This Mathcad CT simulation is based on the paper "Computer Simulation of Current Transformers and Relays For Performance Analysis" by R.Garrett, W.C. Kotheimer, and S.E. Zocholl, presented before the 14th Annual Western Protective Relay Conference, October 20-23, 1987.

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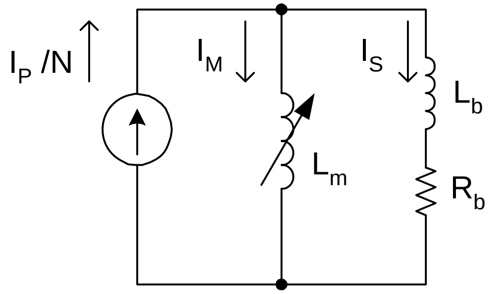
May 7, 2003
Modified by B.K. Johnson

Current Transformer Model



$CY := 6$ Length of simulation in cycles

$I_{mag} := 16000$ RMS magnitude of CT primary current

$X := 38$ Power system inductive reactance component of X/R

$R := 2$ Power system resistance component of X/R

$R_B := 8$ Resistive burden. Reactive burden set to 20% R_B

$X_B := 1$ Reactive Burden in Ohms

$I_{rated} := 5$ CT rated secondary current

$N := 240$ CT turns ratio

$V_{RAT} := 800$ CT "C-Rating"

$Rem := 0.$ Per Unit Remnant Flux

$f := 60$ Frequency (leave units out so plotting works)

$\omega := 2 \cdot \pi \cdot f$ Angular frequency (leave units out so plotting works)

$L_b := \frac{X_B}{\omega}$ Calculate burden inductance (keep unitless to simplify other calculations later) $L_b = 2.653 \times 10^{-3}$

$V_{RAT} := V_{RAT} \cdot (1 - Rem)$ Include effects of remnant flux

$$\left(1+\frac{X}{R}\right)\cdot\frac{I_{mag}}{I_{rated}\cdot N}\cdot\frac{\left|R_B+j\cdot\omega\cdot L_b\right|\cdot 100}{V_{RAT}} = 268.7$$ If this is less than 20, the CT satisfies criterion to avoid saturation entirely

Simulation Time and Indexing

$\Delta t := .00001 \quad i := 1\,..\,\mathrm{ceil}\left(\frac{CY}{f\cdot\Delta t}\right) \qquad t_i := i\cdot\Delta t$ Time step is 10 microseconds

Primary Current Definition

$\theta := \mathrm{atan}\left(\frac{X}{R}\right) \qquad \tau := \frac{X}{\omega\cdot R} \qquad \tau = 0.05 \qquad \theta = 86.987\cdot deg$

$\phi := \theta - 90\cdot deg$

$$I_i := \sqrt{2}\cdot I_{mag}\cdot\left(\sin\left(\omega\cdot t_i+\phi-\theta\right)-e^{\frac{-t_i}{\tau}}\cdot\sin(\phi-\theta)\right)$$

Magnetic Values

$\mu_0 := 4\cdot\pi\cdot 10^{-7} \quad \mu_r := 15000 \qquad B_{sat} := 1.8$

Calculate cross sectional area of CT core

$A := \frac{V_{RAT}}{\omega\cdot N\cdot B_{sat}} \qquad A = 4.912\times 10^{-3}$

Mean core length

$L := .75$

Frolich Equation

$$c := \frac{1}{\mu_0 \cdot \mu_r} \qquad b := \frac{1 - \frac{1}{\sqrt{\mu_r}}}{B_{sat}}$$

- Magnetic Flux Density as a function of primary current:

$$B(I_P) := \frac{I_P}{c + b \cdot |I_P|}$$

- Magnetizing inductance as a function of flux density:

$$L(I_P) := \left(\frac{d}{dI_P} B(I_P)\right) \cdot \frac{A \cdot N^2}{L}$$

Magnetizing inductance with no remnance

$$L_m := L(0) \qquad L_m = 7.111 \qquad X_{m\_prefault} := 2 \cdot \pi \cdot 60Hz \cdot L_m \cdot H \qquad X_{m\_prefault} = 2.681 \cdot k\Omega$$

- Find magnetizing current

1. Use the following differential equation for dI_S and substitute into $I_{S(i)} = I_{S(i-1)} + DI_S$. Recognize that $L_M = L(I_P - N^* I_S)$.
2. Work from:

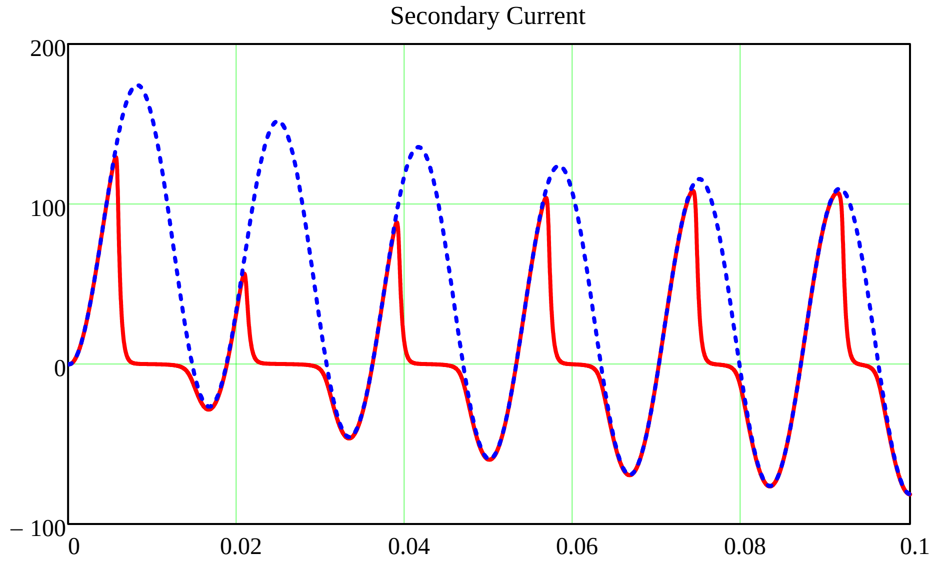
$$L_m \cdot \left(\frac{1}{N} \cdot \frac{d}{dt} I_P - \frac{d}{dt} I_S\right) - L_B \cdot \frac{d}{dt} I_S - R_B \cdot I_S = 0$$

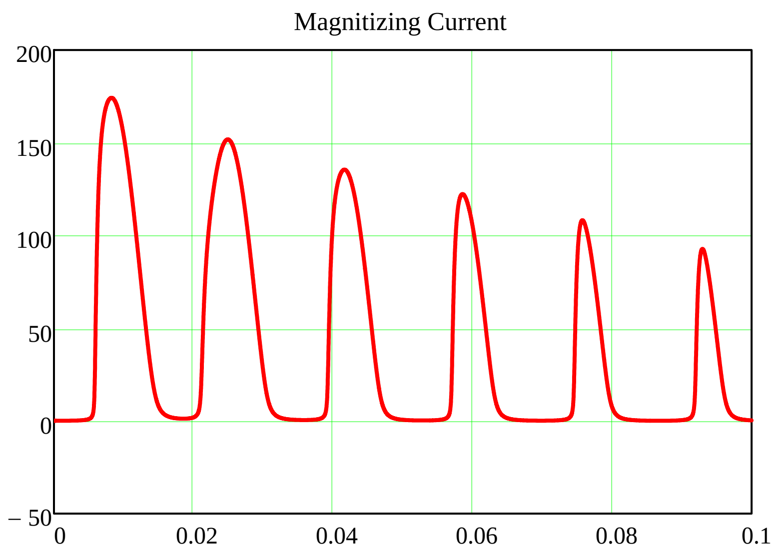
- Define a term for i=0 $\qquad I_{S_0} := 0$

$$I_{S_i} := I_{S_{i-1}} + \left[\frac{\left(L\left(I_{i-1} - N \cdot I_{S_{i-1}}\right)\right) \cdot \left(I_i - I_{i-1}\right)}{N} - R_B \cdot I_{S_{i-1}} \cdot \Delta t\right] \cdot \frac{1}{\left(\frac{X_B}{\omega} + 5 \cdot 10^{-5}\right) + L\left(I_{i-1} - N \cdot I_{S_{i-1}}\right)}$$

