**ECE529: Lecture 37**

MW := 1000kW  \quad MVA := MW  \quad MVAR := MW  \quad pu := 1

**Problem 1:** For the system below we want to increase the power transfer from Bus 1 to Bus 2 by adding a SVC at the midpoint.

A. Calculate the power transfer from bus 1 to bus 2 if the SVC is disconnected.

B. Suppose the SVC is connected through a 18:1 transformer with a leakage reactance of 0.005 Ω on low voltage side, determine the MVAR requirements for the SVC to do the following:
   1. Increase P12 = 650MW with the TCR is not gated.
   2. How much capacitance and inductance is needed to increase P12 to 633MW with the TCR operating with $\sigma = 75$ deg.) Assume delta connection of the TCR and capacitors.
   3. Modify your solution for a 12 pulse SVC (one TCR in Δ and the other in Y)

C. Implement your solution for the 12 pulse SVC in ATP.
A. Calculate the power transfer from bus 1 to bus 2 if the SVC is disconnected.

Sending and Receiving End Voltages

\[ V_{1L} := 345kV \quad V_{1p} := \frac{V_{1L}}{\sqrt{3}} \quad V_{1p} = 199.186-kV \]

\[ V_{2L} := 345kV \quad V_{2p} := \frac{V_{2L}}{\sqrt{3}} \quad V_{2p} = 199.186-kV \]

Sending and Receiving End Angles

\[ \theta_1 := 0\text{deg} \quad \theta_2 := -17.605\text{deg} \]

Line Reactance \( X_{\text{line}} := 60\text{ohm} \)

\[ P_{12} := \frac{|V_{1L}| \cdot |V_{2L}| \cdot \sin(\theta_1 - \theta_2)}{X_{\text{line}}} \quad P_{12} = 599.991\cdot\text{MW} \]

B. Suppose the SVC is connected through a 18:1 transformer with a leakage reactance of \( 0.005 \Omega \) on low voltage side, determine the MVAR requirements for the SVC to do the following:

(1) Increase \( P_{12}=650\text{MW} \) with the TCR is not gated.

\[ X_{\text{half}} := \frac{X_{\text{line}}}{2} \quad X_{\text{half}} = 30\cdot\text{ohm} \]

- Lossless line, so the following relationships holds:

\[ P_{12\max} := 650\text{MW} \]

\[ P_{1m} := P_{12\max} \quad p_{m2} := P_{12\max} \]
\[ \theta_m := \frac{\theta_2 - \theta_1}{2} \quad \theta_m = -8.803 \cdot \text{deg} \]

- Assuming that the angles at the ends of the line to not change with change in \( P_{12} \), start from:

\[
P_{1m} = \frac{|V_{1L}| \cdot |V_{mL}| \cdot \sin(\theta_1 - \theta_m)}{X_{\text{half}}} = 3 \frac{|V_{1p}| \cdot |V_{mp}| \cdot \sin(\theta_1 - \theta_m)}{X_{\text{half}}}
\]

- Then

\[
V_{mp} := \frac{P_{1m} \cdot X_{\text{half}}}{3 \cdot V_{1p} \cdot \sin(\theta_1 - \theta_m)} \quad V_{mp} = 213.246 \cdot \text{kV}
\]

\[
V_{mL} := \sqrt{3} \cdot V_{mp} \quad V_{mL} = 369.353 \cdot \text{kV}
\]

\[
\text{OverVolt} := \frac{V_{mL}}{V_{1L}} \quad \text{OverVolt} = 107.059 \% \]

\[
Q_{m1} := \frac{V_{mL}^2}{X_{\text{half}}} - \left( \frac{V_{mL} \cdot V_{1L} \cdot \cos(\theta_m - \theta_1)}{X_{\text{half}}} \right) \quad Q_{m1} = 349.861 \cdot \text{MVAR}
\]

\[
Q_{m2} := \frac{V_{mL}^2}{X_{\text{half}}} - \left( \frac{V_{mL} \cdot V_{2L} \cdot \cos(\theta_m - \theta_2)}{X_{\text{half}}} \right) \quad Q_{m2} = 349.861 \cdot \text{MVAR}
\]

\[
Q_{\text{compensator_max}} := Q_{m1} + Q_{m2} \quad Q_{\text{compensator_max}} = 699.721 \cdot \text{MVAR}
\]

This is the net capacitive injection to increase \( P_{12} \) to 650MW.
• On the secondary of the transformer (18:1)

\[ V_{\text{seemax}} := \frac{V_{\text{mL}}}{18} \quad V_{\text{seemax}} = 20.52 \cdot \text{kV} \quad \text{line to line} \]

\[ X_{\text{xfmr\_low}} := 0.005 \text{ohm} \quad \frac{X_{\text{xfmr\_low}}}{2\pi \cdot 60 \text{Hz}} = 0.0133 \cdot \text{mH} \]

