

ECE 526

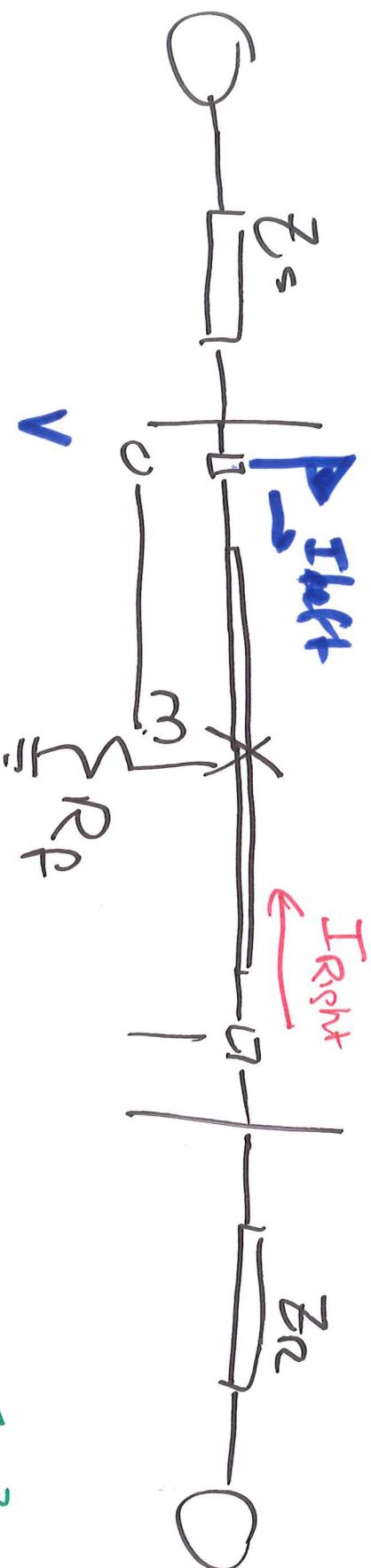
PROTECTION OF  
POWER SYSTEMS II

SESSION no. 7

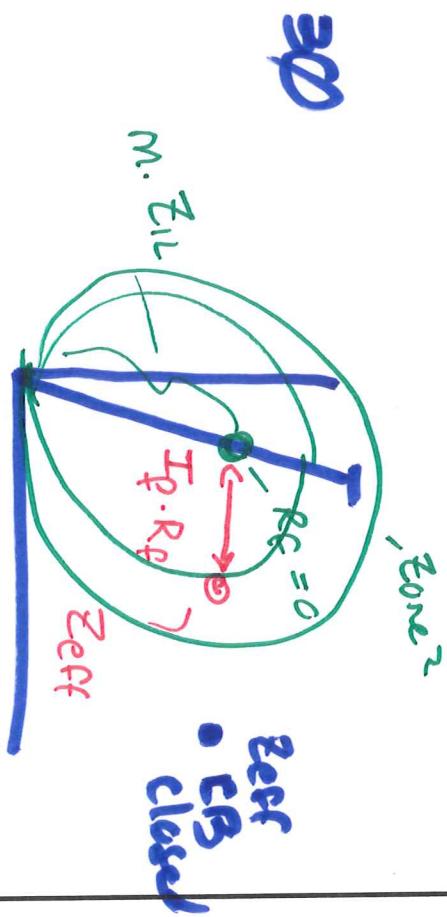
## Fault Type Selection

- Block phase to ground elements  
for DLG faults to avoid overreach
- Single pole tripping - more input
- Relay target
- Fault locator loop selection

- ~~Angle~~ Angle difference between  $I_o$  and  $I_r$ 
  - Single ended calculation
  - Distance
- Double-ended
  - Line current differential protection

