_{\text{Neutral}}}$$

$$I_{R\_sec} := \frac{I_R}{CTR_{\text{phase}}}$$

$$\text{Torque}_{\text{REF}} := \text{Re}(I_{N\_sec} \cdot \overline{I_{R\_sec}})$$

$$\text{Torque}_{\text{REF}} = -229.715$$

- Negative torque implies fault is out of zone.



Alternate approach:

$$I_{OP} := |I_{N\_sec} + I_{R\_sec}| \quad I_{OP} = 0.323$$

$$I_{RT} := |I_{N\_sec}| + |I_{R\_sec}| \quad I_{RT} = 30.315$$

$$\frac{I_{OP}}{I_{RT}} = 0.011$$

Note, if we don't reverse the polarity on the phase CT currents we get:

$$I_{OP} := |I_{N\_sec} - I_{R\_sec}| \quad I_{OP} = 30.314$$

$$\text{Then } \frac{I_{OP}}{I_{RT}} = 1 \quad \text{and the relay would operate.}$$