• Since the TCR is not gated, this leaves capacitor only. Assume it is connected in Delta, so mult by 3

• The capacitative will also need to supply some reactive power to transformer leakage

\[
Q_{\text{comp}} = \frac{3 \cdot V_{\text{mL}}^2}{X_{\text{eff}}} = \frac{3 \cdot V_{\text{mL}}^2}{X_{\text{xfmr\_low}} - X_c}
\]

• Solving for effective compensating reactance:

\[ X_{\text{eff}} := 3 \cdot \frac{V_{\text{seemax}}^2}{Q_{\text{compensator\_max}}} \quad X_{\text{eff}} = 1.805 \Omega \]

\[ X_{c1} := X_{\text{eff}} + X_{\text{xfmr\_low}} \quad X_{c1} = 1.81 \Omega \]

\[ C_{\text{comp}} := \frac{1}{2 \cdot \pi \cdot 60 \text{Hz} \cdot X_{c1}} \quad C_{\text{comp}} = 1465.3203 \cdot \mu F \]

• If we split the capacitance into two banks (one associated with each 6 pulse bridge). Capacitors in parallel add, so we divide by two.

\[ \frac{C_{\text{comp}}}{2} = 732.66 \cdot \mu F \]
Now calculate how much inductance is needed for $P_{12}=633\text{MW}$

\[
P_{12} := 633\text{MW}
\]

\[
P_{1m} := P_{12} \quad P_{m2} := P_{12}
\]

\[
\theta_m := \frac{\theta_2 - \theta_1}{2}
\]

Assuming that the angles at the ends of the line do not change with change in $P_{12}$

\[
V_{mp} := \frac{P_{1m} \cdot X_{half}}{3 \cdot V_{1p} \cdot \sin(\theta_1 - \theta_m)}
\]

\[
V_{mp} = 207.669\text{kV}
\]

\[
V_{mL} := \sqrt{3} \cdot V_{mp} \quad V_{mL} = 359.693\text{kV}
\]

\[
\text{OverVolt} := \frac{V_{mL}}{V_{1L}} \quad \text{OverVolt} = 104.259\% 
\]

\[
Q_{m1} := \frac{V_{mL}^2}{X_{half}} - \left( \frac{V_{mL} \cdot V_{1L} \cdot \cos(\theta_m - \theta_1)}{X_{half}} \right) 
\]

\[
Q_{m1} = 224.889\text{MVAR} 
\]

\[
Q_{m2} := \frac{V_{mL}^2}{X_{half}} - \left( \frac{V_{mL} \cdot V_{2L} \cdot \cos(\theta_m - \theta_2)}{X_{half}} \right) 
\]

\[
Q_{m2} = 224.889\text{MVAR} 
\]

\[
Q_{\text{compensator}} := Q_{m1} + Q_{m2} 
\]

\[
Q_{\text{compensator}} = 449.778\text{MVAR} \quad \text{This is the net capacitive contribution.}
\]
On the secondary of the transformer (18:1)

\[ V_{sec} := \frac{V_{mL}}{18} \quad V_{sec} = 19.983 \cdot \text{kV} \]

\[ X_{eff633} := 3 \cdot \frac{V_{sec}^2}{Q_{\text{compensator}}} \quad X_{eff633} = 2.663 \Omega \quad \text{Multiply by 3 for delta} \]

Subtract off transformer, leaving parallel LC

\[ X_{SVC} := X_{eff633} + X_{\text{xfmr_low}} \quad X_{SVC} = 2.668 \Omega \]

Now solve for \( X_{\text{tcr}} \) from:

\[-j \cdot X_{SVC} = j \cdot X_{tcr} \cdot \left( \frac{-j \cdot X_{c1}}{j \cdot X_{tcr} - j \cdot X_{c1}} \right) = 0\]

\[ X_{tcr,\sigma} := \text{Find}(X_{tcr,\sigma}) \quad X_{tcr,\sigma} = 5.629 \Omega \]

This is \( X_{tcr} \) for \( \sigma = 75 \text{ deg} \), now find actual value of \( XL \) and then \( L \)

\[ \sigma_{633} := 75 \text{deg} \]

\[ X_L := X_{tcr,\sigma} \cdot \frac{\sigma_{633} - \sin(\sigma_{633})}{\pi} \]

\[ X_L = 0.615 \Omega \]

\[ L_{tcr} := \frac{X_L}{2 \cdot \pi \cdot 60 \text{Hz}} \quad L_{tcr} = 1.63 \text{mH} \]
Since there will be two TCR's in parallel, we need to double this value if they are both delta connected

\[ L_{12\text{pulse}} := 2 \cdot L_{\text{tcr}} \quad L_{12\text{pulse}} = 3.261 \cdot \text{mH} \]

