University of Idaho

ECE 529
Utility Applications of Power Electronics

Session 18

Two Axis (Park's) Transformation

Imitation Measured Voltages:

Define array of time and define angular frequency:

$$t := 0\text{sec}, \underbrace{0.0001\text{sec}}_{\Delta t} .. \frac{6}{60\text{Hz}}$$ - 6 cycles at 60Hz

$$\omega_0 := 2\cdot\pi\cdot 60\text{Hz} \qquad \omega(t) := \omega_0$$

Voltage as a function of time

$$V_{mag} := 15\text{kV} \qquad v_a(t) := \sqrt{2}\cdot V_{mag}\cdot\cos(\omega(t)\cdot t)$$

$$v_b(t) := \sqrt{2}\cdot V_{mag}\cdot\cos(\omega(t)\cdot t - 120\text{deg})$$

$$v_c(t) := \sqrt{2}\cdot V_{mag}\cdot\cos(\omega(t)\cdot t + 120\text{deg})$$

Transform measured voltage and currents to the stationary dq0 ($\alpha\beta$) reference frame:

- Use equations from the Clarke Transformation instead of matrix for now

Remember d time

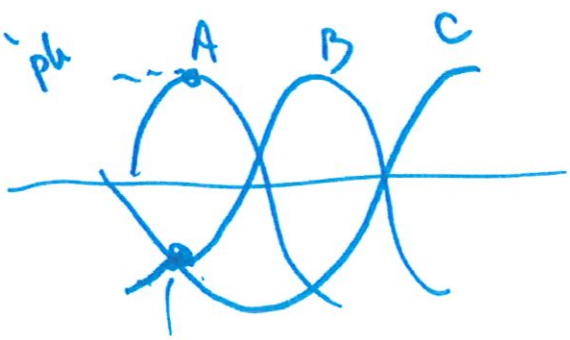
$$v_{ds}(t) := \frac{2}{3}\cdot\left(v_a(t) - 0.5\cdot v_b(t) - 0.5\cdot v_c(t)\right)$$

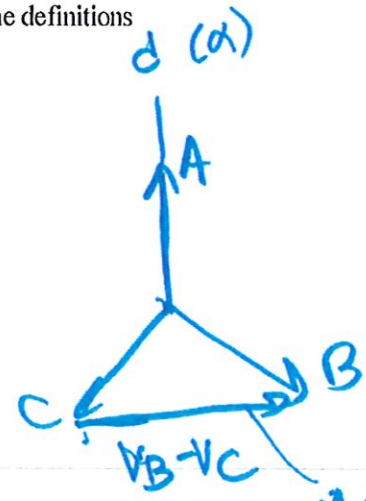
V_α

$$v_{qs}(t) := \frac{\left(v_b(t) - v_c(t)\right)}{\sqrt{3}}$$ Q axis 180 out of phase with some definitions

V_β

1pu ph





90° off from A

$\frac{1}{\sqrt{3}}$ pu/pu

$-\frac{1}{2}$

usually ignore for converters

$$V_0(t) = k\left(V_a(t) + V_B(t) + V_d(t)\right)$$

$\frac{1}{3}$ in symmetrical components (also some $\frac{1}{\sqrt{3}}$)

$$v_{dr2}(t) := v_{qs}(t)\cdot\sin(\theta r(t))$$

positive sequence bias due to $\theta r = 2\pi f(t)$

$$v_{dr}(t) := v_{dr1}(t) + v_{dr2}(t)$$

$$v_{qr1}(t) := v_{ds}(t)\cdot\sin(\theta r(t))$$

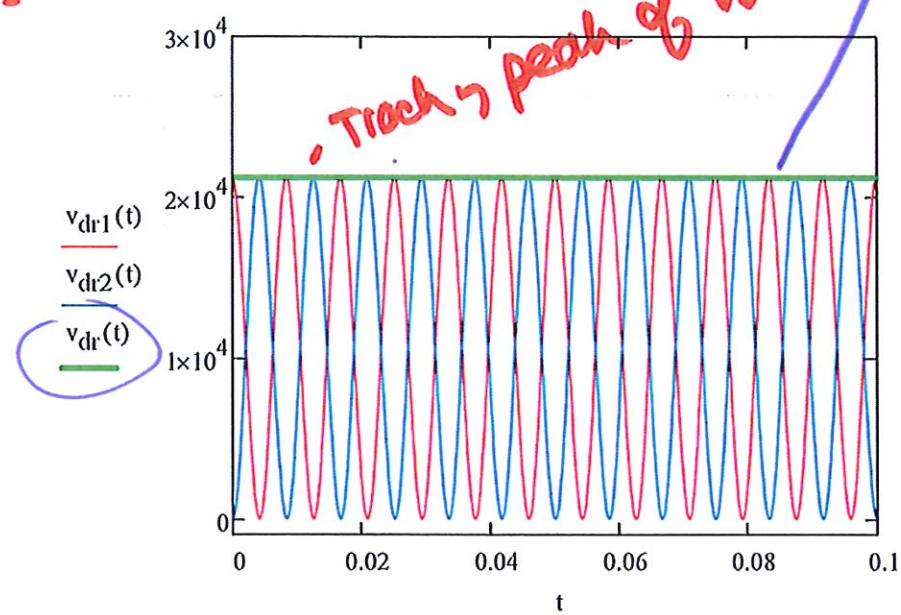
$$v_{qr2}(t) := v_{qs}(t)\cdot\cos(\theta r(t))$$