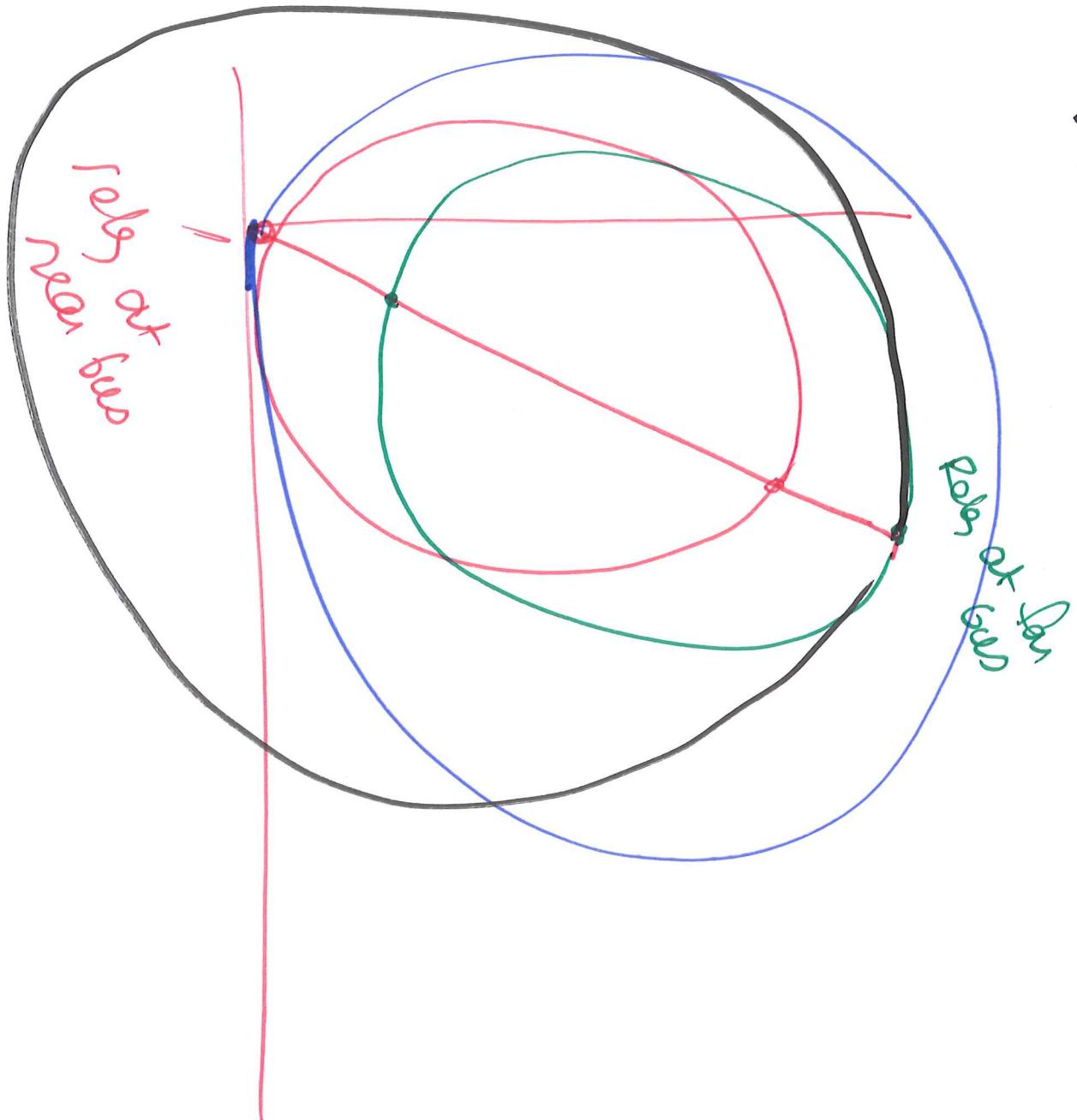


$$V = I_L \cdot m Z_{RL} + I_f \cdot R_f$$



$$V_{\text{rel}} = I_L (m Z_{RL}) + \frac{I_f \cdot R_f}{(I_{\text{left}} + I_{\text{right}})}$$

$(I_{\text{left}} + I_{\text{right}})$



Now apply a forward SLG fault

$$\mathcal{T}_0 + \frac{V_a}{I_{a\_k0l}} V_a := 6.56V \cdot e^{-j \cdot 0.55\text{deg}}$$

$$V_b := 69.32V \cdot e^{-j \cdot 125.06\text{deg}}$$

$$V_c := 71.44V \cdot e^{j \cdot 123.88\text{deg}}$$

$$\mathcal{T}_0 + \frac{V_a}{I_{a\_k0l}} V_a := 6.56V \cdot e^{-j \cdot 0.55\text{deg}}$$