- For the TCR that is Y connected we need to perform a Δ to Y conversion (divide by 3)

\[ L_Y := \frac{L_{12\text{pulse}}}{3} \quad L_Y = 1.087 \cdot \text{mH} \]

Simulation parameters:

- Since the transformer is connected delta - wye - wye the turns ratio is 31.177:1 (the sqrt 3 needs to be added)

\[ \sqrt{3} \cdot 18 = 31.177 \]

- The transformer reactances are added to the transformer secondaries. Since there are two transformers, the reactance are doubled so the effective reactance meets the problem specifications:

\[ \frac{X_{\text{xfmr\_low}}}{2\pi \cdot 60\text{Hz}} = 0.0133 \cdot \text{mH} \quad 2 \cdot \frac{X_{\text{xfmr\_low}}}{2\pi \cdot 60\text{Hz}} = 0.0265 \cdot \text{mH} \]

C. Implement your solution for the 12 pulse SVC in ATP. (see ATPDraw file for better view).
Power Circuit
Notes about the power circuit part of the ATPDraw file:

1. Note that the wye connected TCR is grounded, and that it is connected to a wye grounded transformer.
   a. This allows triplen harmonic currents to flow.
   b. The connection from this TCR to the power system must be delta to keep the triplen harmonics off the power system (this will be a problem if the TCR's a not loaded equally (but this is a problem in general with TCR's).
2. The main power transformer is a three winding transformer connected with the HV winding in delta, and the secondary and tertaries ea in wye.
3. The capacitors in each 6 pulse SVC has a small resistance added in series to provide damping (it tends to ring with the TCR and line inductances. The added resistance is 0.01 ohm per phase. Note that the size of the resistance makes a big difference in the performance since the capacitive reactance is only 1.8ohm. When $\alpha = 90$ degrees, $P_{12}$ should be 650MW. If this added resistance is 0, the system achieves this. If $R = 0.1$ ohm, $P_{12} = 660$MW, of $R = 0.01$ ohm, $P_{12} = 651.5$MW. This has a bigger effect when the TCR is firing too as will be noted below. In practice this is resolved with a closed loop controller, rather than the open loop control shown here.
4. I have added a wye grounded resistance of 10kOhm on each TCR circuit. This primary matters on the delta connected TCR, as it provides a balanced ground reference point (this is more to satisfy ATP numerical behavior as anything else.

Controls

1. Synchronization and firing circuit (shown above) is similar to that used in homework 2
2. The delta connected TCR gets synchronization reference from the line to line voltage across each phase leg of the TCR
3. The wye connected TCR gets synchronization reference from the line to ground voltage on each phase.
4. Initially operate with alpha = 90 degrees (sigma=0), and then move to calculated alpha of 52.5 degrees at $t = 100$ ms.
Simulation Results

The simulations start out with $\alpha = 90$ deg (this corresponds to $\sigma = 0$ deg so the TCR's don't conduct) and then there is a step change to $\alpha = 52.5$ deg (corresponds to $\sigma = 75$ deg). This covers the two operating conditions used for performing the calculations.

- Firing angle $\alpha$

- Single phase TCR currents (one phase on each TCR)

- The current starts out at zero while $\alpha = 90$deg
- Single phase TCR currents (one phase on each TCR) after $\alpha=52.5$ deg

![Graph showing single phase TCR currents](image)

Note:
1. The phase shift between the currents due to the wye and delta connected TCRs
2. The magnitude difference (this difference is $\sqrt{3}$) again due to the difference between the wye and delta connections.

- Line current between the two 6 pulse SVC's and the transformer winding (includes the capacitor current) for each SVC
  1. First both currents over the entire simulation period.
  2. During the initial part of the simulation the TCR is not gated and the capacitor currents dominate.
  3. Note that the currents are identical when the TCR's are not conducting

![Graph showing line current between two SVCs](image)
- Line current Drawn by delta connected TCR + capacitors

- Line current Drawn by wye connected TCR + capacitors

- Harmonic currents from delta SVC

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<th>Phase</th>
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THD=16.505%

- Harmonic currents from SVC with wye connected TCR

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THD=36.21%

- Note that the wye connected TCR has significant triplen harmonic
  1. This is why the transformer winding connected to it is wye (to complete the circuit path)
  2. The primary winding is delta to keep these off the power system
- Note the magnitude and angles of the 5th and 7th harmonics especially

- Phase relationship between the currents drawn by the TCR's plus capacitors (note the fundamental components are in phase)

- For comparison, here is the same waveforms without the capacitors in the circuit
The two waveforms have nearly the same fundamental component magnitudes and angles.

- Current on this line side of the transformer. Note the transient when the thyristors start firing.
  1. This shows much better damping if there is more resistance in the circuit (for example in the capacitors or the line inductances)
  2. This also improves if the firing angle changes in a ramp instead of a step
  3. Harmonic content (after TCR's gated): Harm. Amplitude Phase
Note that power transfer without the TCR gated is slightly higher than calculated, and significantly higher than calculated after the TCR is gated.