$$v_{qr}(t) := -v_{qr1}(t) + v_{qr2}(t)$$

rotating terms

constant in balanced 3Ø steady-state

Track peak of Va



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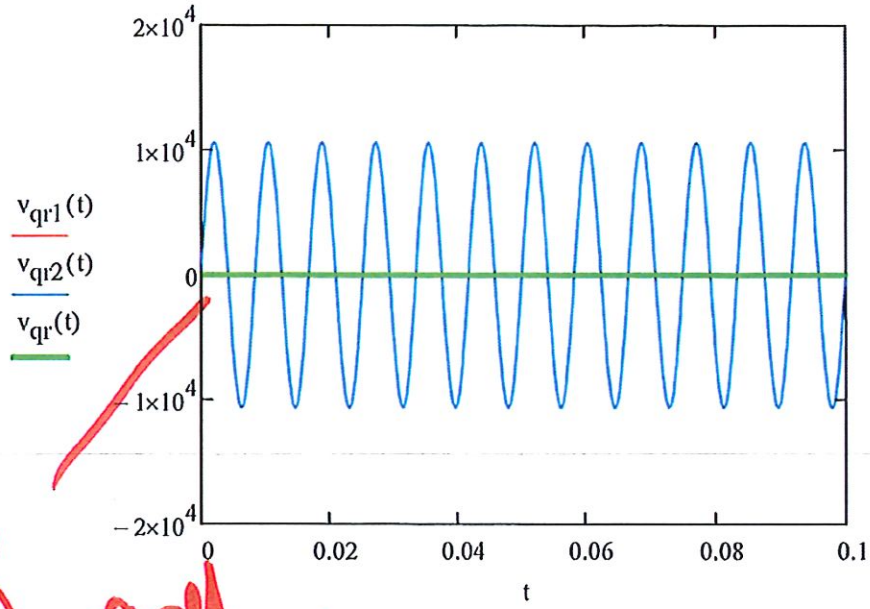

0 ← 90° off from peak of va (v_d)

Park's Transformation in Matrix Form

$\theta(t) := \omega_0 \cdot t$ synchronously rotating reference frame, note that this is generally shifted by $\pi/2$ for rotating machines.

alternate form uses $\sqrt{2/3}$ instead of 2/3

$$P(t) := \frac{2}{3} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \cos(\theta(t)) & \cos\left(\theta(t) - \frac{2\cdot\pi}{3}\right) & \cos\left(\theta(t) + \frac{2\cdot\pi}{3}\right) \\ -\sin(\theta(t)) & -\sin\left(\theta(t) - \frac{2\cdot\pi}{3}\right) & -\sin\left(\theta(t) + \frac{2\pi}{3}\right) \end{pmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

$\theta = 0$ means stationary frame

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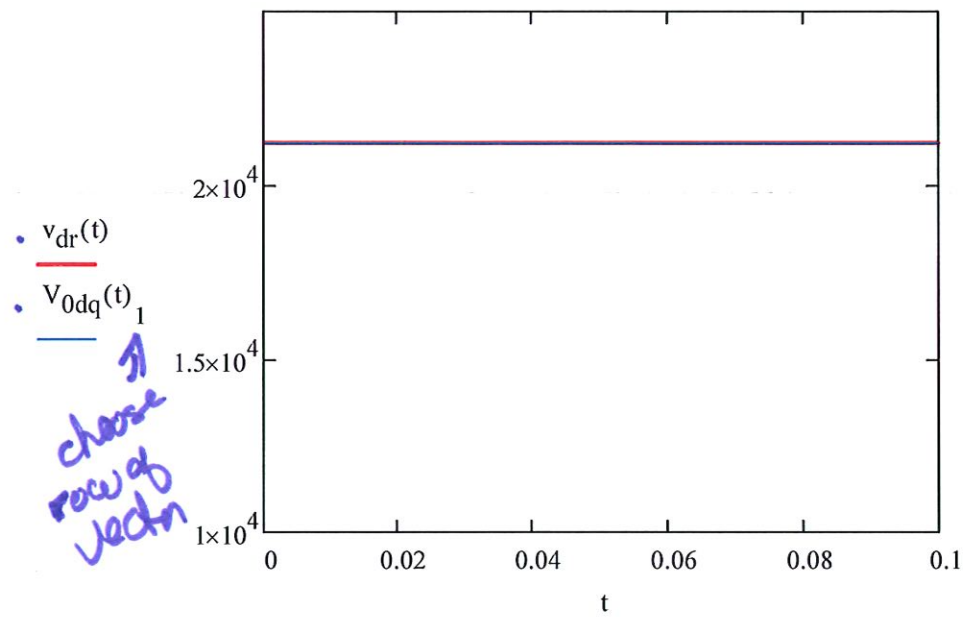
$$V_{0dq}(t) := P(t)\cdot\begin{pmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{pmatrix}$$