- This is  $I_a + k_0 * 3I_0$

$$V_{a1\_mem} := 67V \cdot e^{j \cdot 0\text{deg}}$$

$$k_{v1\_mem} := 1$$

$$V_{p\_mem} := V_a - k_{v1\_mem} \cdot V_{a1\_mem}$$

$$V_{p\_mem} = (-60.44 - 0.06i) V$$

$$Z_{p\_mem} := \frac{V_{p\_mem}}{I_{a\_k0l}}$$

$$|Z_{p\_mem}| = 4.3 \Omega$$

$$\arg(Z_{p\_mem}) = -95.62\text{-deg}$$

$$V_{cross\_pol} := V_b + V_c$$

$$k_{cross\_pol} := -1$$

$$V_{p\_cross\_pol} := V_a - k_{cross\_pol} \cdot V_{cross\_pol}$$

$$V_{p\_cross\_pol} = (-73.08 + 2.51i) V$$

$$Z_{p\_cross} := \frac{V_{p\_cross\_pol}}{I_{a\_k0l}}$$

$$|Z_{p\_cross}| = 5.2 \Omega$$

$$\arg(Z_{p\_cross}) = -97.64\text{-deg}$$

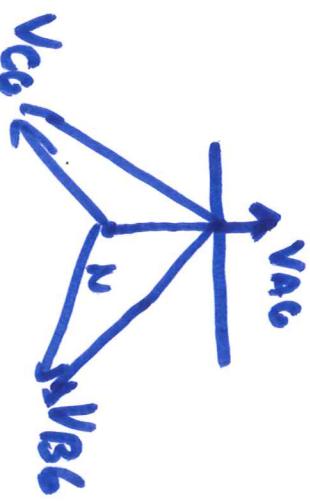
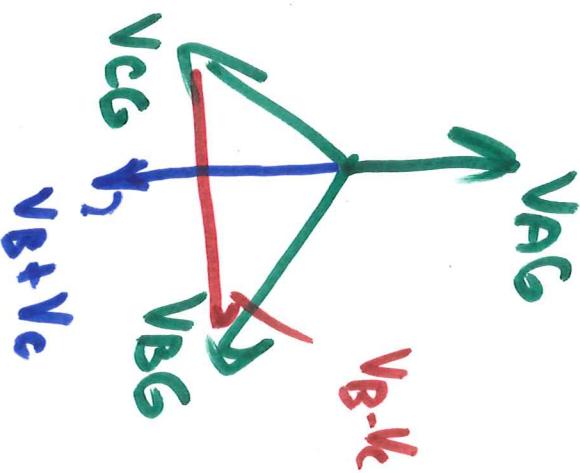
$$Z_{offset\_mem\_fwd} := 0.5 \cdot (Z_{line} + Z_{p\_mem})$$

$$Z_{radius\_mem\_fwd} := 0.5 \cdot |Z_{line} - Z_{p\_mem}|$$

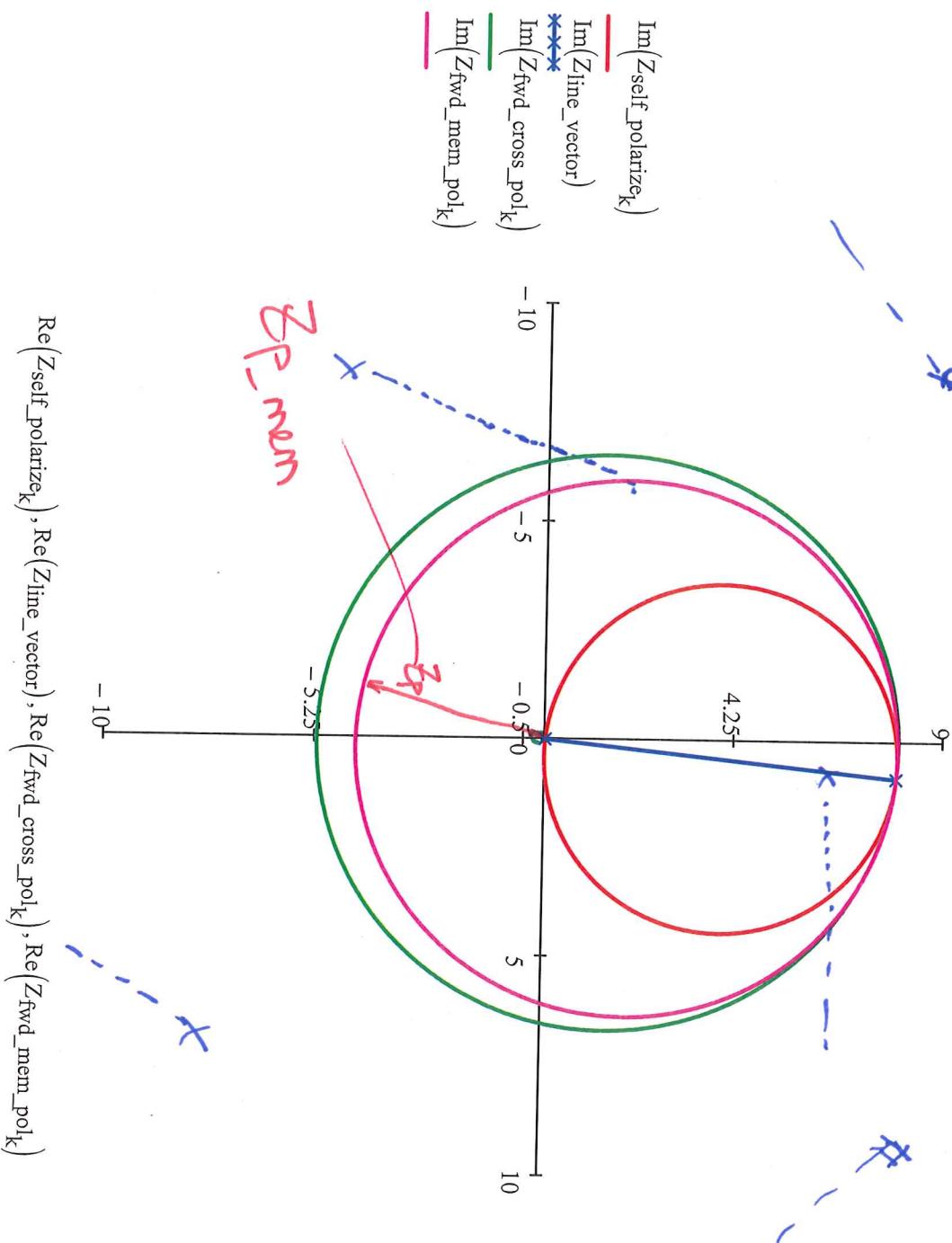
$$Z_{fwd\_mem\_pol_k} := Z_{offset\_mem\_fwd} + Z_{radius\_mem\_fwd} \cdot e^{j \cdot 0.5 \cdot k \cdot \text{deg}}$$

$$Z_{offset\_cross\_fwd} := 0.5 \cdot (Z_{line} + Z_{p\_cross})$$

$$Z_{radius\_cross\_fwd} := 0.5 \cdot |Z_{line} - Z_{p\_cross}|$$



$$Z_{\text{fwd\_cross\_pol}_k} := Z_{\text{offset\_cross\_fwd}} + Z_{\text{radius\_cross\_fwd}} \cdot e^{j \cdot 0.5 \cdot k \cdot \text{deg}}$$



