

ECE 526

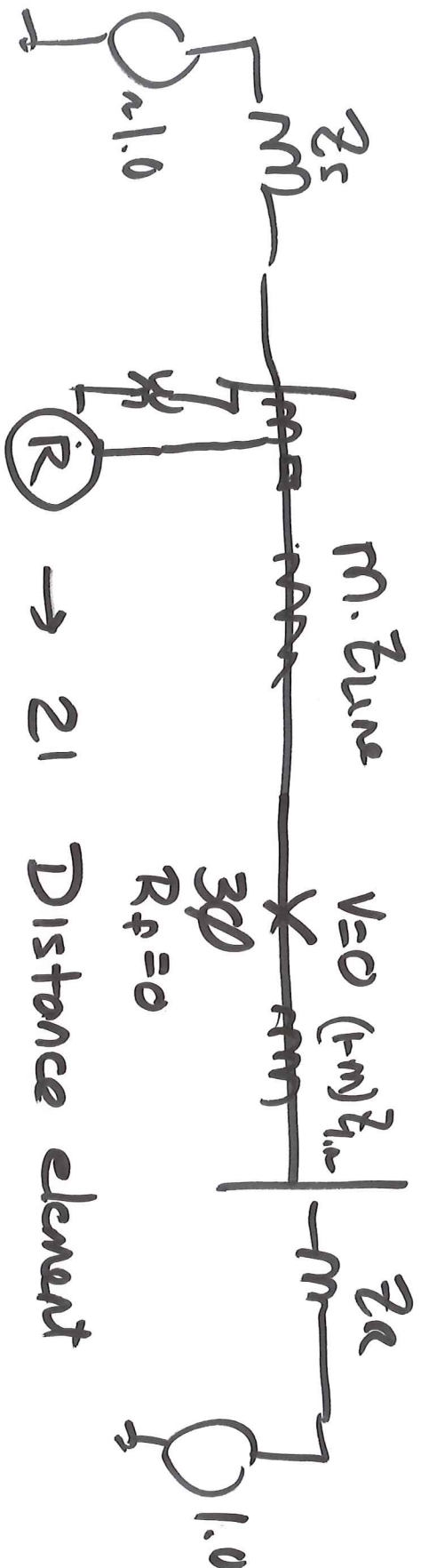
PROTECTION OF  
POWER SYSTEMS II

SESSION no. 4

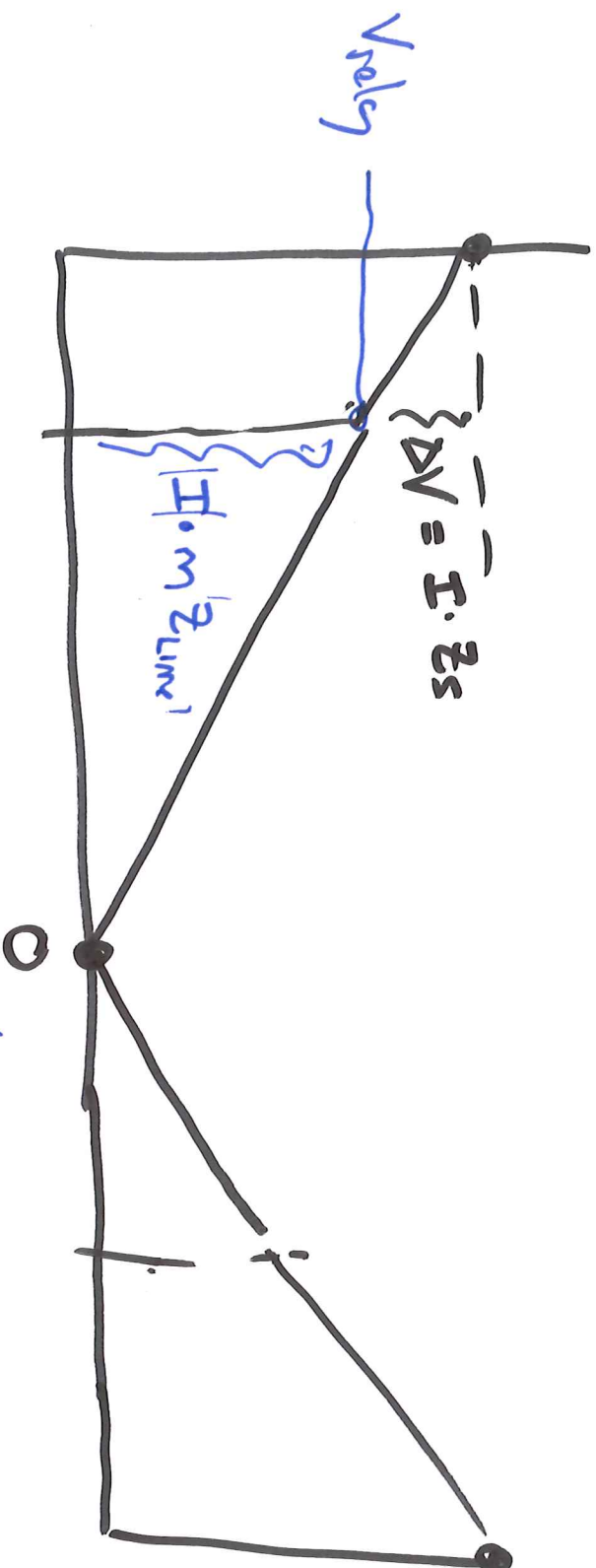
Scheme that largely independent  
of  $Z_{SRC}$  (impedance behind the  
relay)