vector

- Compare to earlier results



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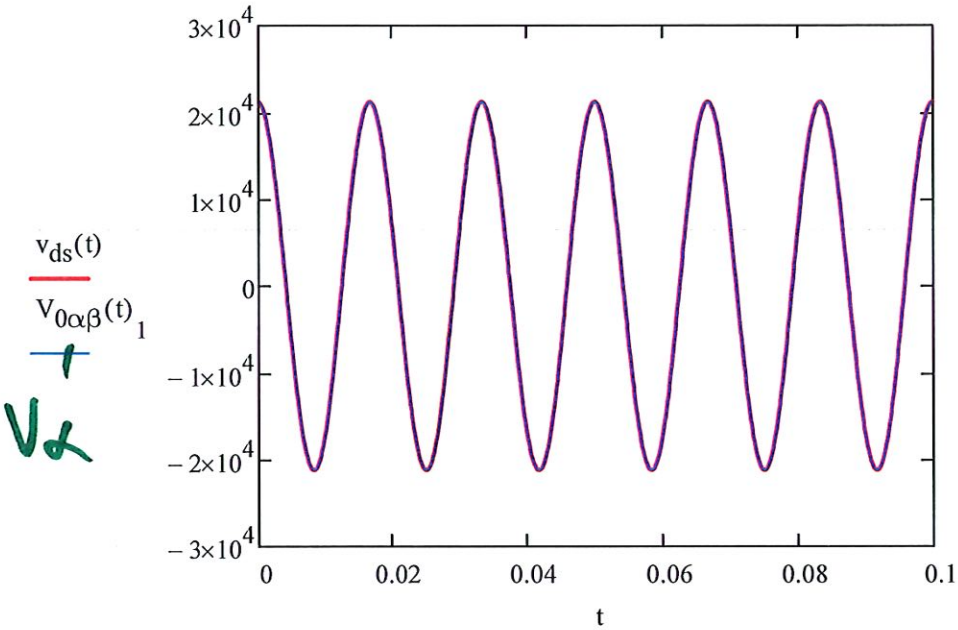
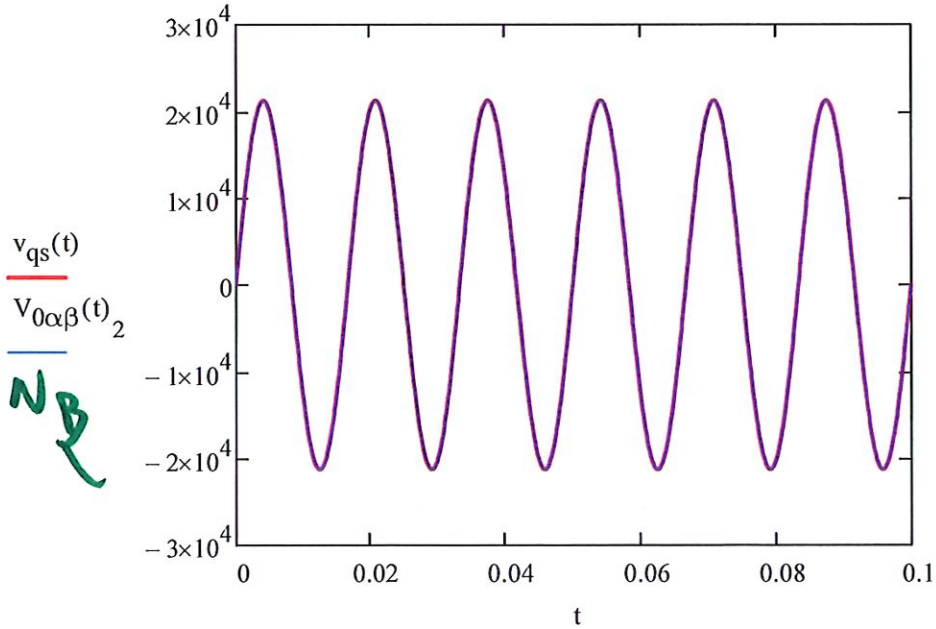
Or if we redefine or reference angle to be stationary in time:

$$V_{0\alpha\beta}(t) := P(0)\cdot\begin{pmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{pmatrix}$$