Notice the oscillation in the "steady-state" power transfer after the TCR is gated. This will be much larger without the resistors.

This error increases as the damping resistance in series with the capacitor gets larger.

- Power transfer with resistance of 0.1 ohm in series with the capacitors:
Notice improved damping, but increased error in power transfer

Since the calculations all neglected resistance in the circuit, they would need to be updated to include resistance. A closed loop control will also improve things.

**Problem 2**: Design 5th and 7th harmonic filters for a 132 kV system such that they supply 30MVAR capacitive (at 60Hz). Assume
the inductors have an X/R ratio of 20, the capacitors have no resistance. Specify L to the nearest 0.1mH and C to the nearest 0.1 μFarad. Connect capacitors in Delta.

If the filters are connected in parallel with a harmonic current source (load) and the impedance looking into the power system is \( Z = 1 + j12 \, \Omega \) (at 60Hz) what percentage of the harmonic current goes into the filter and what percentage goes to the power system?

Test your filters in ATP. Rather than modeling a full SVC, just implement harmonic current sources at the appropriate frequencies (implement one at a time).

\[
\begin{align*}
V_{LL} & := 132kV \\
\omega_{60} & := 2\cdot\pi\cdot60Hz \\
\text{Qcap} & := 30\text{MVAR} \\
R_{\text{system}} & := 1\text{ohm} \\
X_{\text{system}} & := 12\text{ohm} \\
L_{\text{system}} & := \frac{X_{\text{system}}}{2\cdot\pi\cdot60Hz} \\
f_5 & := 5\cdot60Hz \\
f_7 & := 7\cdot60Hz
\end{align*}
\]

Fifth Harmonic Filter
- Neglect resistance since the X/R ratio is high
- Use \( \text{Xc} \) and \( C \) for Wye connected and then convert to delta later

\[
Q_{\text{cap}} = \frac{(-V_{LL})^2}{XL - XC_{\text{wye}}} = \frac{(-V_{LL})^2}{\omega_{60}L_5 - \frac{1}{\omega_{60}C_{5\text{wye}}}}
\]
- Derive an expression only in terms of \( C_{5\text{wye}} \) and known frequency terms
\[ f_5 = \frac{1}{2 \cdot \pi \cdot \sqrt{L_5 \cdot C_{\text{wye}}}} \]

\[ L_5 = \frac{1}{C_{\text{wye}} \cdot (2 \cdot \pi \cdot f_5)^2} \]  
substitute into the Qcap equation

\[ Q_{\text{cap}} = \frac{-V_{\text{LL}}^2}{\omega_{60} \cdot \frac{1}{C_{\text{wye}} \cdot (2 \cdot \pi \cdot f_5)^2} - \frac{1}{\omega_{60} \cdot C_{\text{wye}}}} \]

\[ -Q_{\text{cap}} = \frac{V_{\text{LL}}^2 \left( \omega_{60} \cdot C_{\text{wye}} \cdot (2 \cdot \pi \cdot f_5)^2 \right)}{\omega_{60}^2 - (2 \cdot \pi \cdot f_5)^2} \]

Express this in terms of the capacitance:

\[ C_{\text{wye}} := \frac{-Q_{\text{cap}} \left( \omega_{60}^2 - (2 \cdot \pi \cdot f_5)^2 \right)}{V_{\text{LL}}^2 \left( \omega_{60} \cdot (2 \cdot \pi \cdot f_5)^2 \right)} \]

\[ C_{\text{wye}} = 4.384 \mu F \]

Convert to delta by dividing \( C_{\text{wye}} \) by 3 (this is the same as \( 3 \cdot X_{\text{c5wye}} = X_{\text{c5delta}} \)):

\[ C_{\text{delta}} := \frac{C_{\text{wye}}}{3} \]

\[ C_{\text{delta}} = 1.461 \mu F \]

\[ L_5 := \frac{1}{C_{\text{wye}} \cdot (2 \cdot \pi \cdot f_5)^2} \]

\[ L_5 = 64.192 \text{ mH} \]
Now express L and C within specified tolerances.

\[ L_{\text{set}} := 64.2 \text{mH} \quad C_{\text{delta_set}} := 1.5 \mu\text{F} \]

\[ f_{5_0} := \frac{1}{2 \cdot \pi \cdot \sqrt{L_{\text{set}} \cdot 3 \cdot C_{\text{delta_set}}}} \quad f_{5_0} = 296.105 \frac{1}{s} \]

effectively detuned to slightly below 300 Hz.