→ Basic idea

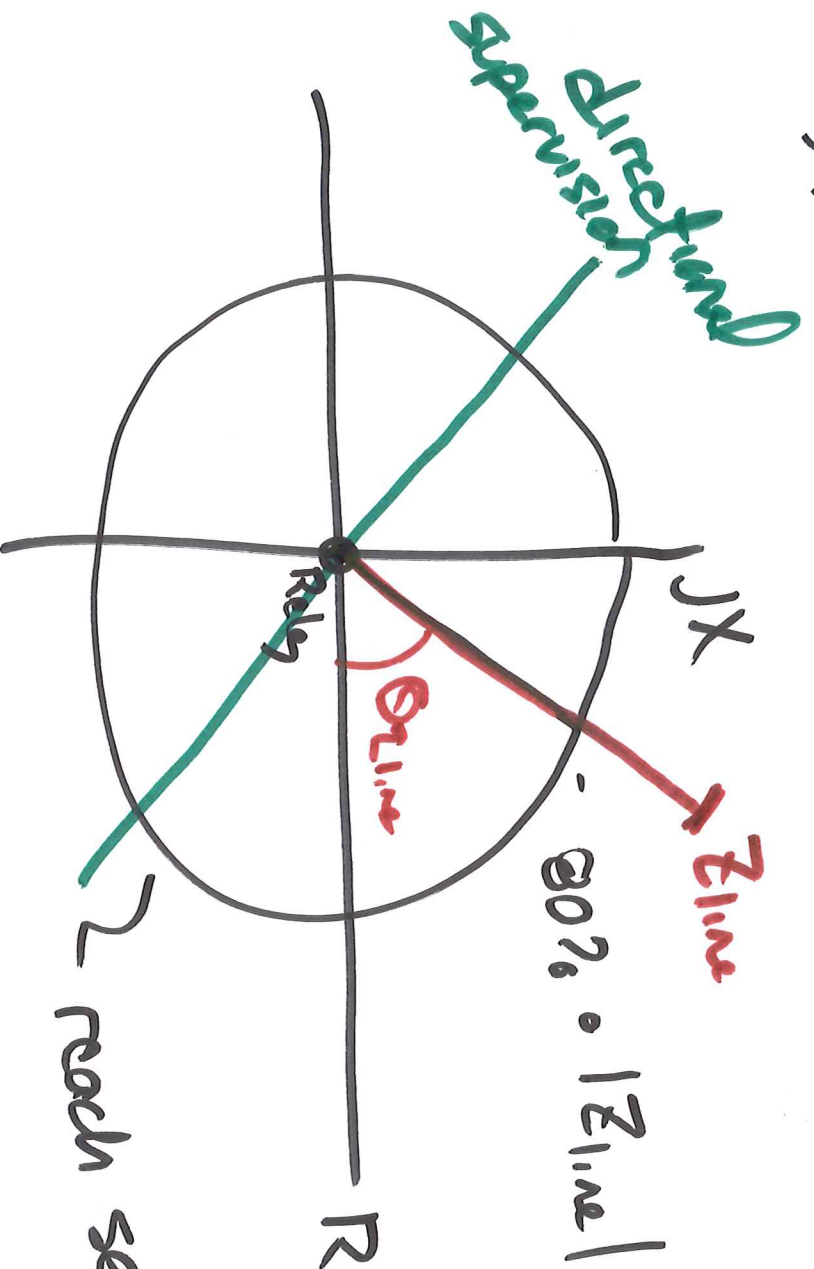
$$Z_{eff} = \frac{V}{I}$$



→ 21 Distance element



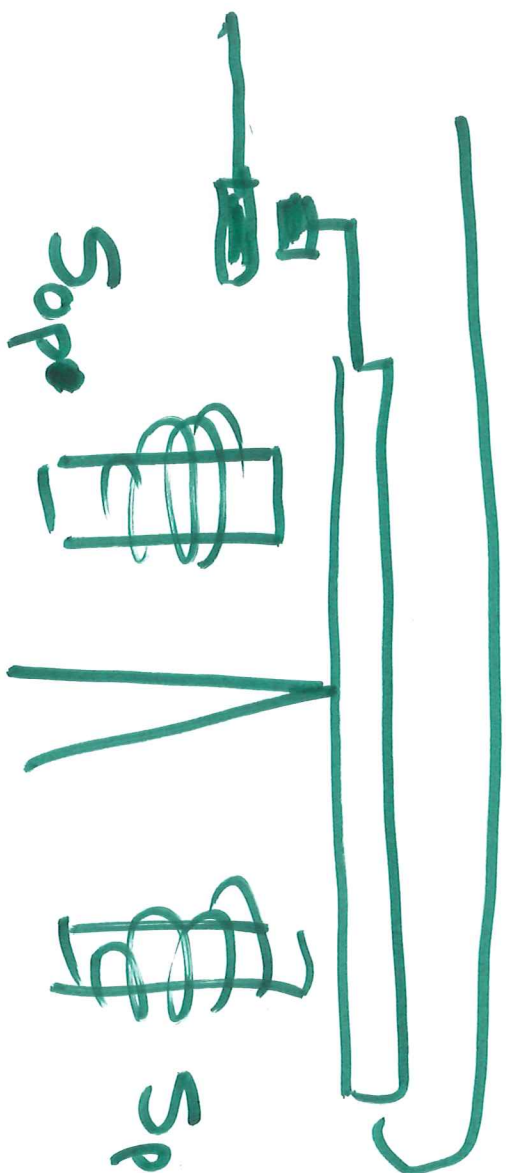
$$V_{relg} = I \cdot n \cdot Z_{line} \rightarrow \frac{V}{I} = n \cdot Z_{line}$$