0 sec





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Now add some currents:

$$I_{mag} := 500A \qquad i_a(t) := \sqrt{2}\cdot I_{mag}\cdot\cos(\omega(t)\cdot t - 30deg)$$

$$i_b(t) := \sqrt{2}\cdot I_{mag}\cdot\cos(\omega(t)\cdot t - 150deg)$$

$$i_c(t) := \sqrt{2}\cdot I_{mag}\cdot\cos(\omega(t)\cdot t + 90deg)$$

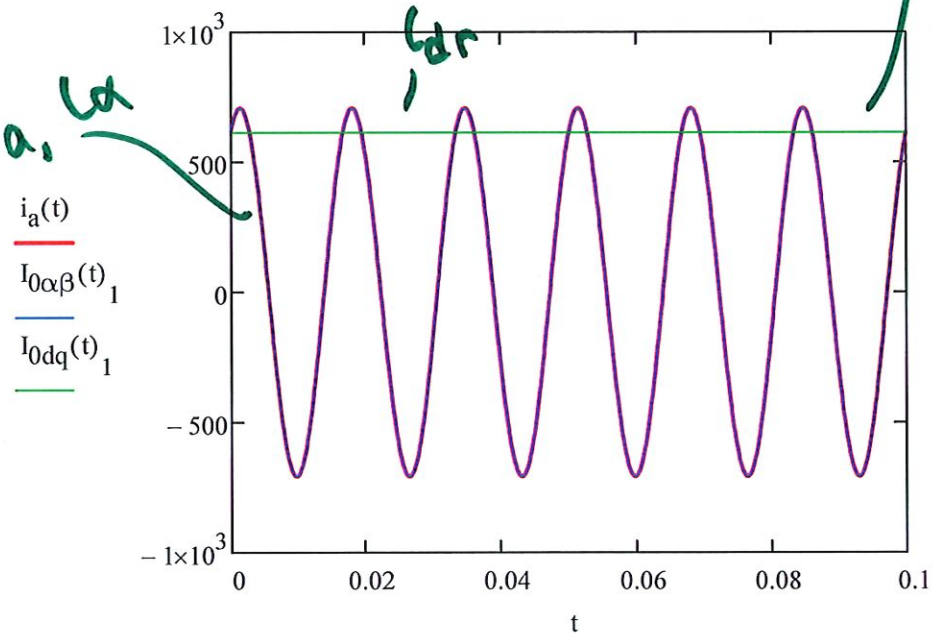
$$I_{0dq}(t) := P(t)\cdot\begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix} \qquad I_{0\alpha\beta}(t) := P(0)\cdot\begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix}$$

Osc

i 30° past peak

i_{dr}

i_a, i_α



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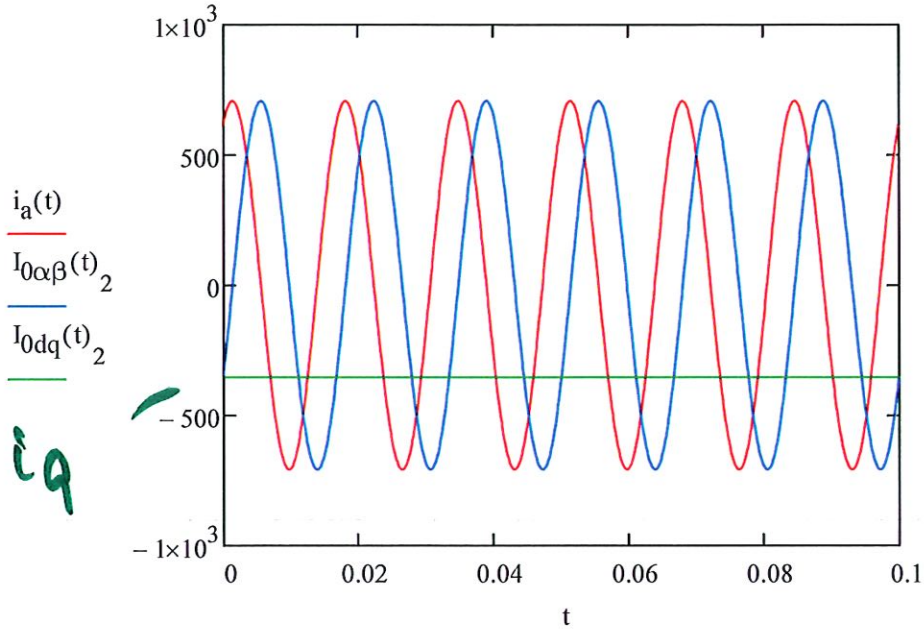
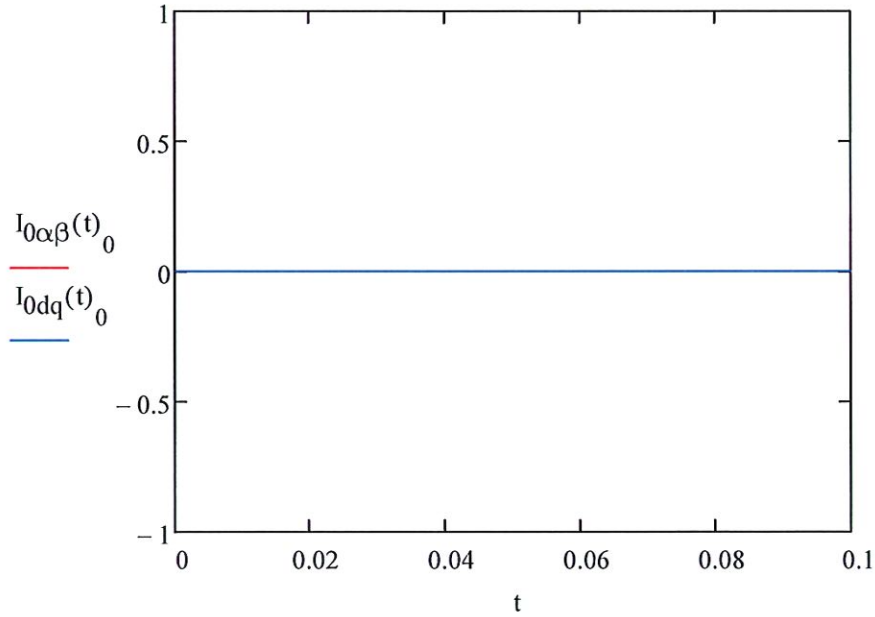