\[ R_L := \frac{\omega_{60} \cdot L_{\text{set}}}{20} \quad R_L = 1.21 \Omega \]

\[ S_{\text{filter}_5} := \frac{V_{LL}^2}{\left[ R_L + j \left( \frac{\omega_{60} \cdot L_{\text{set}} - \frac{1}{\omega_{60} \cdot 3 \cdot C_{\text{delta_set}}}}{\omega_{60}^2} \right) \right]} \]

\[ S_{\text{filter}_5} = (0.066 - 30.825i) \cdot \text{MVA} \]

\[ \text{Im}(S_{\text{filter}_5}) = -30.825 \cdot \text{MVAR} \quad \text{Close, would be exact without rounding.} \]

Repeat for the 7th Harmonic Filter

\[ -Q_{\text{cap}} = \frac{V_{LL}^2 \left[ \omega_{60} \cdot C_{\text{7wye}} \cdot (2 \cdot \pi \cdot f_7)^2 \right]}{\omega_{60}^2 - (2 \cdot \pi \cdot f_7)^2} \]
Express this in terms of the capacitance:

$$C_{7\text{wye}} := \frac{-Q_{\text{cap}} \left[ \omega_{60}^2 - (2\cdot\pi\cdot f_7)^2 \right]}{V_{\text{LL}2}^2 \left[ \omega_{60}(2\cdot\pi\cdot f_7)^2 \right]} \quad C_{7\text{wye}} = 4.474\cdot\mu F$$

Convert to delta by dividing $C_{7\text{wye}}$ by 3 (this is the same as $3\cdot X_{\text{c7wye}} = X_{\text{c7delta}}$):

$$C_{7\text{delta}} := \frac{C_{7\text{wye}}}{3} \quad C_{7\text{delta}} = 1.491\cdot\mu F$$

$$L_7 := \frac{1}{C_{7\text{wye}} \cdot (2\cdot\pi\cdot f_7)^2} \quad L_7 = 32.096\cdot\text{mH}$$

Now express $L$ and $C$ within specified tolerances.

$$L_{7\text{set}} := 32.1\text{mH} \quad C_{7\text{delta}_\text{set}} := 1.5\mu F$$

$$f_{7\_o} := \frac{1}{2\cdot\pi\sqrt{L_{7\text{set}} \cdot 3C_{7\text{delta}_\text{set}}}} \quad f_{7\_o} = 418.756\text{ Hz} \quad \text{detuned to slightly below 420 Hz.}$$

$$R_{L7} := \frac{\omega_{60}\cdot L_{7\text{set}}}{20} \quad R_{L7} = 0.605 \Omega$$

$$S_{\text{filter}_7} := \frac{V_{\text{LL}2}^2}{\left[ R_{L7} + j\left( \omega_{60}\cdot L_{7\text{set}} - \frac{1}{\omega_{60}\cdot 3C_{7\text{delta}_\text{set}}} \right) \right]}$$
\[ S_{\text{filter}_7} = (0.032 - 30.179i) \cdot \text{MVA} \]

\[ \text{Im}(S_{\text{filter}_7}) = -30.179 \cdot \text{MVAR} \quad \text{Close, would be exact without rounding.} \]

**Now do current divider relationships:**

First, see what percentage of 5th harmonic current enters the 5th harmonic filter, the 7th harmonic filter, and the power system.

\[ Z_{\text{system}}(f) := R_{\text{system}} + j \cdot 2 \cdot \pi \cdot f \cdot L_{\text{system}} \]

\[ Z_{\text{filter}_5}(f) := R_{L5} + j \cdot \left( 2 \cdot \pi \cdot f \cdot L_{5\text{set}} - \frac{1}{2 \cdot \pi \cdot f \cdot 3 \cdot C_{5\delta \text{set}}} \right) \]

\[ Z_{\text{filter}_7}(f) := R_{L7} + j \cdot \left( 2 \cdot \pi \cdot f \cdot L_{7\text{set}} - \frac{1}{2 \cdot \pi \cdot f \cdot 3 \cdot C_{7\delta \text{set}}} \right) \]

\[ Z_{\text{system}}(f5) = (1 + 60i) \, \Omega \quad \text{Ignoring skin effect on the resistance.....} \]

\[ Z_{\text{filter}_5}(f5) = (1.21 + 3.122i) \, \Omega \]

\[ Z_{\text{filter}_7}(f5) = (0.605 - 57.385i) \, \Omega \]

1. If the 7th harmonic filter is neglected:

\[ \text{PercentFilter}_{5\text{th}}(f5) := \frac{|Z_{\text{system}}(f5)|}{|Z_{\text{system}}(f5) + Z_{\text{filter}_5}(f5)|} \quad \text{PercentFilter}_{5\text{th}}(f5) = 95.01\% \]

The 5th harmonic filter will take 95% of the 5th harmonic current, only 5% of the 5th harmonic current will enter the power system.