Simple  
basic  
impedance  
elements

% of  $Z_{line}$   
for our trip  
threshold

reach setting



$S_{op}$

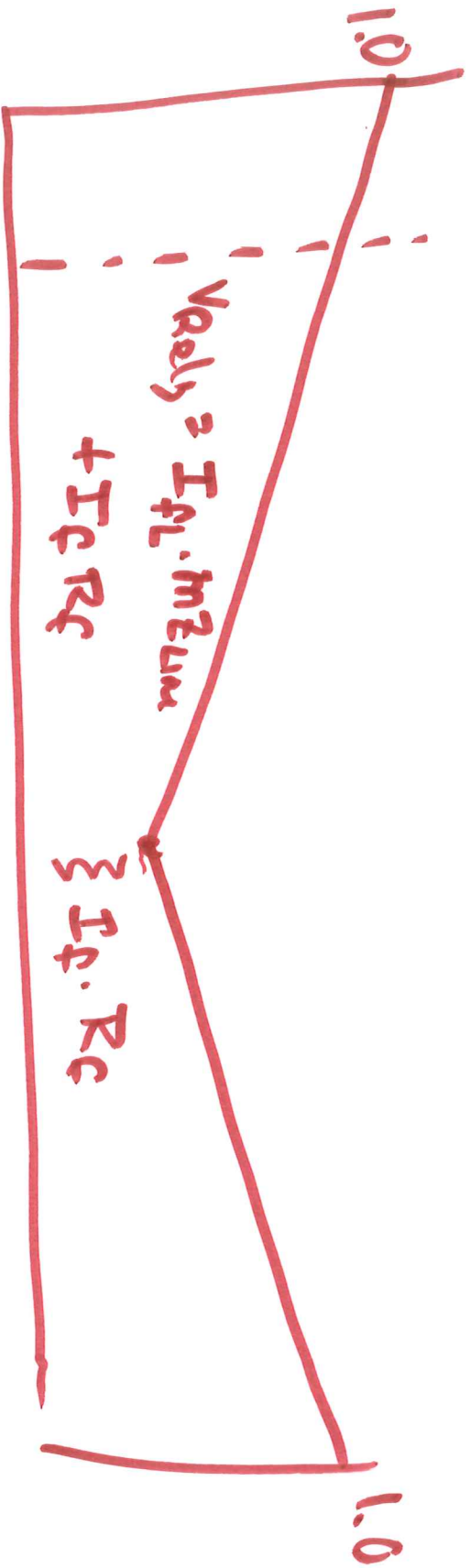
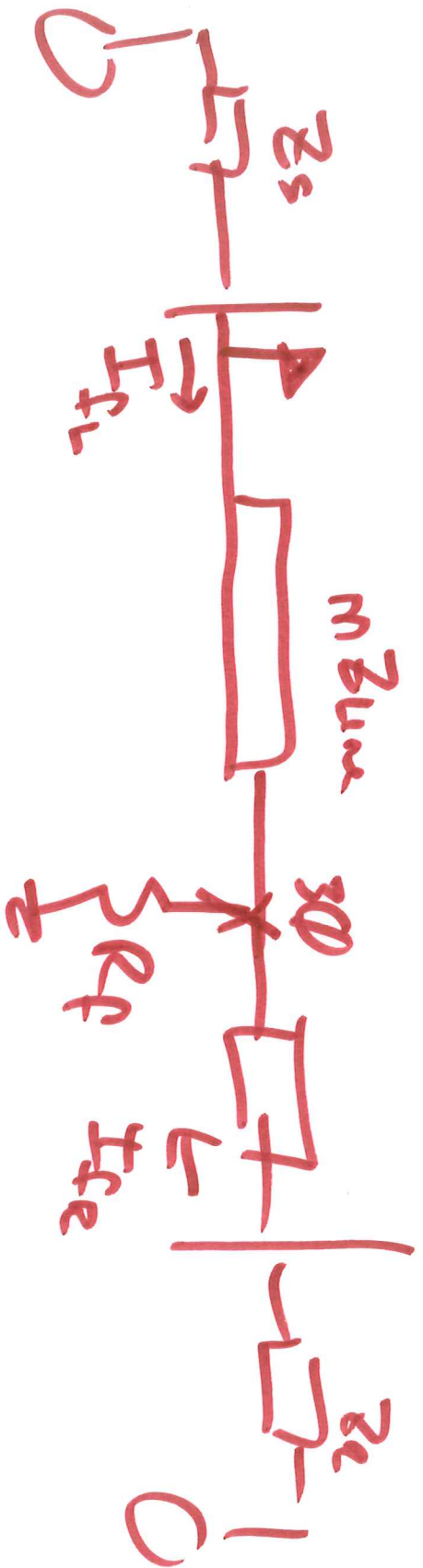
$S_{pol} \rightarrow S_{restraint}$