•  Reverse fault

$$V_a := 9.01 V \cdot e^{-j \cdot 3.43 \text{deg}}$$

$$V_b := 69.21 V \cdot e^{-j \cdot 124.77 \text{deg}}$$

$$V_c := 71.15 V \cdot e^{j \cdot 123.70 \text{deg}}$$

- This is  $I_a + k_0 * 3I_0$

$$I_{REV\_a\_k0Ir} := 18.01 A \cdot e^{j \cdot 93.59 \text{deg}}$$

$$V_{cross\_polREV} := V_b + V_c$$

$$V_{p\_mem\_polREV} := V_a - k_{V1\_mem} \cdot V_{al\_mem}$$

$$V_{p\_mem\_polREV} = (-58.01 - 0.54i) V$$

$$V_{p\_cross\_polREV} := V_a - k_{cross\_pol} \cdot V_{cross\_pol}$$

$$V_{p\_cross\_polREV} = (-70.65 + 2.03i) V$$

$$Z_{p\_memREV} := \frac{V_{p\_mem\_polREV}}{I_{REV\_a\_k0Ir}}$$

$$|Z_{p\_memREV}| = 3.22 \Omega \quad \arg(Z_{p\_memREV}) = 86.94 \cdot \text{deg}$$

$$Z_{p\_crossREV} := \frac{V_{p\_cross\_polREV}}{I_{REV\_a\_k0Ir}}$$

$$|Z_{p\_crossREV}| = 3.92 \Omega \quad \arg(Z_{p\_crossREV}) = 84.77 \cdot \text{deg}$$

$$Z_{offset\_mem\_rev} := 0.5 \cdot (Z_{line} + Z_{p\_memREV}) \quad Z_{radius\_mem\_rev} := 0.5 \cdot |Z_{line} - Z_{p\_memREV}|$$

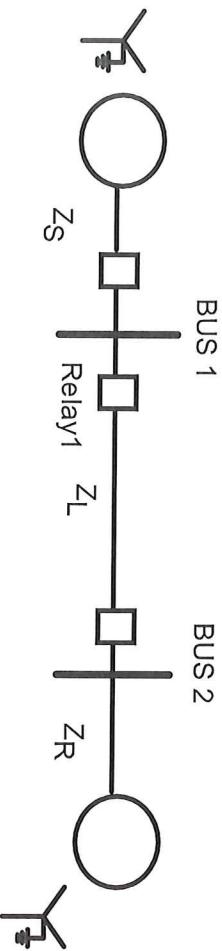
$$Z_{rev\_mem\_pol_k} := Z_{offset\_mem\_rev} + Z_{radius\_mem\_rev} \cdot e^{j \cdot 0.5 \cdot k \cdot \text{deg}}$$

$$Z_{offset\_cross\_rev} := 0.5 \cdot (Z_{line} + Z_{p\_crossREV}) \quad Z_{radius\_cross\_rev} := 0.5 \cdot |Z_{line} - Z_{p\_crossREV}|$$

$$Z_{rev\_cross\_pol_k} := Z_{offset\_cross\_rev} + Z_{radius\_cross\_rev} \cdot e^{j \cdot 0.5 \cdot k \cdot \text{deg}}$$

**ECE 526****Distance Element Examples**

The impedances for the system below are given in secondary ohms. The zone 1 reach of the relay at BUS 1 is set to cover 85% of the line.



$V_{secLL} := 120V$  Note that this is a line to line voltage.

$$Z_{L1} := 6\text{ohm}\cdot e^{j \cdot 85\text{deg}} \quad Z_{S1} := 2.5\text{ohm}\cdot e^{j \cdot 85\text{deg}} \quad Z_{R1} := j \cdot 3\text{ohm} \quad \theta_{L1} := \arg(Z_{L1}) \quad \theta_{L1} = 85\text{.deg}$$

$$Z_{L2} := Z_{L1}$$

$$Z_{S2} := Z_{S1}$$

$$Z_{R2} := Z_{R1}$$

$$Z_{L0} := 3 \cdot Z_{L1}$$

$$Z_{S0} := 3 \cdot Z_{S1}$$

$$Z_{R0} := 3 \cdot Z_{R1}$$

A. With the breaker at Bus 2 open calculate how much fault resistance can be present before the the distance element is unable to see the a SLG fault on phase A in Zone 1 for faults at the following locations: (a) 10% of the line, (b) 50% of the line and (c) 80% of the line if the relay is

- (1) self polarized,
- (2) cross polarized (use  $V_B + V_C$ )
- (3) positive sequence memory polarized (use the prefault source voltage)