2. If the 7th harmonic filter is included:
Now need to consider the 7th harmonic filter in parallel with the system impedance (for the current divider with the 5th harmonic filter) and in parallel with the system impedance (for the current divider with the system).

\[
\text{ParaFilt7\_and\_System}(f) := \left( \frac{1}{|Z_{\text{system}}(f)|} + \frac{1}{|Z_{\text{filter7}}(f)|} \right)^{-1}
\]

\[
\text{ParaFilt7\_and\_System}(f_5) = 29.335 \, \Omega
\]

\[
\text{ParaFilt7\_and\_Filt5}(f) := \left( \frac{1}{|Z_{\text{filter5}}(f)|} + \frac{1}{|Z_{\text{filter7}}(f)|} \right)^{-1}
\]

\[
\text{ParaFilt7\_and\_Filt5}(f_5) = 3.163 \, \Omega
\]

\[
\text{ParaFilt5\_and\_System}(f) := \left( \frac{1}{|Z_{\text{system}}(f)|} + \frac{1}{|Z_{\text{filter5}}(f)|} \right)^{-1}
\]

\[
\text{ParaFilt5\_and\_System}(f_5) = 3.171 \, \Omega
\]

\[
\text{PercentFilter5th2}(f_5) := \frac{\text{ParaFilt7\_and\_System}(f_5)}{\text{ParaFilt7\_and\_System}(f_5) + |Z_{\text{filter5}}(f_5)|}
\]

\[
\text{PercentFilter5th2}(f_5) = 89.756 \%
\]

\[
\text{PercentSystem2}(f_5) := \frac{\text{ParaFilt7\_and\_Filt5}(f_5)}{\text{ParaFilt7\_and\_Filt5}(f_5) + |Z_{\text{system}}(f_5)|}
\]

\[
\text{PercentSystem2}(f_5) = 5.008 \%
\]

\[
\text{PercentFilter7th2}(f_5) := \frac{\text{ParaFilt5\_and\_System}(f_5)}{\text{ParaFilt5\_and\_System}(f_5) + |Z_{\text{filter7}}(f_5)|}
\]

\[
\text{PercentFilter7th2}(f_5) = 5.236 \%
\]
The system and the 7th harmonic filter each draw roughly equal amounts of 5th harmonic current.

Repeat for the 7th harmonic case:

\[ Z_{\text{system}}(f_7) = (1 + 84i) \Omega \text{ignoring skin effect on the resistance...} \]

\[ Z_{\text{filter}5}(f_7) = (1.21 + 85.211i) \Omega \]

\[ Z_{\text{filter}7}(f_7) = (0.605 + 0.501i) \Omega \]

1. If the 5th harmonic filter is neglected:

\[
\text{PercentFilter}_{7\text{th}}(f_7) := \frac{|Z_{\text{system}}(f_7)|}{|Z_{\text{system}}(f_7)| + |Z_{\text{filter}7}(f_7)|} \quad \text{PercentFilter}_{7\text{th}}(f_7) = 99.074\% 
\]

\[
\text{PercentSystem}_{7\text{th}}(f_7) := \frac{|Z_{\text{filter}7}(f_7)|}{|Z_{\text{system}}(f_7)| + |Z_{\text{filter}7}(f_7)|} \quad \text{PercentSystem}_{7\text{th}}(f_7) = 0.926\% 
\]

2. If the 5th harmonic current is included:

Now need to consider the 5th harmonic filter in parallel with the system impedance (for the current divider with the 7th harmonic filter) and in parallel with the system impedance (for the current divider with the system).

\[ \text{ParaFilt}_{5\text{ and } 7\text{th}}(f_7) = 42.304 \Omega \]

\[
\text{ParaFilt}_{7\text{ and } Filt5}(f_7) := \left( \frac{1}{Z_{\text{filter}5}(f_7)} + \frac{1}{Z_{\text{filter}7}(f_7)} \right)^{-1} \quad \text{ParaFilt}_{7\text{ and } Filt5}(f_7) = (0.598 + 0.502i) \Omega 
\]

\[
\text{PercentFilter}_{7\text{th}2}(f_7) := \frac{|\text{ParaFilt}_{5\text{ and } System}(f_7)|}{|\text{ParaFilt}_{5\text{ and } System}(f_7)| + |Z_{\text{filter}7}(f_7)|} \quad \text{PercentFilter}_{7\text{th}2}(f_7) = 98.177\% 
\]
PercentSystem2(f7) := \frac{|\text{ParaFilt7\_and\_Filt5}(f7)|}{|\text{ParaFilt7\_and\_Filt5}(f7)| + |Z\text{system}(f7)|} \quad \text{PercentSystem2}(f7) = 0.921\% 

PercentFilter5th2(f7) := \frac{|\text{ParaFilt7\_and\_System}(f7)|}{|\text{ParaFilt7\_and\_System}(f7)| + |Z\text{filter5}(f7)|} \quad \text{PercentFilter5th2}(f7) = 0.905\% 

As a check, the percentages should add up to 100%.