$$S_R = I$$

$$S_R = \frac{V}{Z_R}$$

2 reach setting

fault with  $R_f$

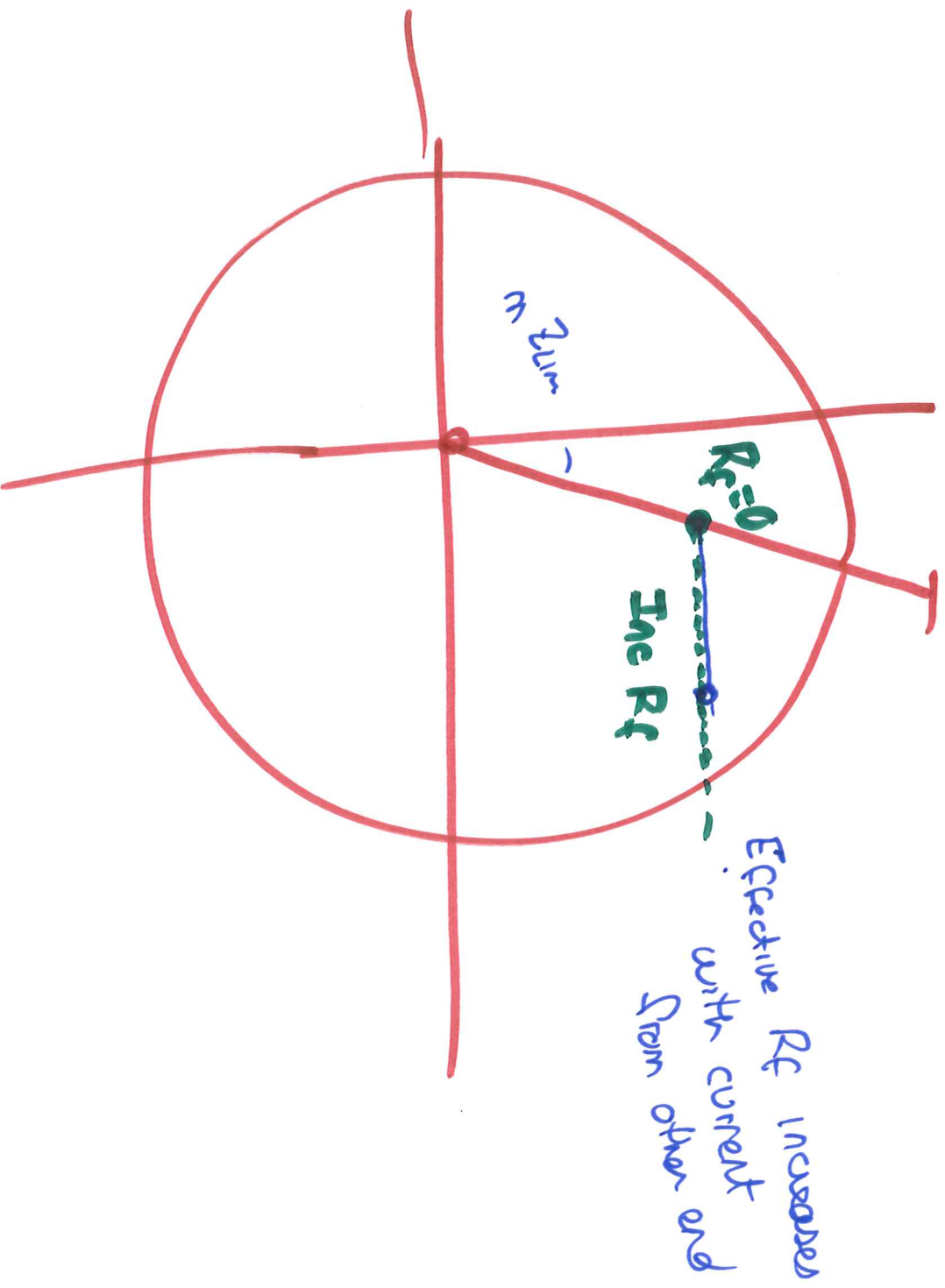


**Polygon characteristic or Quadrilateral**

The diagram illustrates the 'Polygon characteristic' or 'Quadrilateral' protection scheme. It features a red quadrilateral plotted on a coordinate system where the horizontal axis represents Resistance ( $R$ ) and the vertical axis represents Reactance ( $X$ ). A diagonal line originating from the center is labeled  $Z_{line}$ . The top-right vertex of the quadrilateral is labeled  $x_{reach}$ , and the bottom-left vertex is labeled  $R$ . A dashed line also originates from the center.

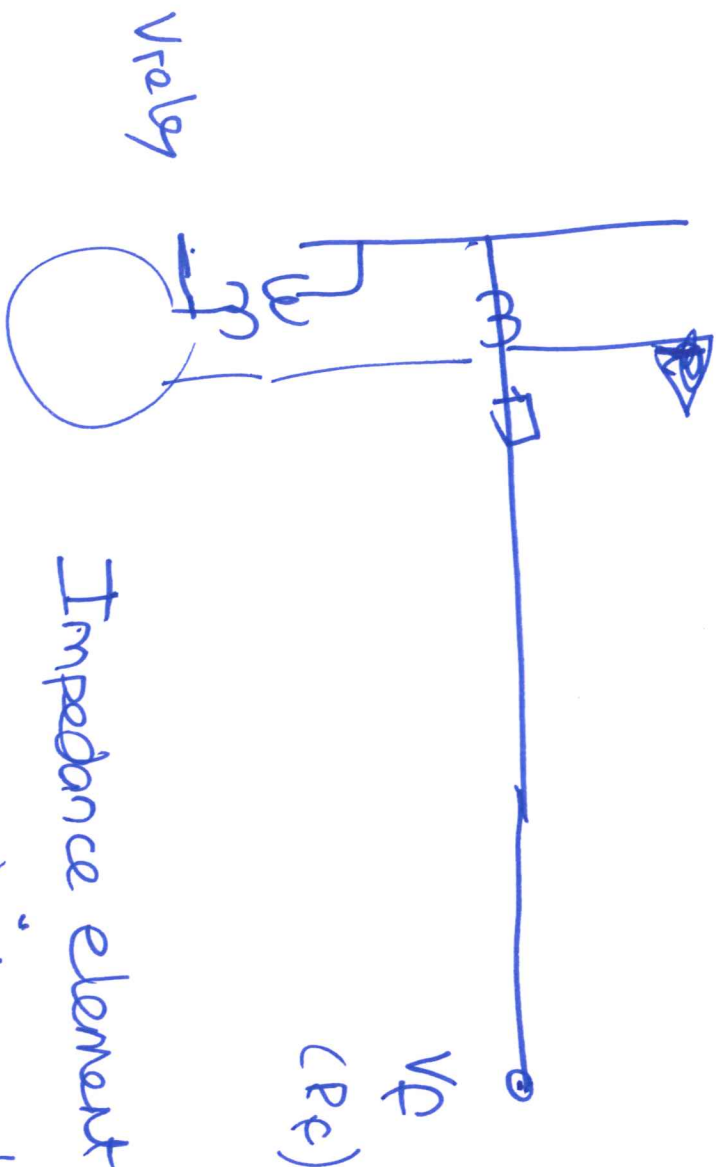
Polygon characteristic  
or Quadrilateral







# rho distance (modified impedance element)



$$R/p.l : V_{pol} = V_{relg}$$

$$OP : \delta V = I_p \cdot Z_R - V_{relg}$$

(PC)

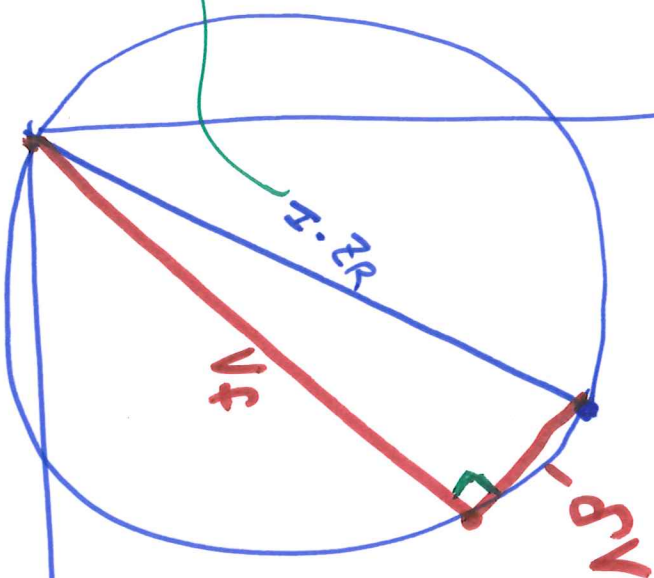
$Z_{reach}$

Impedance element:

- Add "Line drop compensation"
- Correcting for voltage across the line to reach

$X \cdot I$

"Voltage Plane"