1×10^3
500
0
− 500
− 1×10^3
0
0.02
0.04
0.06
0.08
0.1
t
i_a(t)
I_0αβ(t)_2
I_0dq(t)_2
iq


1
0.5
0
− 0.5
− 1
0
0.02
0.04
0.06
0.08
0.1
t
I_0αβ(t)_0
I_0dq(t)_0

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$$v_{odq} = P\, v_{abc}$$

$$v_{abc} = P^{-1} v_{odq}$$

$$i_{abc} = P^{-1} i_{odq}$$

$$v_a i_a + v_b i_b + v_c i_c = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$= \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix}^T \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}$$

$$\left[P^{-1} v_{odq}\right]^T \left[P^{-1} i_{odq}\right]^T$$

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work through algebra

$$\left[V_{odq}\right]^T \underbrace{P\,P^{-1}}_{\frac{3}{2}[I]} \left[I_{odq}\right]$$

$$P = \frac{2}{3}\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ & & \end{bmatrix}$$

$$\frac{3}{2}\left[V_0 \; V_d \; V_q\right]\begin{bmatrix} I_0 \\ I_d \\ I_q \end{bmatrix} = \frac{3}{2}\left[V_0 i_0 + V_d i_d + V_q i_q\right]$$

in stationary or rotating . . —

Now calculate real and reactive power

$MW := 1000kW \qquad MVA := MW \qquad MVAR := MW$

- Phasor form first:

V_{LN} $V_a := V_{mag} \cdot e^{j \cdot 0deg}$

$I_a := I_{mag} \cdot e^{-j \cdot 30deg}$ } phasors

$p(t) = v_a i_a + v_b i_b + v_c i_c$

$P_{3ph} := 3 \cdot Re\left(V_a \cdot \overline{I_a}\right) \qquad P_{3ph} = 19.486 \cdot MW$

$Q_{3ph} := 3 \cdot Im\left(V_a \cdot \overline{I_a}\right) \qquad Q_{3ph} = 11.25 \cdot MW$

- Now in DQ reference frames:

$v_0 \; i_0 \quad v_d \; i_d \quad v_q \; i_q$

$$P_{0dq}(t) := \frac{3}{2} \cdot \left(V_{0dq}(t)_0 \cdot I_{0dq}(t)_0 + V_{0dq}(t)_1 \cdot I_{0dq}(t)_1 + V_{0dq}(t)_2 \cdot I_{0dq}(t)_2 \right)$$

