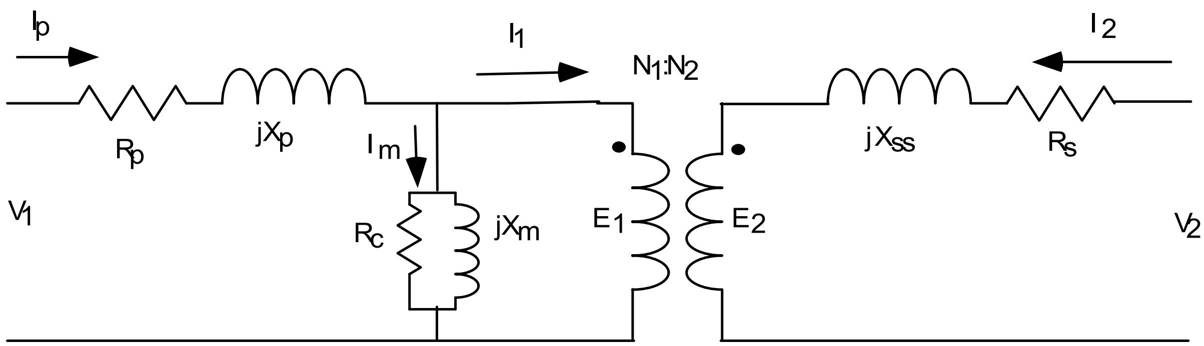
ECE 320: Lecture 10

Notes

Transformer equivalent circuit:



- Where Rp, Rs (or R1 and R2) are winding resistance
- Xp and Xs (or X1 and X2) are leakage reactance (inductance). Function of magnetic material, geometry of the core.
- Xm is magnetizing reactance. This is a non-linear inductance, that varies with the voltage appl across the inductance. Function of magnetic material properties.
- Rc represents core losses
- The transformer in the middle is an ideal transformer where the following relationships hold:

$$\frac{E1}{E2} = \frac{N1}{N2} \qquad \text{or} \qquad \frac{E1}{N1} = \frac{E2}{N2}$$

and

$$N1 \cdot I1 = -N2 \cdot I2 \qquad \text{or} \qquad N1 \cdot I2 + N2 \cdot I2 = 0 \qquad \text{or} \qquad \frac{I1}{I2} = \frac{-N2}{N1}$$

If we treat I2 as flowing out of the polarity mark, then the negative sign goes away.

Combining these: $S1 = V1 \cdot \overline{I1}$

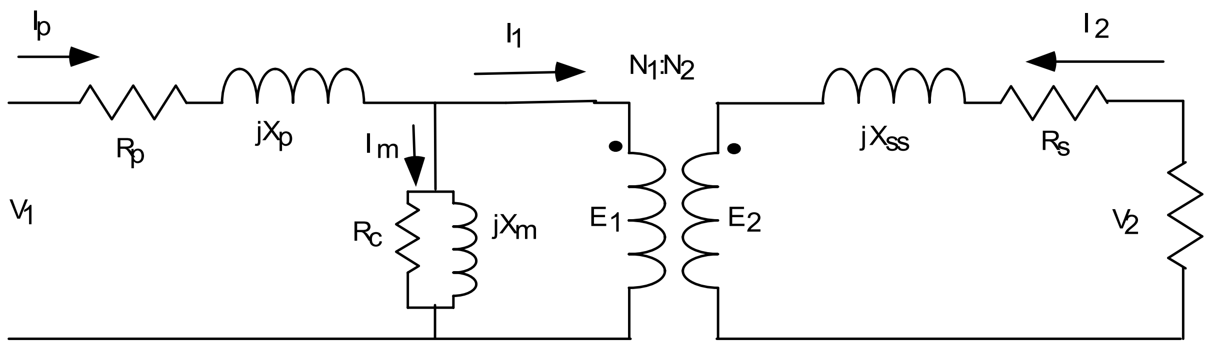
$$S2 = V2 \cdot \overline{I2} = \left[\left(\frac{N2}{N1}\right) \cdot V1\right] \cdot \overline{\left[\left(\frac{-N1}{N2}\right) I1\right]} = -V1 \cdot \overline{I1}$$

- Note that Ip is not equal to I1, since there are currents in the magnetizing and core loss branch The magnitude of this current is generally 2-5% of the rated load current.

- The rated load current is based on the Volt-Ampere rating of the transformer and the rated voltages (primary and secondary). These are generally given on the tranformer nameplate.

- Impedances can also be referred across the ideal transformer. This can make analysis easier, and also make it easier to determine parameters through testing.