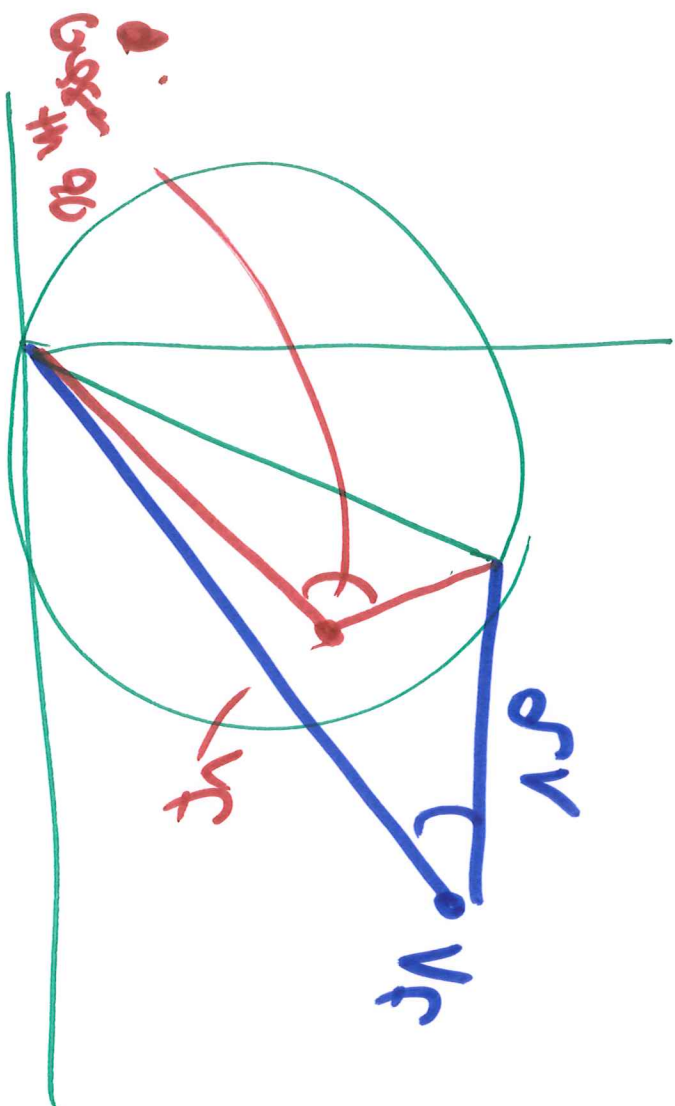
Measured  
by

$E_R = R \cdot I_{line}$   
R is desired  
reach

when on circle  
 $\delta V$  is perpendicular to  $V_f$

$$|V_f| |\delta V| \cos(\angle V_f - \angle \delta V) = 0$$

$R \cdot I$



$\delta V$  is line drop compensated voltage

$$\delta V = V_{bp} = I \cdot Z - V_c$$

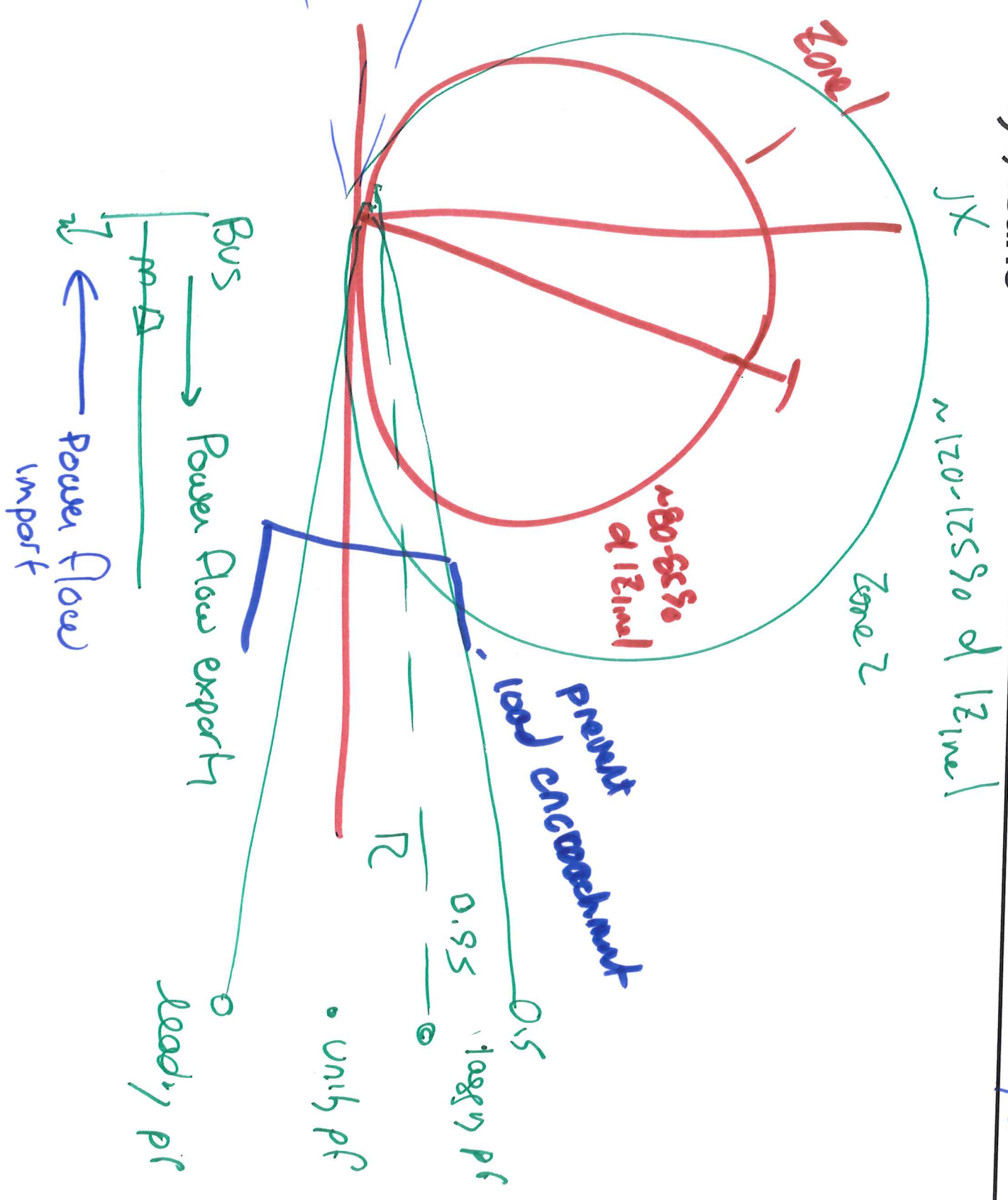
measured at relay

$$Z = r + j \cdot Z_{line}$$

reach

$$V_{pol} = V_t - \text{polarizing Component}$$

$$\text{on circle } \delta V \cdot V_{pol}^* = 0$$

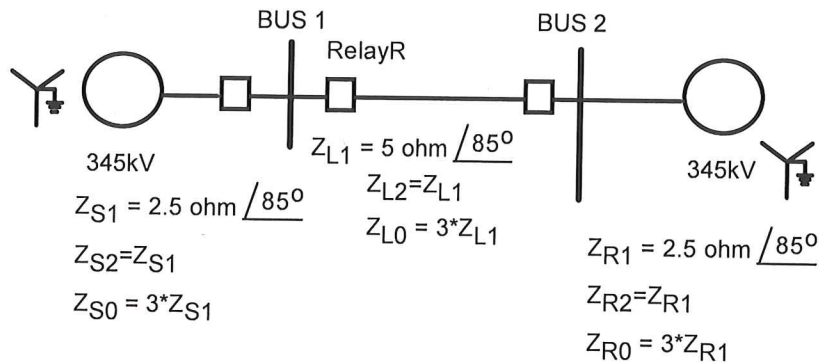




## ECE 526: Homework #1

**Due Session 6 (January 31)**

1. A distance relay is installed at Bus1 as indicated to protect the line from Bus1 to Bus 2. Set zone 1 to protect 80% of the length of the line and zone 2 to protect 125% of the length of the line. The impedance values are in secondary ohms.