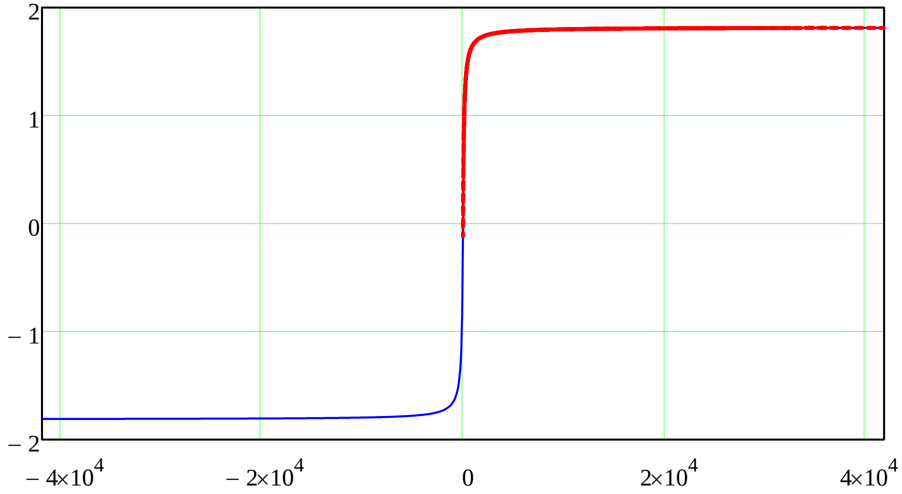
Calculate Magnetizing Current

$$I_{M_i} := \frac{I_i}{N} - I_{S_i}$$

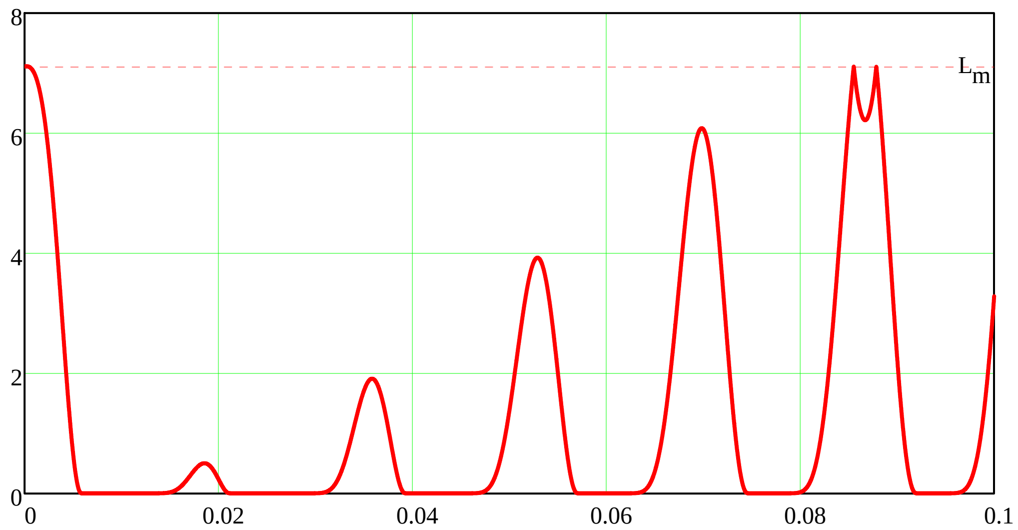


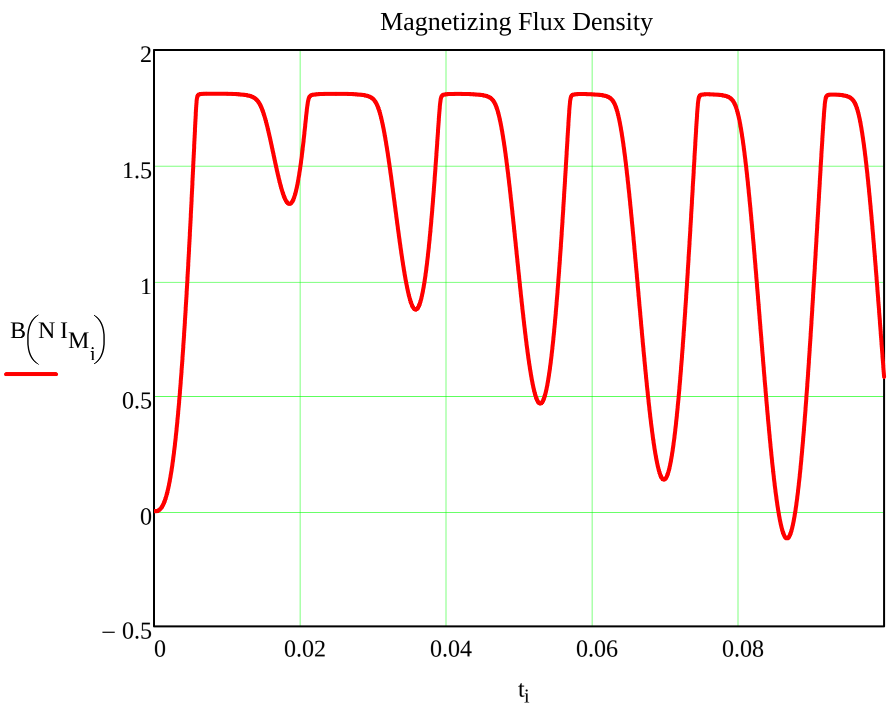

Magnitizing Current
200
150
100
50
0
− 50
0
0.02
0.04
0.06
0.08
0.1

Magnitizing Current on B-H Curve



Magnitizing Branch Inductance




Magnetizing Flux Density
2
1.5
1
0.5
0
– 0.5
0
0.02
0.04
0.06
0.08
t_i
$B\left(N\, I_{M_i}\right)$

Simple Digital Filter Calculation

$RS := 16$

Enter the number of samples per cycle of the relay

$LPW := \text{floor}\left(\frac{2}{60 \cdot \Delta t \cdot RS}\right)$

Calculate the number of samples to create an averaging LP filter with at cutoff frequency at 1/2 the sampling frequency.

