University of Idaho

ECE 525

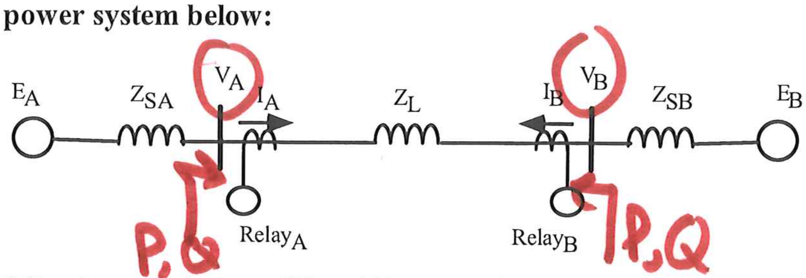
POWER SYSTEM PROTECTION AND RELAYING

SESSION no. 7

ECE 525: Homework #1

Solutions

1. Given the power system below:



Where the following are given as CT and VT secondary quantities::

$E_{SA} := 70V \cdot e^{j \cdot 0deg}$ $\quad E_{SB} := 70V \cdot e^{-j \cdot 30deg}$

$Z_{SA1} := 1.5ohm \cdot e^{j \cdot 87deg}$ $\quad Z_{SA2} := Z_{SA1}$ $\quad Z_{SA0} := 5ohm \cdot e^{j \cdot 87deg}$

$Z_{SB1} := 0.8ohm \cdot e^{j \cdot 83deg}$ $\quad Z_{SB2} := Z_{SB1}$ $\quad Z_{SB0} := 2.5ohm \cdot e^{j \cdot 83deg}$

$Z_{L1} := 5ohm \cdot e^{j \cdot 82deg}$ $\quad Z_{L0} := 18ohm \cdot e^{j \cdot 82deg}$ $\quad Z_{L2} := Z_{L1}$

The current trransformer ratios are: $CTR := \frac{1200}{5}$

The voltage transformer ratios are: $VTR := \frac{132.8kV}{70V}$ Line-to-neutral

A. Calculate the source voltages, line and source impedances and line current referred to primary values based on the information given above. Also find the line currents in secondary Amps accounting for CT polarity.

$Z_{SA1\_prim} := \frac{VTR}{CTR} \cdot Z_{SA1}$ $\quad \left| Z_{SA1\_prim} \right| = 11.86\,\Omega$ $\quad \arg\left(Z_{SA1\_prim}\right) = 87 \cdot deg$

$Z_{SA2\_prim} := \frac{VTR}{CTR} \cdot Z_{SA2}$ $\quad \left| Z_{SA2\_prim} \right| = 11.86\,\Omega$ $\quad \arg\left(Z_{SA2\_prim}\right) = 87 \cdot deg$

$Z_{SA0\_prim} := \frac{VTR}{CTR} \cdot Z_{SA0}$ $\quad \left| Z_{SA0\_prim} \right| = 39.52\,\Omega$ $\quad \arg\left(Z_{SA0\_prim}\right) = 87 \cdot deg$

$Z_{SB1\_prim} := \frac{VTR}{CTR} \cdot Z_{SB1}$ $\quad \left| Z_{SB1\_prim} \right| = 6.32\,\Omega$ $\quad \arg\left(Z_{SB1\_prim}\right) = 83 \cdot deg$

$Z_{SB2\_prim} := \frac{VTR}{CTR} \cdot Z_{SB2}$ $\quad \left| Z_{SB2\_prim} \right| = 6.32\,\Omega$ $\quad \arg\left(Z_{SB2\_prim}\right) = 83 \cdot deg$

$Z_{SB0\_prim} := \frac{VTR}{CTR} \cdot Z_{SB0}$ $\quad \left| Z_{SB0\_prim} \right| = 19.76\,\Omega$ $\quad \arg\left(Z_{SB0\_prim}\right) = 83 \cdot deg$

D. For the conditions of part A, calculate the effective imedance measured by Relay A and Relay B in terms of secondary values.