$$\text{CTR} = \frac{800\text{A}}{5\text{A}} \quad \text{PTR} = \frac{345\text{kV}}{120\text{V}}$$

A. For an unfaulted condition, how much load current can flow (unity power factor as measured at the relay) from BUS 1 to BUS 2 without the zone 2 element picking up if the distance relay is a simple impedance relay? What if the power factor is 0.8 lagging? Assume BUS 1 has a voltage of 1.0 pu. Does it matter if power is going from BUS 1 to BUS 2 or from BUS 2 to BUS 1?

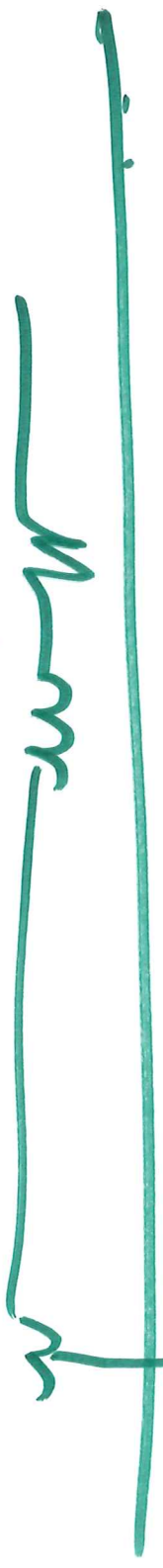
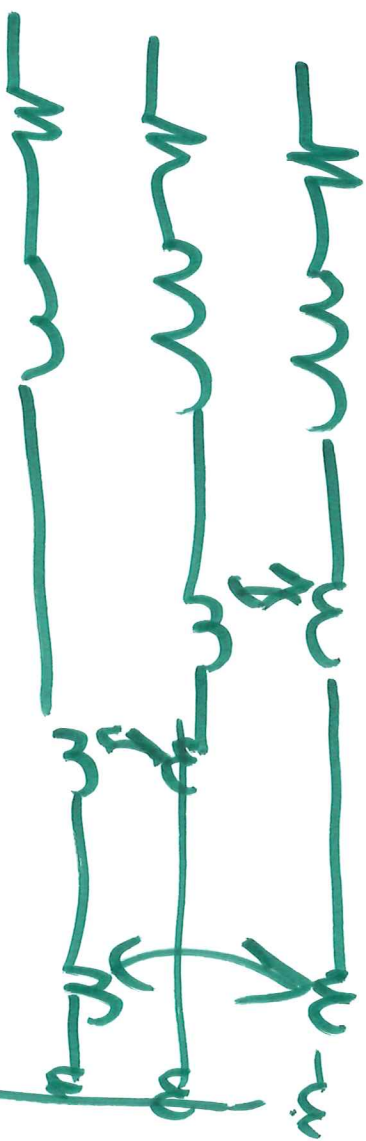
B. Repeat if a mho relay is used instead. *- 80% reach*

C. With the breaker at bus open calculate what the following mho elements will calculate for a SLG (AG) fault at 70% of the way down the line (AG, BG, CG, AB, BC, CA) if the fault resistance is zero. Plot your results against a mho circle. You can use a fault program to calculate the voltages and currents seen by the relay if you wish to do so). Repeat with  $R_f = 1 \text{ ohm}$  and with  $R_f = 4 \text{ ohms}$

D. Repeat part C if the circuit breaker is closed.

*LL, 3φ, DLG  
( $R_f = 0$ )*

$V/I$



earth

note  $z_0 > z_1$

$$\begin{bmatrix} z_s & z_m & z_n \\ z_n & z_s & z_n \\ z_n & z_n & z_s \end{bmatrix} \rightarrow \begin{bmatrix} z_0 & & \\ & z_1 & \\ & & z_2 \end{bmatrix}$$



## ECE 526: Lecture 4

### Derivation of $k_0$ zero sequence current (or residual current) correction factor for ground fault elements

For most transmission lines the zero sequence impedance is not equal to the positive sequence impedance. As a result, a simply  $V/I$  calculation at the relay will not give the correct effective impedance for a fault involving ground.

#### Derivation

Voltage seen at the relay for a SLG fault on phase A (based on the figure below):

Fault location is  $m\%$  of the way down the line.

$$V_A = I_A \cdot m \cdot Z_{as} + \underline{V_{mBA}} + \underline{V_{mCA}} + I_A \cdot R_F \quad (1)$$

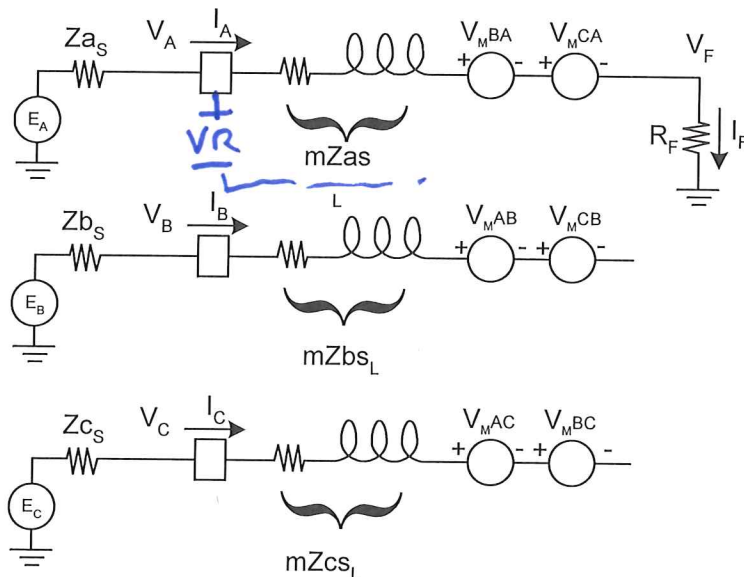
Where:

$Z_{as}$  is the self impedance of phase A of the line

$$V_{mBA} = m \cdot Z_m \cdot I_B$$

$$V_{mCA} = m \cdot Z_m \cdot I_C$$

- Note that these terms represent the induced voltages due to inductive mutual coupling between phase A and either phase B or phase C.
- $Z_m$  is the mutual impedance (assuming equal spacing between conductors)



We can rewrite equation (1) as follows (note that  $I_A \cdot Z_m$  is both added and subtracted)

$$V_A = I_A \cdot m \cdot (Z_{as} - Z_m) + (I_A + I_B + I_C) \cdot m \cdot Z_m + I_A \cdot R_f \quad (2)$$

Now think back to the impact of the symmetrical components transformation on the impedances. Start with a 3x3 impedance matrix for the line:

$$Z_{\text{line}} = \begin{pmatrix} Z_{as} & Z_m & Z_m \\ Z_m & Z_{bs} & Z_m \\ Z_m & Z_m & Z_{cs} \end{pmatrix}$$

Since the  $Z_m$  terms are all equal, we would expect  $Z_{as} = Z_{bs} = Z_{cs} = Z_s$

Now if we convert this to the sequence domain with the transformation matrix:

$$A_{012} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \quad A_{012}^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$Z_{012} = A_{012}^{-1} \cdot Z_{\text{line}} \cdot A_{012}$$

Resulting in:

$$Z_{012} = \begin{pmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{pmatrix}$$

where we would find that:

$$Z_1 = Z_2 = Z_s - Z_m$$

$$Z_0 = Z_s + 2 \cdot Z_m$$

So if we look at equation (2) again (repeated here) to try to express the result in terms of  $Z_1$  and  $Z_0$ :

$$V_A = I_A \cdot m \cdot (Z_{as} - Z_m) + (I_A + I_B + I_C) \cdot m \cdot Z_m + I_A \cdot R_f$$

- First we see that:

$$Z_1 = Z_{as} - Z_m$$

- Now we want to be able to replace  $Z_m$  with a function of  $Z_1$  and  $Z_0$ . We can work from the two equations above. From the first equation:

$$Z_s = Z_1 + Z_m$$

Substitute this into the second equation resulting in:

$$Z_0 = Z_1 + 3 \cdot Z_m$$

then  $Z_m = \frac{1}{3} \cdot (Z_0 - Z_1)$

Now rewrite the voltage equation:

$$V_A = I_A \cdot m \cdot (Z_1) + (I_A + I_B + I_C) \cdot m \cdot \left[ \frac{1}{3} \cdot (Z_0 - Z_1) \right] + I_A \cdot R_f$$

We can factor out the "m" term:

$$V_A = m \cdot \left[ I_A \cdot Z_1 + (I_A + I_B + I_C) \cdot \left[ \frac{1}{3} \cdot (Z_0 - Z_1) \right] \right] + I_A \cdot R_f$$

We also know that the residual current (or the the current through neutral of the CT's) is:

$$I_R = 3 \cdot I_0 = I_A + I_B + I_C$$

*"residual" current*

Therefore:

$$V_a = m \cdot \left[ I_A \cdot (Z_1) + (I_R) \cdot \left[ \frac{1}{3} \cdot (Z_0 - Z_1) \right] \right] + I_A \cdot R_f \quad (4a)$$

or in terms of the zero sequence current:

$$V_a = m \cdot \left[ I_A \cdot (Z_1) + (3 \cdot I_0) \cdot \left[ \frac{1}{3} \cdot (Z_0 - Z_1) \right] \right] + I_A \cdot R_f \quad (4b)$$

Which can be rearranged and written as:

$$V_a = m \cdot Z_1 \cdot \left[ I_A + I_R \cdot \left( \frac{1}{3} \cdot \frac{Z_0 - Z_1}{Z_1} \right) \right] + I_A \cdot R_f \quad (5)$$

- Note that the term multiplying the residual current,  $I_R$ , needs to be divided by  $Z_1$  since the entire term in the outer brackets is multiplied by  $Z_1$ .

Define:

$$k_0 = \frac{Z_0 - Z_1}{3Z_1}$$

- Zero sequence compensation factor

$$V_a = m \cdot Z_1 \cdot (I_A + k_0 \cdot I_R) + I_A \cdot R_f \quad (5a)$$

Or in terms of  $I_0$ :

$$V_a = m \cdot Z_1 \cdot (I_A + k_0 \cdot 3 \cdot I_0) + I_A \cdot R_f \quad (5b)$$

So the phase A to ground relay element could be viewed as calculating:

$$Z_{AG} = \frac{V_a}{I_a + k_0 \cdot I_R}$$

- Effective for fault involving ground on phase A

Self polarized

Done by "relay"

Note that the fault resistance is not going to be part of the equation since it is an unknown

This will give the effective impedance for a fault on phase A and include the effect of the common mode (zero sequence) current on other phases as well as zero sequence imbalance in the lines.

Similarly

$$Z_{BG} = \frac{V_b}{I_b + k_0 \cdot I_R}$$

$$Z_{CG} = \frac{V_c}{I_c + k_0 \cdot I_R}$$

### Example

Consider a phase A SLG fault at 80% of the length of the line in front of the relay. Assume that the source impedance is zero. Assume no load current, unfaulted phases have no current.

$$Z_1 := 1 \text{ ohm} \cdot e^{j \cdot 85 \text{ deg}}$$

$$V_a := j \cdot \frac{120 \text{ V}}{\sqrt{3}}$$

Ideal source

$$Z_2 := Z_1$$

$$Z_0 := 3.5 \text{ ohm} \cdot e^{j \cdot 80 \text{ deg}}$$

↑  $V_A$

$$k_0 := \frac{Z_0 - Z_1}{3 \cdot Z_1}$$

$$|k_0| = 0.84$$

$$\arg(k_0) = -6.99 \cdot \text{deg}$$

$$I_0 := \frac{V_a}{0.8 \cdot (Z_1 + Z_2 + Z_0)}$$

$$|I_0| = 15.76 \text{ A}$$

$$\arg(I_0) = 8.18 \cdot \text{deg}$$

Apparent impedance using only phase A current:

$$I_{af} := 3 \cdot I_0$$

$$Z_{meas} := \frac{V_a}{I_{af}} \quad |Z_{meas}| = 1.47 \Omega$$

$$\arg(Z_{meas}) = 81.82 \cdot \text{deg}$$

Notice the simply calculating V/I does not give the correct result, which should be  $0.8 \cdot Z_1$

Now use the term with the zero sequence correction factor:

$$Z_{AG} := \frac{V_a}{I_{af} + k \cdot 3I_0} \quad |Z_{AG}| = 0.8 \Omega \quad \arg(Z_{AG}) = 85 \cdot \text{deg}$$

Now look at what happens on the unfaulted phase to ground elements:

$$Z_{BG} := \frac{V_a \cdot e^{-j \cdot 120 \text{deg}}}{0 + k \cdot 3I_0} \quad |Z_{BG}| = 1.75 \Omega \quad \arg(Z_{BG}) = -31.19 \cdot \text{deg}$$

$$Z_{CG} := \frac{V_a \cdot e^{j \cdot 120 \text{deg}}}{0 + k \cdot 3I_0} \quad |Z_{CG}| = 1.75 \Omega \quad \arg(Z_{CG}) = -151.19 \cdot \text{deg}$$

So notice that the unfaulted phases will see a fault impedance due to the zero sequence coupling between currents.