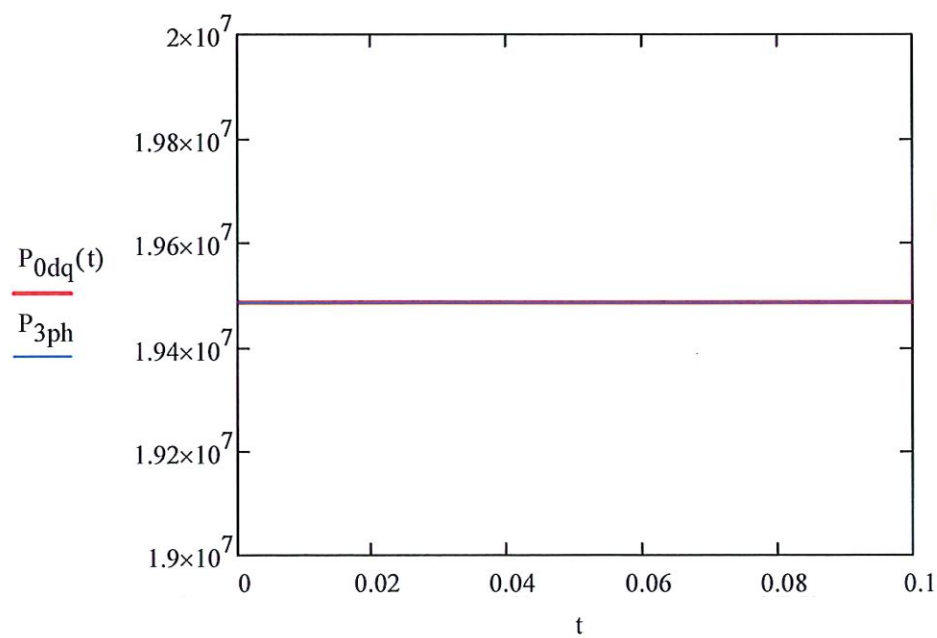
3wire

If we use V_{POI} as our phase A ref for V_d

Simplifies to

$P = \frac{3}{2}(V_d i_d)$

  - Note: we need the 3/2 term because of 2/3 constant in transformation matrix.
  - This will differ if use SQRT(2/3)



$3/2\left[v_q i_d - v_d i_q\right]$

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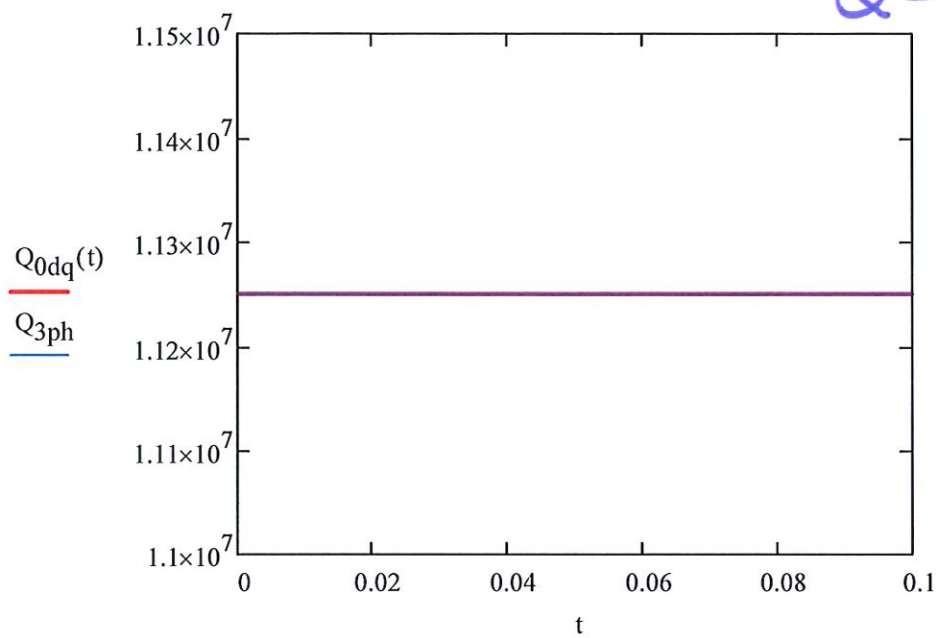
$$Q_{0dq}(t) := \frac{3}{2}\cdot\left(V_{0dq}(t)_2\cdot I_{0dq}(t)_1 - V_{0dq}(t)_1\cdot I_{0dq}(t)_2\right)$$

If V_q is zero due to synchronization loop

$Q = \frac{3}{2}\left(-V_d i_q\right)$

- Note: we need the 3/2 term because of 2/3 constant in transformation matrix.
- This will differ if use SQRT(2/3)