7 2/2

- Line to ground element:
  - Assume balanced three phase: $I_{0\_RA} := 0A$ $I_{0\_RB} := 0A$

$$k_0 := \frac{Z_{L0} - Z_{L1}}{Z_{L0}} \qquad k_0 = 0.72$$

$$V_{AG\_sec\_relayA} := \frac{V_{A\_BUSA\_LG}}{VTR} \qquad \left|V_{AG\_sec\_relayA}\right| = 68.95\,V$$

$$\arg\left(V_{AG\_sec\_relayA}\right) = -6.09 \cdot deg$$

$$Z_{AG\_eff\_RelayA} := \frac{V_{AG\_sec\_relayA}}{I_{RelayA\_sec} + k_0 \cdot I_{0\_RA}} \qquad \left|Z_{AG\_eff\_RelayA}\right| = 13.88\,\Omega$$

$$\arg\left(Z_{AG\_eff\_RelayA}\right) = 2.05 \cdot deg$$

0 – balanced

- Phase to phase element

$$I_{RelayA\_sec\_phaseB} := I_{RelayA\_sec} \cdot e^{-j \cdot 120deg}$$

$$V_{AG\_sec\_relayA\_phaseB} := V_{AG\_sec\_relayA} \cdot e^{-j \cdot 120deg}$$

$$Z_{AB\_relayA} := \frac{V_{AG\_sec\_relayA} - V_{AG\_sec\_relayA\_phaseB}}{I_{RelayA\_sec} - I_{RelayA\_sec\_phaseB}} \qquad \left|Z_{AB\_relayA}\right| = 13.88\,\Omega$$

$$\arg\left(Z_{AB\_relayA}\right) = 2.05 \cdot deg$$

Compare to line impedance $\left|Z_{L1}\right| = 5\,\Omega$ $\arg\left(Z_{L1}\right) = 82 \cdot deg$

- Line to ground element at relay B

$$V_{AG\_sec\_relayB} := \frac{V_{A\_BUSB\_LG}}{VTR} \qquad \left|V_{AG\_sec\_relayB}\right| = 69.09\,V$$

$$\arg\left(V_{AG\_sec\_relayB}\right) = -26.81 \cdot deg$$

$$Z_{AG\_eff\_RelayB} := \frac{V_{AG\_sec\_relayB}}{I_{RelayB\_sec} + k_0 \cdot I_{0\_RB}} \qquad \left|Z_{AG\_eff\_RelayB}\right| = 13.91\,\Omega$$

$$\arg\left(Z_{AG\_eff\_RelayB}\right) = 161.32 \cdot deg$$

$$I_{RelayB\_sec\_phaseB} := I_{RelayB\_sec} \cdot e^{-j \cdot 120deg}$$

$$V_{AG\_sec\_relayB\_phaseB} := V_{AG\_sec\_relayB} \cdot e^{-j \cdot 120deg}$$

- Line to line element at Relay B

$$Z_{AB\_relayB} := \frac{V_{AG\_sec\_relayB} - V_{AG\_sec\_relayB\_phaseB}}{I_{RelayB\_sec} - I_{RelayB\_sec\_phaseB}}$$