PercentFilter7th2(f7) + PercentFilter5th2(f7) + PercentSystem2(f7) = 100\%

Note that the percentage to the 7th harmonic filter goes down slightly, since some of the current now enters the 5th harmonic filter.

Test your filters in ATP. Rather than modeling a full SVC, just implement harmonic current sources at the appropriate frequencies (implement one at a time).

- ATP test system (\(\Delta t = 1\ \mu\text{sec}\)): 
Determine source voltage such that filter bus is at 1:0pu to verify reactive power:

First calculate the current the filters should draw (based on the design parameters, not the desired results)

\[
I_{\text{filt5}} := \frac{V_{\text{LL}}}{\sqrt{3} \cdot Z_{\text{filter5}}(60\text{Hz})} \quad |I_{\text{filt5}}| = 134.823\, \text{A} \quad \arg(I_{\text{filt5}}) = 89.877\, \text{deg}
\]

\[
I_{\text{filt7}} := \frac{V_{\text{LL}}}{\sqrt{3} \cdot Z_{\text{filter7}}(60\text{Hz})} \quad |I_{\text{filt7}}| = 131.997\, \text{A} \quad \arg(I_{\text{filt7}}) = 89.94\, \text{deg}
\]

\[
I_{60\text{Hz}} := I_{\text{filt5}} + I_{\text{filt7}}
\]

Now calculate at the source if the filters draw this current:

\[
V_{\text{source}} := \frac{V_{\text{LL}}}{\sqrt{3}} + I_{60\text{Hz}} \cdot Z_{\text{system}}(60\text{Hz}) \quad |V_{\text{source}}| = 73.009\, \text{kV} \quad \arg(V_{\text{source}}) = 0.213\, \text{deg}
\]
As a check: Fourier spectrum from simulation result:

\[ V_{\text{source}} \sqrt{2} = 103.251 \text{kV} \]

\[
V_{\text{source\_pk}} := 103.251 \text{kV} \cdot e^{-j \cdot 89.797 \text{deg}}
\]

\[
\begin{array}{ccc}
\text{Harm.} & \text{Amplitude} & \text{Phase (Deg)} \\
1 & 76212 & 0 \\
3 & 0.16617 & -176.67 \\
5 & 7.0396 & 88.985 \\
7 & 0.080307 & 24.445 \\
9 & 0.043343 & 12.562 \\
\end{array}
\]

\[
\frac{76.212 \text{kV}}{\sqrt{3}} = 1 \cdot \text{pu}
\]

Fifth Harmonic Current Source Case

- The 5th harmonic source is a balanced three phase (negative phase rotation) source with that has 10A peak.

**Current through the 5th harmonic filter:**

<table>
<thead>
<tr>
<th>Harm.</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>134.82</td>
<td>89.851</td>
</tr>
<tr>
<td>3</td>
<td>0.16617</td>
<td>-176.67</td>
</tr>
<tr>
<td>5</td>
<td>7.0396</td>
<td>88.985</td>
</tr>
<tr>
<td>7</td>
<td>0.080307</td>
<td>24.445</td>
</tr>
<tr>
<td>9</td>
<td>0.043343</td>
<td>12.562</td>
</tr>
</tbody>
</table>

THD=5.2595%

- The fundamental current is very close to the current calculated above:

\[ |I_{1\_filt5}| = 134.823 \text{ A} \]

\[ \arg(I_{1\_filt5}) = 89.877 \text{ deg} \]

- The percentage of the 7th harmonic current is:
\[
\frac{7.0396A}{10A} = 99.555\% \\
\frac{10A}{\sqrt{2}}
\]

Compared to calculated value of: PercentFilter5th2(f5) = 89.756\%

**Current through the 7th harmonic filter:**

<table>
<thead>
<tr>
<th>Harm.</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>132.02</td>
<td>89.941</td>
</tr>
<tr>
<td>3</td>
<td>0.096796</td>
<td>156.71</td>
</tr>
<tr>
<td>5</td>
<td>0.30847</td>
<td>-98.809</td>
</tr>
<tr>
<td>7</td>
<td>0.060121</td>
<td>-101.11</td>
</tr>
<tr>
<td>9</td>
<td>0.017849</td>
<td>-78.287</td>
</tr>
<tr>
<td></td>
<td>THD=0.94753%</td>
<td></td>
</tr>
</tbody>
</table>

- The fundamental current is very close to the current calculated above:
  \[
  |I_{1\_filt7}| = 131.997 A \quad \arg(I_{1\_filt7}) = 89.94\\text{deg}
  \]
- The percentage of the 7th harmonic current is:
  \[
  \frac{0.30847A}{10A} = 4.362\% \\
  \frac{10A}{\sqrt{2}}
  \]

Compared to calculated value of: PercentFilter7th2(f5) = 5.236\%

**Source Current**

<table>
<thead>
<tr>
<th>Harm.</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
</table>