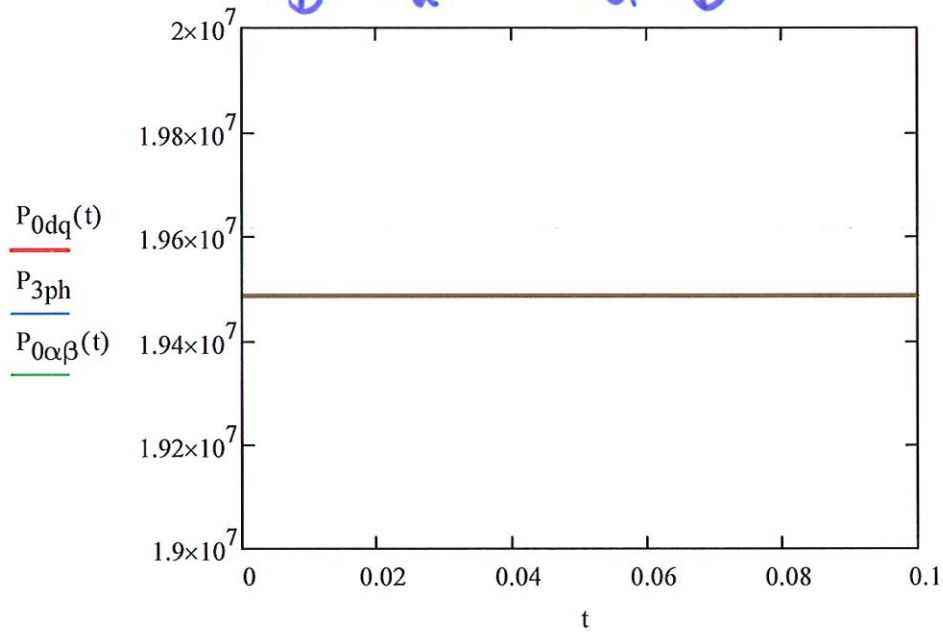
- Finally compare to αβ reference frame

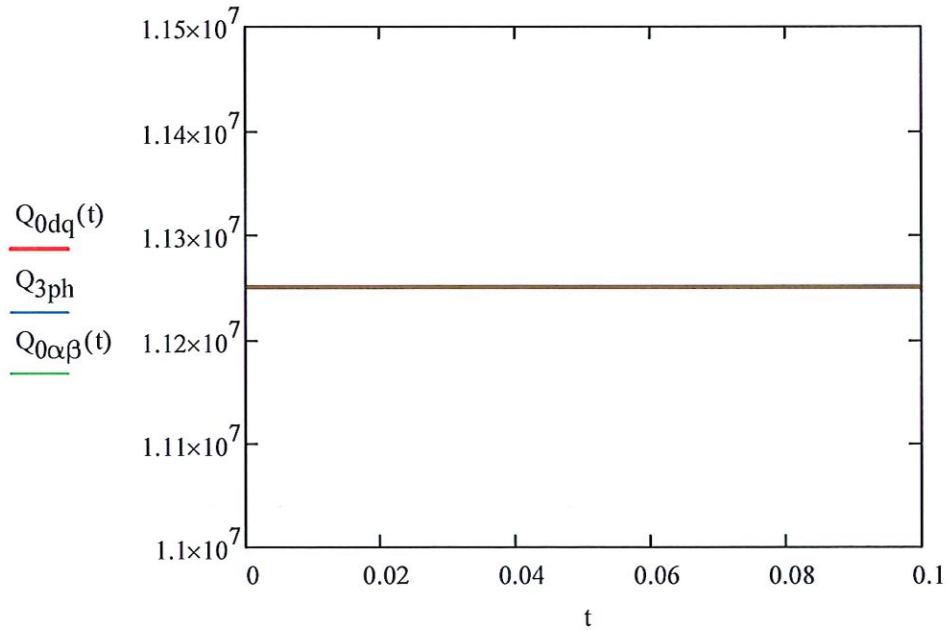
v_0 i_0 v_α i_α v_β i_β

$$P_{0\alpha\beta}(t) := \frac{3}{2}\cdot\left(V_{0\alpha\beta}(t)_0\cdot I_{0\alpha\beta}(t)_0 + V_{0\alpha\beta}(t)_1\cdot I_{0\alpha\beta}(t)_1 + V_{0\alpha\beta}(t)_2\cdot I_{0\alpha\beta}(t)_2\right)$$

$$Q_{0\alpha\beta}(t) := \frac{3}{2}\cdot\left(V_{0\alpha\beta}(t)_2\cdot I_{0\alpha\beta}(t)_1 - V_{0\alpha\beta}(t)_1\cdot I_{0\alpha\beta}(t)_2\right)$$