$LP(a) := \left(1 + \frac{1}{RS}\right) \cdot \sum_{k=0}^{LPW-1} \frac{IS_{a-LPW+k}}{LPW}$

Averaging Filter

$ii := LPW .. \text{ceil}\left(\frac{CY}{f \cdot \Delta t}\right)$

$I_{ii} := LP(ii)$

Calculate filtered current

$S := CY \cdot RS$

Calculate the number of relay samples available in the data and create an index "s" as a row pointer

$s := 2 .. S$

$Ia_s := \text{linterp}\left(t, I, \frac{s}{RS \cdot 60}\right)$

Create a vector "Ia" representing the relay's sampled values

$if := (RS - 1) .. S$

Create a filter index, "if" and apply a full cycle cosine filter, "IF" to vector "Ia"

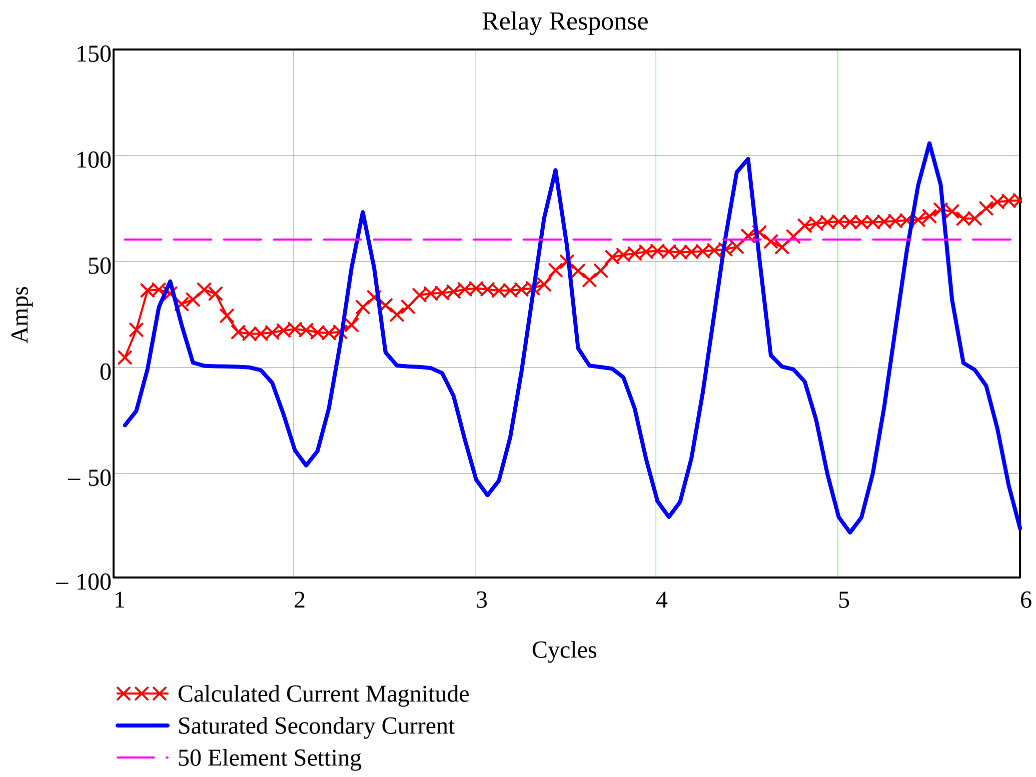
$IF_{if} := \frac{2}{RS} \cdot \sum_{k=0}^{RS-1} \left[\cos\left(k \cdot \frac{2 \cdot \pi}{RS}\right) \cdot Ia_{[if-(RS-1)]+k}\right]$

Cosine Filter

$iv := (RS + 1) .. S$

Create a vector index, "iv" and form a complex vector, "Icpx" from filtered quatities at 90 degree intervals

$Icpx_{iv} := IF_{iv} + j \cdot IF_{iv-\frac{RS}{4}}$


Relay Response
150
100
50
0
− 50
− 100
Amps
1
2
3
4
5
6
Cycles
Calculated Current Magnitude
Saturated Secondary Current
50 Element Setting