For example, consider the secondary circuit with a load in place:



$$I2 = \frac{E2}{Rs + j \cdot Xss + Rload}$$

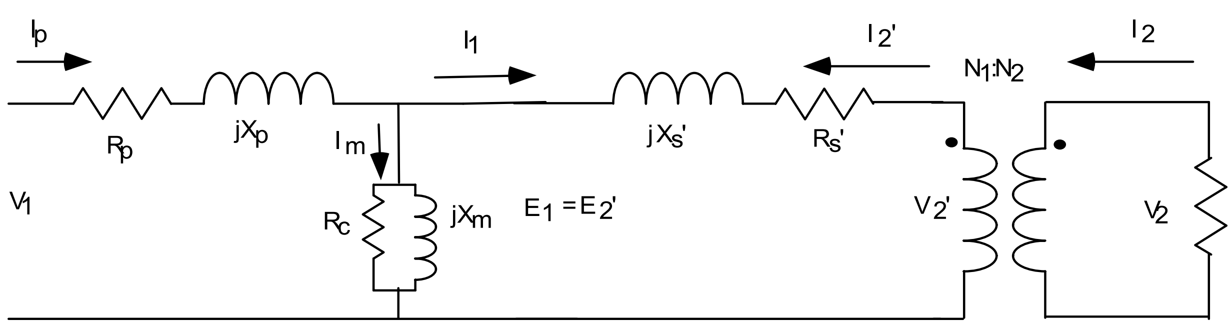
$$Zeq = \frac{E2}{I2}$$

We can express this in terms of the primary quantities

$$Zeq = \frac{\frac{N2}{N1} \cdot E1}{\frac{N1}{N2} \cdot I1} = \left(\frac{N2}{N1}\right)^2 \cdot \frac{E1}{I1}$$

The negative sign is left off for now.

We can use this to refer the secondary impedances across the windings as shown. We can refer only the winding resistance and leakage inductance, or we could also refer the load too. :



Where:

$$E2^1 = \left(\frac{N1}{N2}\right)\cdot E2 \qquad I2^1 = \left(\frac{N2}{N1}\right)\cdot I2$$

$$Xs^1 = \left(\frac{N2}{N1}\right)^2\cdot Xs \qquad Rs^1 = \left(\frac{N2}{N1}\right)^2\cdot Rs$$

Determination of Transformer Parameters from Test Data

The nameplate for a transformer doesn't always include the values for parameters in the equiavlen circuit. They can be determined through 2 tests.

1. Short Circuit Test:

- Short circuit the secondary.
- Apply a small voltage to the primary (it is easy to produce large currents)
- Measure: Vsc, Isc, and Psc
- The large impedance of the shunt branch (Xm and Rc) compared to Xs' and Rs' means that In is negligble, and this test determines the series branch parameters

$$Zseries = \frac{Vp}{Ip} \qquad Zseries = Z1 + Z2^1$$

$$Rseries = \frac{Psc}{Ip^2} \qquad Rseries = R1 + R2^1$$

$$Xseries = \sqrt{Zeries^2 - Rseries^2} \qquad Xseries = X1 + X2^1$$

- Note that this only give the combined Z1 + Z2', and not the relative magnitudes of each. This be determine approximate ratio R1 and R2 by doing dc ohmmeter tests on the windings. A common rule of thumb is to say that Z1 = Z2'
- Note also that secondary quantities are referred to the primary in these calculations.

Alternate method with same results:

$$\theta sc = acos\left(\frac{Psc}{Vsc\cdot Isc}\right) \qquad Rseries = Zseries\cdot \cos(\theta sc) \qquad Xeq = Zseries\cdot \sin(\theta sc)$$

2. Open circuit test:

- Open circuit the secondary.
- Apply rated voltage to the primary (nameplate voltage). It is important for this to be the nameplate voltage, since Xm is voltage dependent.
- Measure: Voc, Ioc, and Poc
- The large impedance of the shunt branch (Xm and Rc) compared to Xp and Rp means that th voltage across Rc and Xm is essentially the same as Voc.

$$Rc = \frac{Voc^2}{Poc}$$

$$Zoc = \frac{Voc}{Ioc} = \frac{Rc\cdot(j\cdot Xm)}{Rc + j\cdot Xm}$$ solve for Xm

Alternate Approach:

$$Yoc = \frac{Ioc}{Voc} \qquad \theta oc = acos\left(\frac{Poc}{Voc\cdot Ioc}\right)$$

$$Rc = \frac{1}{Yoc\cdot cos(\theta oc)}$$

$$Xm = \frac{1}{Yoc\cdot sin(\theta oc)}$$