ECE 320: Lecture 2
Notes

Basic AC Circuit Analysis (Continued)

$$v(t) = V_m \cos(\omega \cdot t + \phi) \quad \text{Volts}$$

(Note, units named for a person always start with capital letters).

$$\vec{V} = |V| \cdot e^{j \cdot \phi} \quad \text{Volts} \quad \text{where} \quad |V| = \frac{Vm}{\sqrt{2}} \quad \text{RMS Phasor}$$

- RMS phasor written using upper case, instaneous time quantity lower case.
- The frequency is assumed in this notation.
- The angle of voltage source is the angle relative to a reference point which is given the angle of 0 degrees (or radians).
- This is especially important when there are a large number of voltage sources in a system. In a large power system there might be hundreds of generators to represent.

Response of AC Circuit

- Now lets connect this voltage source to a series R-L circuit. R-L circuits are the most comm type. Any conductor or wire has inductance, enough to impact the circuit behavior.
- Also, every conductor has parasitic capacitance relative to some reference, how at 60Hz, thi capacitance has a small impact on circuit response unless the conductor is very long.
- We'll look at cases where discrete capacitors are added intentionally.
- The differential equation for this circuit is:

$$v(t) = R \cdot i(t) + L \cdot \frac{d}{dt} i(t)$$

- The resulting current when the source is switched into the circuit is:

$$i(t) = \left[\frac{-Vm}{\sqrt{R^2 + (\omega \cdot L)^2}} \right] \cdot \cos(\phi - \theta) \cdot e^{-\left(\frac{R}{L}\right) \cdot t} + \left[\frac{Vm}{\sqrt{R^2 + (\omega \cdot L)^2}} \right] \cdot \cos(\omega \cdot t + \phi - \theta) \quad \text{Amps}$$

- The first term is a decaying dc offset. The initial amplitude of this offset depends on t angle where the switch is closed. This is a transient term
- The second term is the steady-state component. This is our primary concern in this cla
- Not that we have an additional term in the angle, which describes the R-L circuit:

$$\theta = \operatorname{atan}\left(\frac{\omega \cdot L}{R} \right)$$

- The angle of the current is now: ϕ-θ
- If we only look at the sinusoidal steady-state component, we can compute a RMS magnitude

$$|I| = \left[\frac{\frac{Vm}{\sqrt{2}}}{\sqrt{R^2 + (\omega \cdot L)^2}} \right] \cdot \cos(\omega \cdot t + \phi - \theta) \quad \text{Amps}$$

$$\vec{I} = |I| \cdot e^{j \cdot (\phi - \theta)} \quad \text{Amps}$$

- We define Impedance as:

$$\vec{Z} = \sqrt{R^2 + (\omega \cdot L)^2} \cdot e^{j \cdot \theta} \quad \Omega$$

- This is more commonly written as:

$$\vec{Z} = |Z| \cdot e^{j \cdot \theta} \quad \Omega$$

- Note that frequency is still assumed here, but we no longer have a time varying function
- We can also describe an admittance as:

$$\vec{Y} = \frac{1}{\vec{Z}} = |Y| \cdot e^{-j \cdot \theta} \quad \text{Mhos or Siemens}$$

- Where

$$\vec{Z} = R + j \cdot X \qquad \text{Resistance and Reactance}$$

$$\vec{Y} = G + j \cdot B \qquad \text{Conductance and Susceptance}$$

- In general we can't say that G=1/R or B = 1/X (unless the X or R terms are zero respectively)
- For an pure inductor (R = 0). Note that pure inductors and pure resistors don't really exist, every circuit will have at least some R and some L, but for now neglect the R

$$X = \omega \cdot L \qquad B = \frac{1}{(\omega \cdot L)}$$

- For an pure capacitor (R = 0). Again, most capacitors have some small series resistance, but for now neglect the R (which is done in many cases) :

$$X = \frac{1}{\omega \cdot C} \qquad B = \omega \cdot C$$

- Now lets look at impedance angles. For a pure resistance (X = 0)

$$\theta = 0$$

- For a pure inductance:

$$\theta = 90deg$$

- For a pure capacitance:

$$\theta = -90deg$$

- In general, most cirucit are combination, so the angle of the impedance will somewhere in the range:

$$90deg \le \theta \le 90deg$$

- Starting from:

$$\vec{I} = \frac{\vec{V}}{\vec{Z}}$$

- We can define the **Power Factor Angle** as the angle between the voltage and the current.

$$pf\_angle = \phi_v - \phi_i$$

- For the simple configuration described above we would have:

$$pf\_angle = \theta$$

- We may also use this to define the effective angle computed from measured voltages and currents.
- Just as with the impedance angle we will have:

$$90deg \le pf\_angle \le 90deg$$

unless there is a polarity problem in the measurements.

- We can also define the **Power Factor** as

  pf **=** cos(pf_angle)

- Where (as a result of the limits on the pf_angle):

  $0 \le pf \le 1.0$

- Because of this, when one writes (or types) the value for a power factor, they also need to indicate whether the power factor is leading or lagging.
- A lagging power factor means "I lags V". This is common with an R-L circuit This means that the zero crossing for the current waveform appears after (to th right) the current zero for the voltage.
- A leading power factor means "I leads V". This is common with a series R-C circuit. This means that the zero crossing for the current waveform appears before (to the left) the current zero for the voltage.

Next time:

We will discuss methods for calculating power.