$$\left| Z_{AB\_relayB} \right| = 13.91\,\Omega$$

$$\arg\left(Z_{AB\_relayB}\right) = 161.32 \cdot deg$$

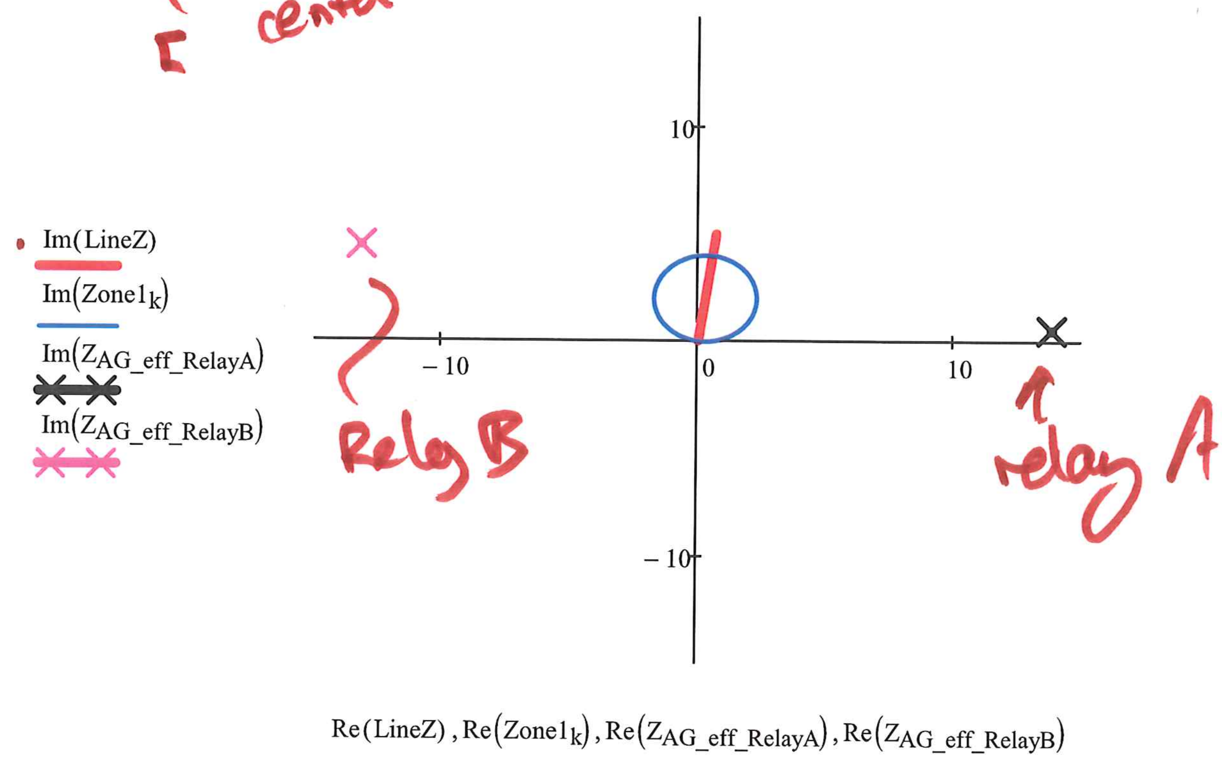
- Not required for assignment, but to help visualize this result, lets plot these in the impedance plane
  - First draw a mho characteristic, and reason the problem out a bit. I'm using Zone 1, set to 80% of the line

$k := 0, 1 .. 719$

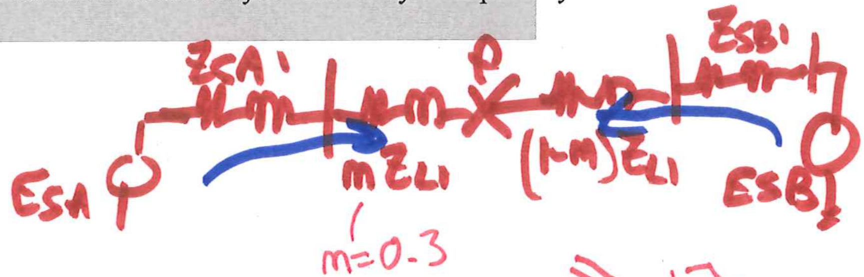
- Create vector:

$$rad_{Mhozone1} := 0.8 \cdot \frac{\left|Z_{L1}\right|}{2} \qquad offset_{Mhozone1} := 0.8 \cdot \frac{Z_{L1}}{2} \qquad LineZ := \begin{pmatrix} 0 \\ Z_{L1} \end{pmatrix}$$

$$Zone1_k := offset_{Mhozone1} + rad_{Mhozone1} \cdot e^{j \cdot k \cdot 0.5deg}$$



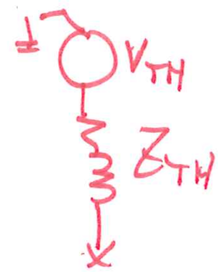
E. Suppose a 3 phase fault occurs 30% of the way from Bus A to Bus B, do the following:
(1) Calculate the total fault current and the current seen at Relay A and Relay B in primary and secondary quantities.