Zero sequence fault current as a function of fault resistance (recall that the  $R_f$  vector starts at 0)

$$I_0(m, R_f) := \frac{V_f}{Z_{Left1}(m) + Z_{Left2}(m) + Z_{Left0}(m) + 3 \cdot R_f}$$

$$I_1(m, R_f) := I_0(m, R_f)$$

$$I_2(m, R_f) := I_0(m, R_f)$$

$$V_1(m, R_f) := V_f - I_1(m, R_f) \cdot Z_{S1}$$

$$V_2(m, R_f) := 0 - I_2(m, R_f) \cdot Z_{S2}$$

$$V_0(m, R_f) := 0 - I_0(m, R_f) \cdot Z_{S0}$$

$$I_{ABC}(m, R_f) := A_{012} \cdot \begin{pmatrix} I_0(m, R_f) \\ I_1(m, R_f) \\ I_2(m, R_f) \end{pmatrix} \quad V_{ABC}(m, R_f) := A_{012} \cdot \begin{pmatrix} V_0(m, R_f) \\ V_1(m, R_f) \\ V_2(m, R_f) \end{pmatrix}$$

*Solution Option 1: Using the m-equations (calculate the effective reach to the fault)*

$$M_{aG} = \frac{\operatorname{Re}(V_a \cdot \overline{V_{a1}})}{\operatorname{Re}\left[\left(Z1\text{mag} \cdot e^{j \cdot \theta Z1L}\right) \cdot \left(I_a + k_0 I_R\right) \cdot \left(\overline{V_{a1}}\right)\right]}$$

$$V_{almem} := \frac{jV_{secLL}}{\sqrt{3}}$$

- Self Polarized Case:

$$\text{Re}(\overline{\text{V}_{ABC}(m, R_f)_0 \cdot \text{V}_{ABC}(m, R_f)_0})$$

$$\text{Mag\_self}(m, R_f) := \frac{\text{Re}[(Z_{1\text{MAG}} \cdot e^{j \cdot Z_{1\text{ANG}}}) \cdot (\text{I}_{ABC}(m, R_f)_0 + k_0 \cdot 3 I_0(m, R_f)) \cdot (\overline{\text{V}_{ABC}(m, R_f)_0})]}{[ ]}$$

- Memory Polarized Case:

$$\text{Re}(\overline{\text{V}_{ABC}(m, R_f)_0 \cdot \text{V}_{almem}})$$

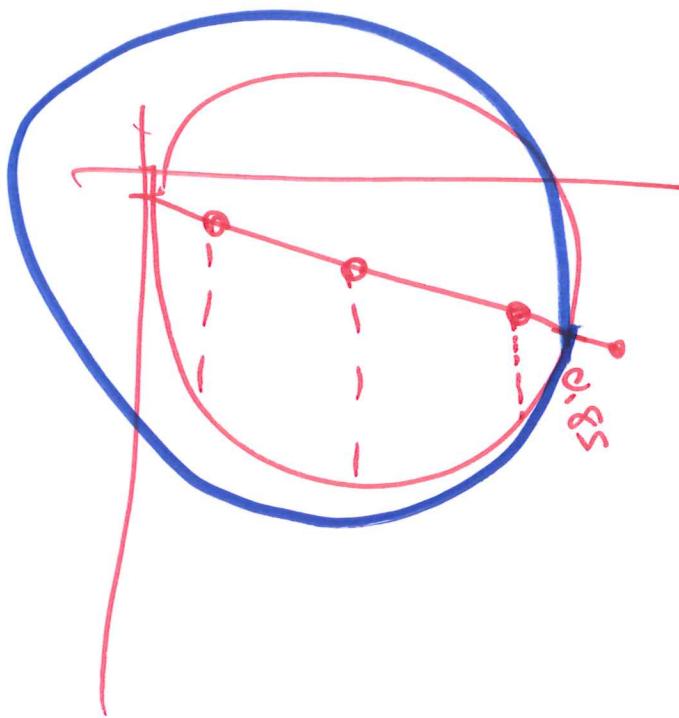
$$\text{Mag\_mem}(m, R_f) := \frac{\text{Re}[(Z_{1\text{MAG}} \cdot e^{j \cdot Z_{1\text{ANG}}}) \cdot (\text{I}_{ABC}(m, R_f)_0 + k_0 \cdot 3 I_0(m, R_f)) \cdot (\overline{\text{V}_{almem}})]}{[ ]}$$

- Cross polarized case

$$\text{Vpol\_cross}(m, R_f) := \text{V}_{ABC}(m, R_f)_1 + \text{V}_{ABC}(m, R_f)_2$$

$$\text{Re}(\overline{\text{V}_{ABC}(m, R_f)_0 \cdot \text{V}_{pol\_cross}(m, R_f)})$$

$$\text{Mag\_cross}(m, R_f) := \frac{\text{Re}[(Z_{1\text{MAG}} \cdot e^{j \cdot Z_{1\text{ANG}}}) \cdot (\text{I}_{ABC}(m, R_f)_0 + k_0 \cdot 3 I_0(m, R_f)) \cdot (\overline{\text{V}_{pol\_cross}(m, R_f)})]}{[ ]}$$



$R_{f\_m} := 1\text{ohm}$

Given

$$\text{Mag\_self}(0.10, R_{f\_m}) = .85$$

→ Find  $(R_{f\_m}) = 3.03648\Omega$

Repeating the process for the other cases:

$$R_{self10A} := 3.0360\text{ohm}$$

$$R_{cross10A} := 7.941\text{ohm}$$

$$R_{self50A} := 4.118\text{ohm}$$

$$R_{cross50A} := 6.234\text{ohm}$$

$$R_{self80A} := 1.700\text{ohm}$$

$$R_{cross80A} := 2.206\text{ohm}$$

$$R_{mem10A} := 6.327\text{ohm}$$

$$R_{mem50A} := 5.423\text{ohm}$$

$$R_{mem80A} := 2.01\text{ohm}$$

- Check results using the resistance values:

Fault at 10% of the line:

$$\text{Mag\_self}(0.1, R_{self10A}) = 0.85$$

$$\text{Mag\_cross}(0.1, R_{cross10A}) = 0.85$$

$$\text{Mag\_mem}(0.1, R_{mem10A}) = 0.85$$

Fault at 50% of the line:

$$\text{Mag\_self}(0.5, R_{self50A}) = 0.85$$

$$\text{Mag\_cross}(0.5, R_{cross50A}) = 0.85$$

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$$Mag\_mem(0.5, R_{mem50A}) = 0.85$$

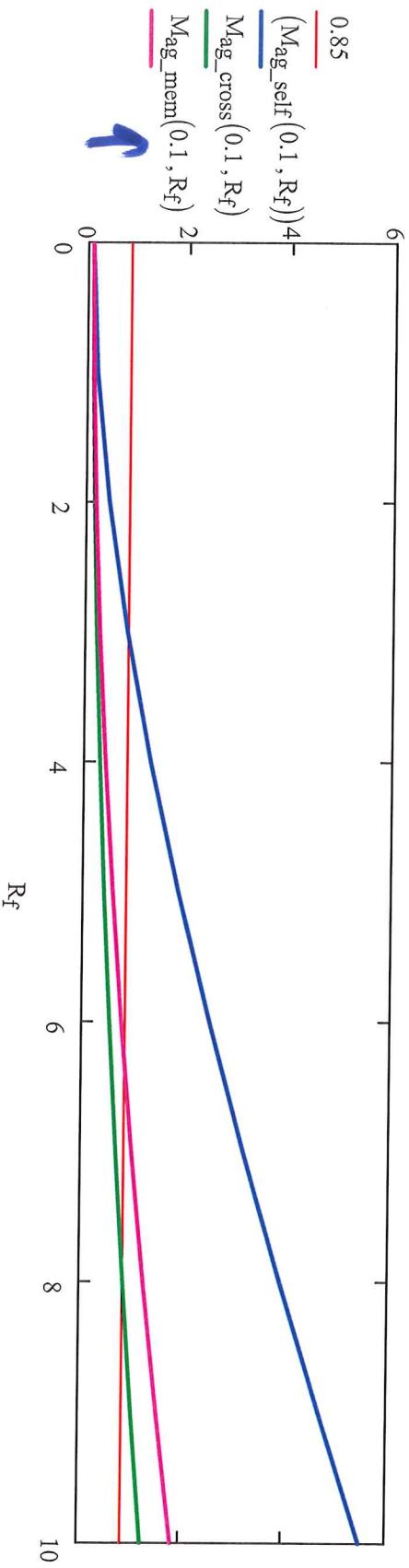
Fault at 80% of the line:

$$Mag\_self(0.8, R_{self80A}) = 0.85$$

$$Mag\_cross(0.8, R_{cross80A}) = 0.85$$

$$Mag\_mem(0.8, R_{mem80A}) = 0.85$$

Plot how quickly  $Mag$  increases with  $R_f$  for different  $V_{pol}$  and different location on line.



$$\text{Zone1k} := \text{offsetMhozone1\_self}(0.5, 0\text{ohm}) + \text{radMhozone1\_self}(0.5, 0\text{ohm}) \cdot e^{j \cdot k \cdot 0.5\text{deg}}$$

$$\text{Line impedance vector for diagram: LineZ} := \begin{pmatrix} 0 \\ Z_{L1} \end{pmatrix}$$

For the self polarized case we want to find R such that:  $|Z_{AG}(m, R_f) - \text{offsetMhozone1}| \leq \text{radMhozone1}$

- In this case we want to be exactly on the circle

Given

$$|Z_{AG}(0.10, R_f) - \text{offsetMhozone1\_self}(0.1, R_f)| = \text{radMhozone1\_self}(0.1, R_f)$$

$$R_{self10\_A\_mho} := \text{Find}(R_f) \quad R_{self10\_A\_mho} = 3.036\Omega$$

earlier we found:  $R_{self10A} = 3.036\Omega$

As a check:

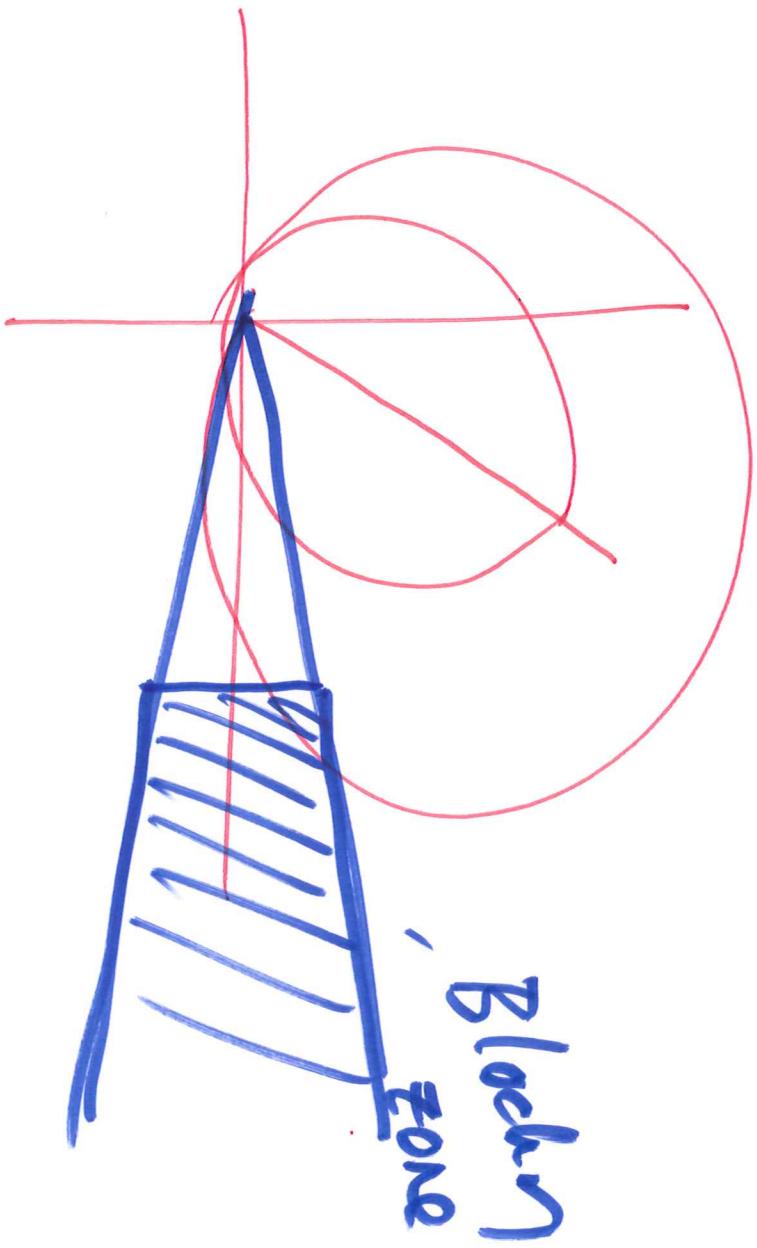
$$|Z_{AG}(0.1, R_{self10\_A\_mho}) - \text{offsetMhozone1\_self}(0.1, R_{self10\_A\_mho})| - \text{radMhozone1\_self}(0.1, R_{self10\_A\_mho}) = 0\Omega$$

- Now for the cross polarized case - now the Mho circle changes, but not the Zag equation

From above we had defined  $V_{pol\_cross}(m, R_f)$

$$k_{cross\_pol} := -1$$

$$V_{p\_cross}(m, R_f) := V_{ABC}(m, R_f)_0 - k_{cross\_pol} \cdot V_{pol\_cross}(m, R_f)$$