For completeness:

*phase to phase elements*

*V<sub>AG</sub>-V<sub>BG</sub>*

*V<sub>AB</sub>  
I<sub>AB</sub>*

*I<sub>A</sub>-I<sub>B</sub>*

$$Z_{AB} := \frac{V_a - V_a \cdot e^{-j \cdot 120 \text{deg}}}{I_{af} - 0A} \quad Z_{CA} := \frac{V_a \cdot e^{j \cdot 120 \text{deg}} - V_a}{0A - I_{af}} \quad \bullet \quad Z_{BC} \text{ not impacted by the fault}$$

Now plot these impedance values against a Mho characteristic set at 80% of the line length.

$$k := 0, 1 \dots 719$$

$$\text{rad}_{\text{Mhozone1}} := 0.8 \cdot \frac{|Z_1|}{2} \quad \text{offset}_{\text{Mhozone1}} := 0.8 \cdot \left( \frac{|Z_1|}{2} \right) e^{j \cdot \arg(Z_1)}$$

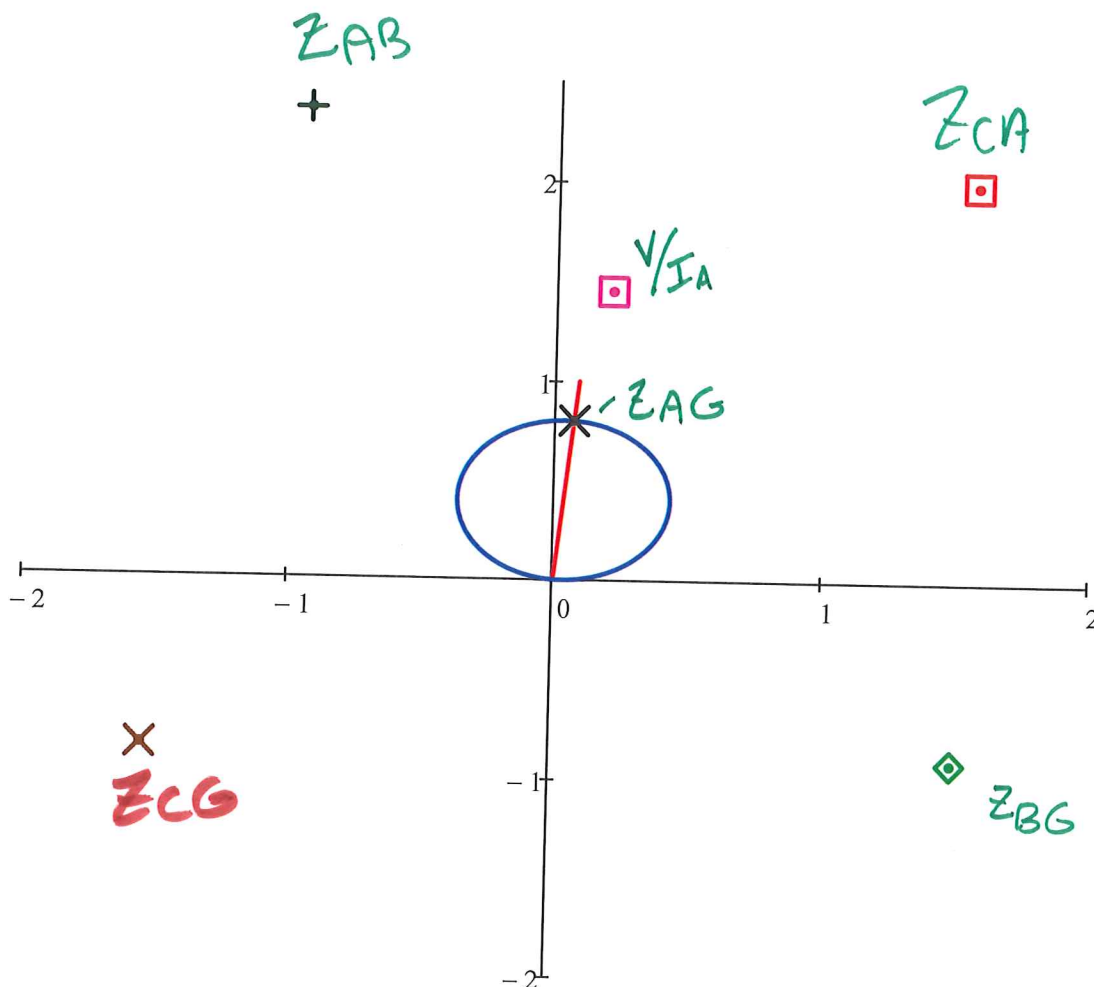
$$\text{Zone1}_k := \text{offset}_{\text{Mhozone1}} + \text{rad}_{\text{Mhozone1}} \cdot e^{j \cdot k \cdot 0.5 \text{deg}}$$

*Zone1<sub>k</sub>*



$$\text{LineZ} := \begin{pmatrix} 0 \\ Z_1 \end{pmatrix}$$

- Im(LineZ)
- Im(Zone1<sub>k</sub>)
- Im(Z<sub>AG</sub>)
- Im(Z<sub>meas</sub>)
- Im(Z<sub>BG</sub>)
- Im(Z<sub>CG</sub>)
- Im(Z<sub>AB</sub>)
- Im(Z<sub>CA</sub>)



$\text{Re}(\text{LineZ}), \text{Re}(\text{Zone1}_k), \text{Re}(Z_{AG}), \text{Re}(Z_{meas}), \text{Re}(Z_{BG}), \text{Re}(Z_{CG}), \text{Re}(Z_{AB}), \text{Re}(Z_{CA})$