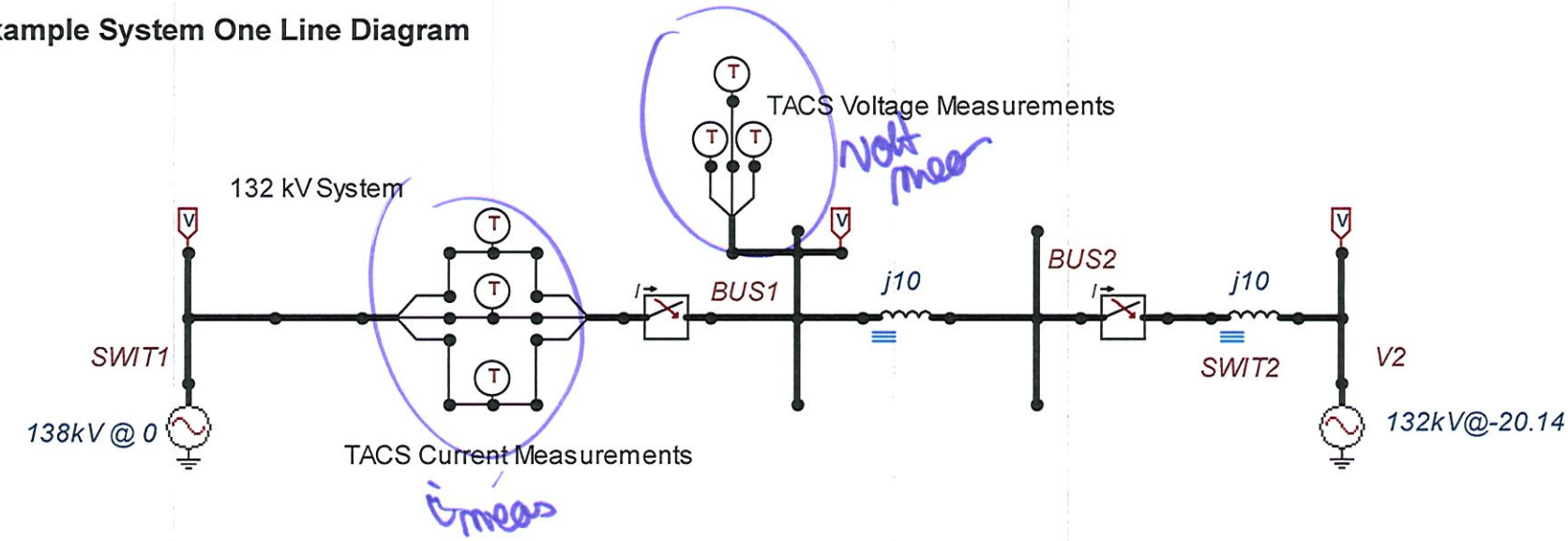
v_β i_α v_α i_β





Implementing Park's Transformer in Transients Program

Example System One Line Diagram

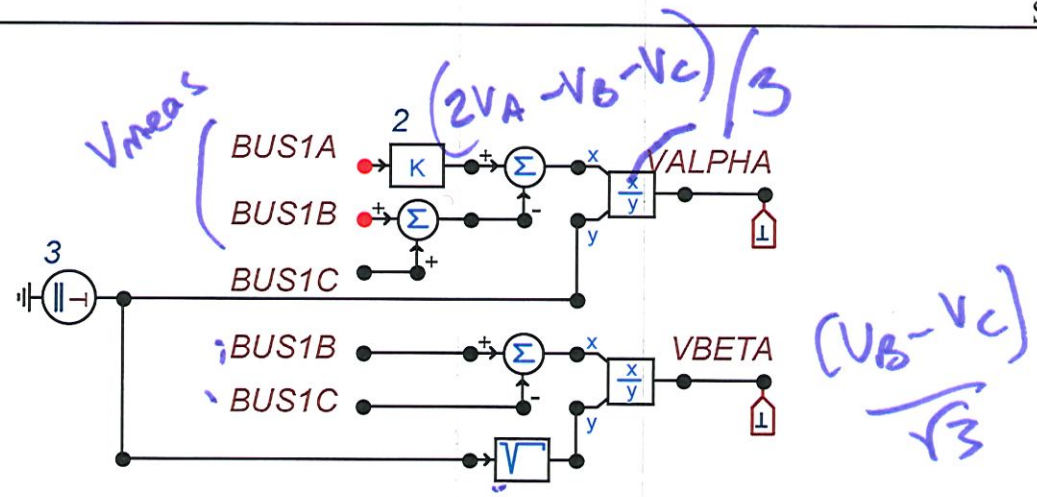


Implementing α–β and d-q transformations in ATPDraw

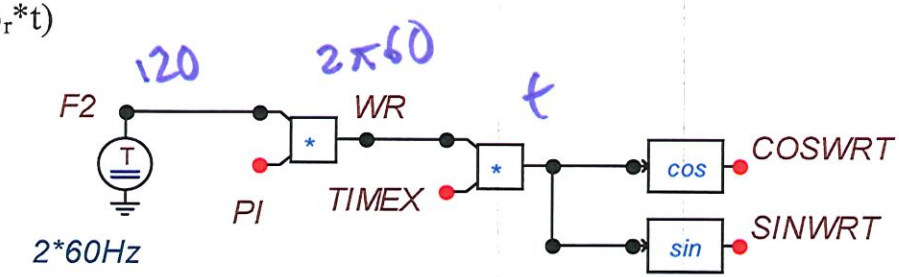
- Create V_α and V_β, where BUS1A, BUS1B and BUS1C are measured voltages

$$v_{ds}(t) := \frac{2}{3}\cdot\left(v_a(t) - 0.5\cdot v_b(t) - 0.5\cdot v_c(t)\right)$$

$$v_{qs}(t) := \frac{\left(v_b(t) - v_c(t)\right)}{\sqrt{3}}$$



- Create $\cos(\omega_r*t)$ and $\sin(\omega_r*t)$



- Transform from stationary to rotating reference frame

$$v_{dr}(t) := v_{ds}(t)\cdot\cos(\theta r(t)) + v_{qs}(t)\cdot\sin(\theta r(t))$$

$$v_{qr}(t) := -\left(v_{ds}(t)\cdot\sin(\theta r(t))\right) + v_{qs}(t)\cdot\cos(\theta r(t))$$