$$Z_{left1} := Z_{SA1} + 0.3 \cdot Z_{L1}$$

$$Z_{right1} := (1 - 0.3) \cdot Z_{L1} + Z_{SB1}$$

$$Z_{thev1} := \left(\frac{1}{Z_{left1}} + \frac{1}{Z_{right1}} \right)^{-1} \qquad Z_{thev1} = (0.2 + 1.76i)\,\Omega$$



- Since we are assuming no load flow, the prefault voltage is the same everywhere in the system. Therefore:
  - Total fault current

$$V_{fault} := E_{SA}$$

- Secondary Amps

$$I_{f\_3ph\_sec} := \frac{V_{fault}}{Z_{thev1}} \qquad \left| I_{f\_3ph\_sec} \right| = 39.63\,A \qquad \arg\left(I_{f\_3ph\_sec}\right) = -83.55 \cdot deg$$

- Primary Amps

$$I_{f\_3ph\_prim} := I_{f\_3ph\_sec} \cdot CTR \qquad \left| I_{f\_3ph\_prim} \right| = 9510.52\,A$$

$$\arg\left(I_{f\_3ph\_prim}\right) = -83.55 \cdot deg$$

- Fault current at Relay A
  - Secondary Amps first:
    - Use current divider

$$I_{f\_RelayA\_3ph\_sec} := I_{f\_3ph\_sec} \cdot \frac{Z_{right1}}{Z_{left1} + Z_{right1}} \qquad \left| I_{f\_RelayA\_3ph\_sec} \right| = 23.36\,A$$

$$\arg\left(I_{f\_RelayA\_3ph\_sec}\right) = -84.5 \cdot deg$$

$$I_{f\_RelayA\_3ph\_prim} := I_{f\_RelayA\_3ph\_sec} \cdot CTR \qquad \left| I_{f\_RelayA\_3ph\_prim} \right| = 5605.34\,A$$

$$\arg\left(I_{f\_RelayA\_3ph\_prim}\right) = -84.5 \cdot deg$$

L7 5/9

$\left|V_{fA\_BUSA\_3ph}\right| = 66.46 \cdot kV \qquad arg\left(V_{fA\_BUSA\_3ph}\right) = -2.5 \cdot deg$

$V_{fA\_BUSB\_3ph} := E_{SA\_prim} - I_{f\_RelayB\_3ph\_prim} \cdot Z_{SB1\_prim}$

$\left|V_{fA\_BUSB\_3ph}\right| = 108.1 \cdot kV \qquad arg\left(V_{fA\_BUSB\_3ph}\right) = -0.19 \cdot deg$

- Note that I used the same source voltage for each since I'm ignoring load flow

- Line to ground element at Relay A:
  - Assume balanced three phase fault: $I_{0\_RA} := 0A \qquad I_{0\_RB} := 0A$

$$V_{f\_AG\_3ph\_sec\_relayA} := \frac{V_{fA\_BUSA\_3ph}}{VTR} \qquad \left|V_{f\_AG\_3ph\_sec\_relayA}\right| = 35.03\,V$$

$$arg\left(V_{f\_AG\_3ph\_sec\_relayA}\right) = -2.5 \cdot deg$$

$$Z_{f\_3ph\_AG\_eff\_RelayA} := \frac{V_{f\_AG\_3ph\_sec\_relayA}}{I_{f\_RelayA\_3ph\_sec} + k_0 \cdot I_{0\_RA}} \qquad \left|Z_{f\_3ph\_AG\_eff\_RelayA}\right| = 1.5\,\Omega \qquad arg\left(Z_{f\_3ph\_AG\_eff\_RelayA}\right) = 82 \cdot deg$$