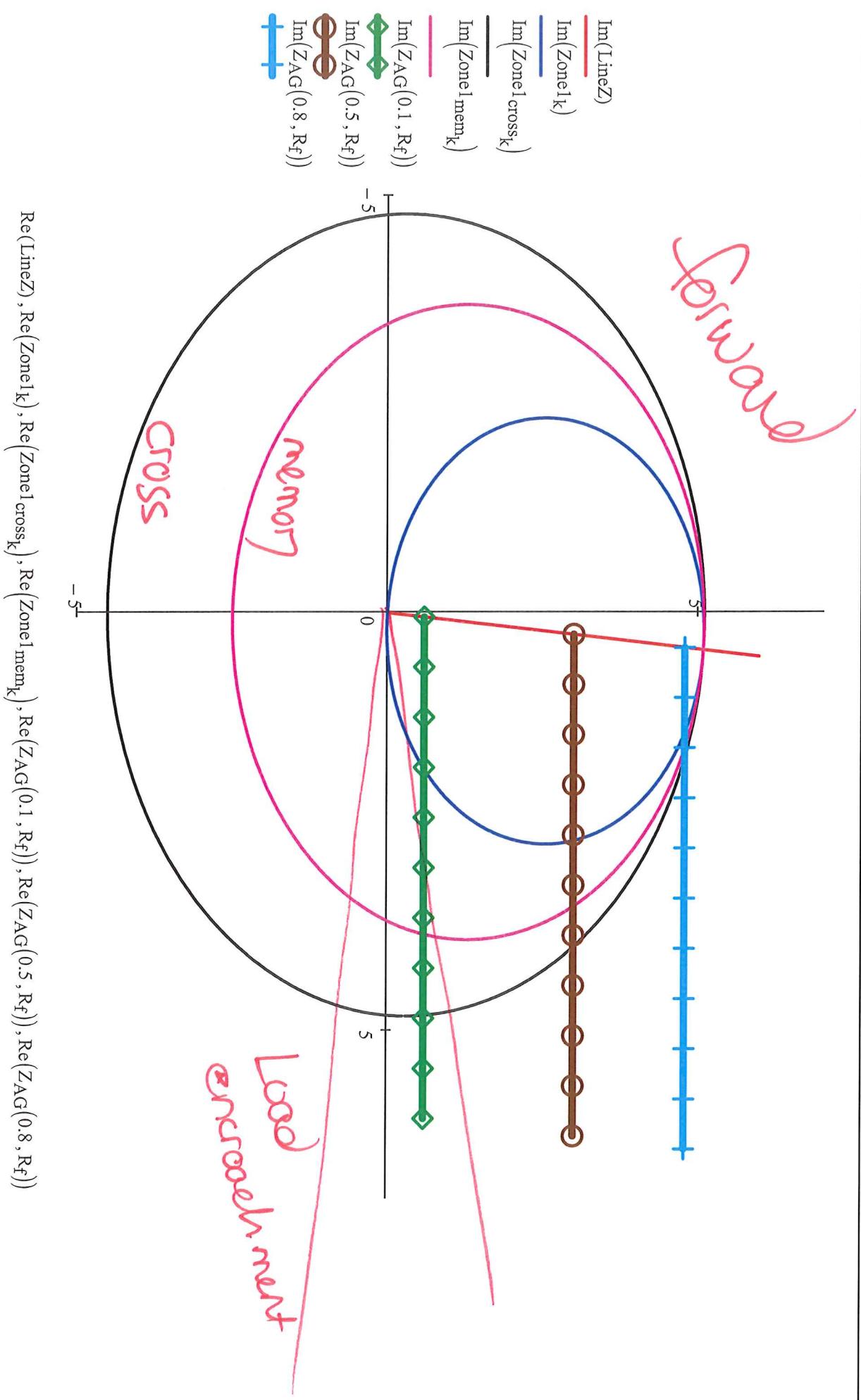


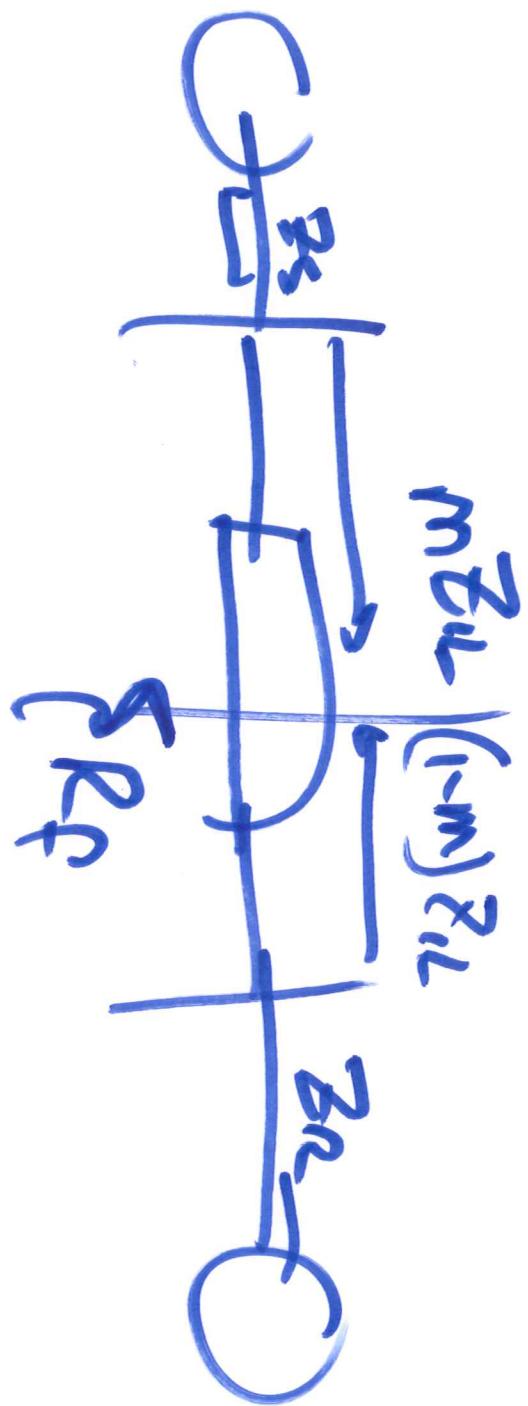
What is  $\bar{Z}_P$  for  
load conditions?

$$\bar{Z}_P = \frac{V_{AG}}{I_{AG}} = \frac{k_{mem} V_{A1mem}}{\bar{I}}$$

$$\Rightarrow 0$$

No expansion





*Solution:* There will be a couple of different effects. First, the effective impedance,  $Z_{A,G}$  for the mho circle, will move to the right, since the infeed effect will be much more pronounced. At the same time, for the cross and memory polarized cases the amount of Mho expansion will also increase significantly since the expanded circle roughly extends back by the source impedance.

$$R_{self10C} := 0.902 \Omega$$

$$R_{self50C} := 0.926 \Omega$$

Big decrease

$$R_{self80C} := 0.305 \Omega$$

$$R_{cross10C} := 5.291 \Omega$$

$$R_{mem10C} := 4.076 \Omega$$

$$R_{cross50C} := 2.697 \Omega$$

$$R_{mem50C} := 2.14 \Omega$$

small decrease

$$R_{cross80C} := 0.702 \Omega$$

$$R_{mem80C} := 0.575 \Omega$$

Infeed by factor 5