- Source current (a little visible distortion)
1. The fundamental current is very close to the current calculated above:

\[
|I_{60Hz}| = 266.82 \, \text{A} \quad \arg(I_{60Hz}) = 89.908 \, \text{deg}
\]

2. The percentage of the 7th harmonic current is:

\[
\frac{0.34449 \, \text{A}}{10 \, \text{A}} = 4.872\%.
\]

Compared to calculated value of: \( \text{PercentSystem2}(f5) = 5.569\% \)

Notice that the percentages of the 5th harmonics currents don't compare very well to the theoretical results (and add to well over 100%). Let's add them as phasors:

\[
I_{5th} := 7.0396 \, \text{A} \, e^{j \cdot 88.985 \, \text{deg}} + 0.30847 \, \text{A} \, e^{-j \cdot 98.809 \, \text{deg}} - 0.34449 \, \text{A} \, e^{-j \cdot 82.093 \, \text{deg}} \quad \text{Subtract Isys due to measurement polarity}
\]

\[
|I_{5th}| = 7.075 \, \text{A} \quad \arg(I_{5th}) = 89.756 \, \text{deg} \quad \text{simulation had I5_rms=7.042 at an angle of -90.23deg}
\]

\[
\sqrt{2} \cdot |I_{5th}| = 10.005 \, \text{A} \quad \text{So percentage of magnitudes might not be the best way...}
\]

- **Seventh Harmonic Current Source Case**
  - Again, the 7th harmonic source has 10A peak, but now with a positive sequence rotation.
Current through the 7th harmonic filter:

<table>
<thead>
<tr>
<th>Harm.</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>131.99</td>
<td>89.907</td>
</tr>
<tr>
<td>3</td>
<td>0.044533</td>
<td>-149.87</td>
</tr>
<tr>
<td>5</td>
<td>0.056157</td>
<td>-98.598</td>
</tr>
<tr>
<td>7</td>
<td>7.0306</td>
<td>90.164</td>
</tr>
<tr>
<td>9</td>
<td>0.020174</td>
<td>29.305</td>
</tr>
</tbody>
</table>

THD=5.3699%

- The fundamental current is very close to the current calculated above:

\[
|I_{1_{filter}}| = 131.997 \text{ A} \quad \arg(I_{1_{filter}}) = 89.94 \text{ deg}
\]

- The percentage of the 7th harmonic current is:

\[
\frac{7.0306 \text{ A}}{10 \text{ A} / \sqrt{2}} = 99.428\%\]

Compared to calculated value of: \( \text{PercentFilter7th}(f7) = 98.177\% \)

Current through the 5th harmonic filter:
<table>
<thead>
<tr>
<th>Harm.</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>134.84</td>
<td>89.844</td>
</tr>
<tr>
<td>3</td>
<td>0.049522</td>
<td>163.89</td>
</tr>
<tr>
<td>5</td>
<td>0.06254</td>
<td>33.347</td>
</tr>
<tr>
<td>7</td>
<td>0.048741</td>
<td>16.446</td>
</tr>
<tr>
<td>9</td>
<td>9.3974E-03</td>
<td>-68.074</td>
</tr>
</tbody>
</table>

THD=0.42168%

- The fundamental current is very close to the current calculated above:

\[ |I_{1_{\text{filt}5}}| = 134.823 \text{ A} \quad \arg(I_{1_{\text{filt}5}}) = 89.877 \text{ deg} \]

- The percentage of the 7th harmonic current is:

\[
\frac{0.048741 \text{ A}}{10 \text{ A}} \times \frac{\sqrt{2}}{\sqrt{2}} = 0.689\% 
\]

Compared to calculated value of: \(\text{PercentFilter5th2}(7) = 0.905\%\)

**Source Current**
### Harmonics Table

<table>
<thead>
<tr>
<th>Harm.</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>266.81</td>
<td>89.907</td>
</tr>
<tr>
<td>3</td>
<td>0.08622</td>
<td>179.61</td>
</tr>
<tr>
<td>5</td>
<td>0.048949</td>
<td>-26.105</td>
</tr>
<tr>
<td>7</td>
<td>0.028382</td>
<td>-118.81</td>
</tr>
<tr>
<td>9</td>
<td>0.021815</td>
<td>14.557</td>
</tr>
</tbody>
</table>

**THD = 0.15502%**

- The fundamental current is very close to the current calculated above:

\[
|I_{60Hz}| = 266.82 \text{ A} \quad \arg(I_{60Hz}) = 89.908 \text{ deg}
\]

- The percentage of the 7th harmonic current is:

\[
\frac{0.028382 \text{ A}}{10\text{ A}} = 0.401\% \quad \frac{2}{\sqrt{2}}
\]

Compared to calculated value of: PercentSystem2(f7) = 0.921\%