- Phase to phase element at Relay A

$$I_{f\_RelayA\_3ph\_sec\_ph\_B} := I_{f\_RelayA\_3ph\_sec} \cdot e^{-j \cdot 120deg}$$

$$V_{f\_AG\_3ph\_sec\_relayA\_phB} := V_{f\_AG\_3ph\_sec\_relayA} \cdot e^{-j \cdot 120deg}$$

$$Z_{f\_3ph\_AB\_relayA} := \frac{V_{f\_AG\_3ph\_sec\_relayA} - V_{f\_AG\_3ph\_sec\_relayA\_phB}}{I_{f\_RelayA\_3ph\_sec} - I_{f\_RelayA\_3ph\_sec\_ph\_B}}$$

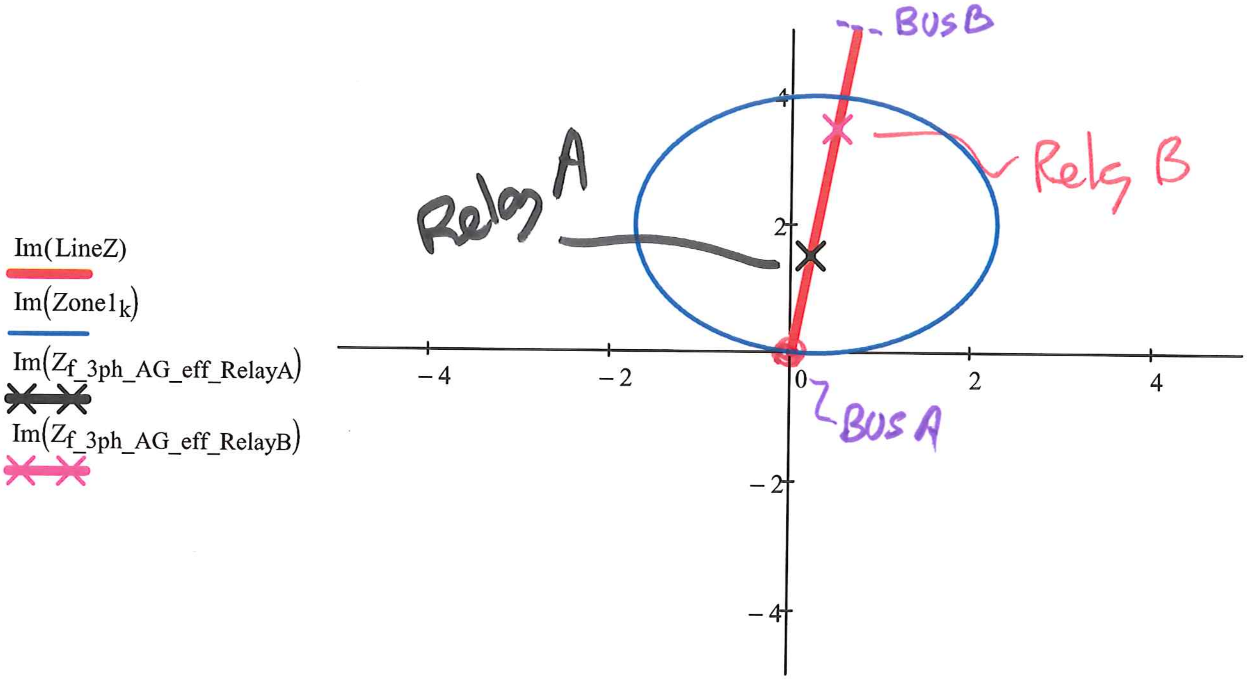
$$\left|Z_{f\_3ph\_AB\_relayA}\right| = 1.5\,\Omega$$

$$arg\left(Z_{f\_3ph\_AB\_relayA}\right) = 82 \cdot deg$$

Compare to line impedance $\quad 0.3\left|Z_{L1}\right| = 1.5\,\Omega \quad arg\left(0.3 \cdot Z_{L1}\right) = 82 \cdot deg$

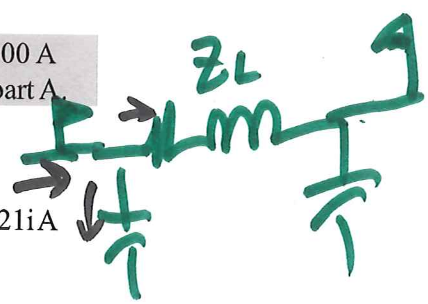
So consistent results

- Not required for assignment, but to help visualize this result, lets plot these in the impedance plane. Compare this figure to the one above with the load currents. Note also the change in scale



$\mathrm{Re(LineZ)}, \mathrm{Re}(\mathrm{Zone1}_k), \mathrm{Re}(Z_{f\_3ph\_AG\_eff\_RelayA}), \mathrm{Re}(Z_{f\_3ph\_AG\_eff\_RelayB})$

(4) What Iop and Irt be for a differential element (assume a charging current of 100 A capacitive divided equally between each end), for the fault and load flow of part A.



$$I_{cap\_secondary\_busA} := \frac{\frac{100}{2} A \cdot e^{j \cdot 90deg}}{CTR} \qquad I_{cap\_secondary\_busA} = 0.21i\,A$$

$$I_{cap\_secondary\_busB} := \frac{\frac{100}{2} A \cdot e^{j \cdot 90deg}}{CTR} \qquad I_{cap\_secondary\_busB} = 0.21i\,A$$

- Load flow case (only do for 1 phase since same for all three)

$$I_{OP\_loadflow} := \left| I_{RelayA\_sec} + I_{RelayB\_sec} + I_{cap\_secondary\_busA} + I_{cap\_secondary\_busB} \right|$$

$$I_{OP\_loadflow} = 0.42\,A$$

$\Sigma = 0$

If $\left(1 + \frac{X}{R}\right)\left(I_{f_{normalize}}\right)\left(Z_{B_{normalize}}\right) < 20$

→ $Z_B / Z_{STANDARD}$

Guaranteed "not to saturate" at C-class CT → significantly

⇒ C-Class

Error < 10%

- 20 × nominal current (5A)
- Standard Burden → (ex) 8Ω C800

C800 $20 \times 5 \cdot 8\,\Omega = 800V$

L7 8/9 ~~L6 11/11~~

CT doesn't saturate if 20 × Irated ∘ Zst < 800

C800

Normalized equat.

assumes worst → $\left(1+\frac{X}{R}\right)\cdot\frac{I_{mag}}{I_{rated}N}\cdot\frac{\left|R_B + j\cdot\omega\cdot L_b\right|\cdot 100}{V_{RAT}} = 268.7$ If this is less than 20, the CT satisfies criterion to avoid saturation entirely

Ifault; 5.240; Isec / Irated; (VRATED / 100)

Simulation Time and Indexing

$\Delta t := .00001 \quad i := 1 .. \text{ceil}\left(\frac{CY}{f\cdot\Delta t}\right) \qquad t_i := i\cdot\Delta t$ Time step is 10 microseconds

Primary Current Definition

$\theta := \text{atan}\left(\frac{X}{R}\right) \qquad \tau := \frac{X}{\omega\cdot R} \qquad \tau = 0.05 \qquad \theta = 86.987\cdot\text{deg}$

$\phi := \theta - 90\cdot\text{deg}$

$$I_i := \sqrt{2}\cdot I_{mag}\cdot\left(\sin\left(\omega\cdot t_i + \phi - \theta\right) - e^{\frac{-t_i}{\tau}}\cdot\sin(\phi - \theta)\right)$$

Magnetic Values

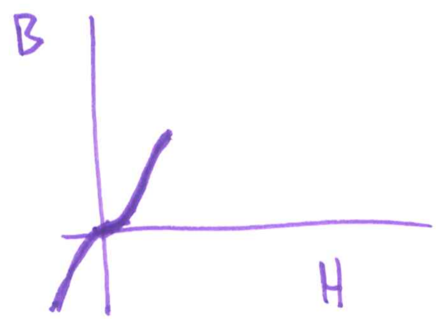
$\mu_o := 4\cdot\pi\cdot 10^{-7} \qquad \mu_r := 15000 \qquad B_{sat} := 1.8$

Calculate cross sectional area of CT core

$A := \frac{V_{RAT}}{\omega\cdot N\cdot B_{sat}} \qquad A = 4.912\times 10^{-3}$

Mean core length

$L := .75$


B
H