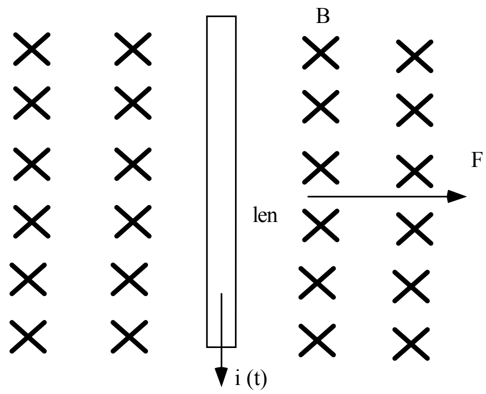
# ECE 320: Lecture 21

**Notes**

## *Electromechanical Energy Conversion*

- Last week we saw how current flowing through a magnetic field can produce force.




F = i (len x B)

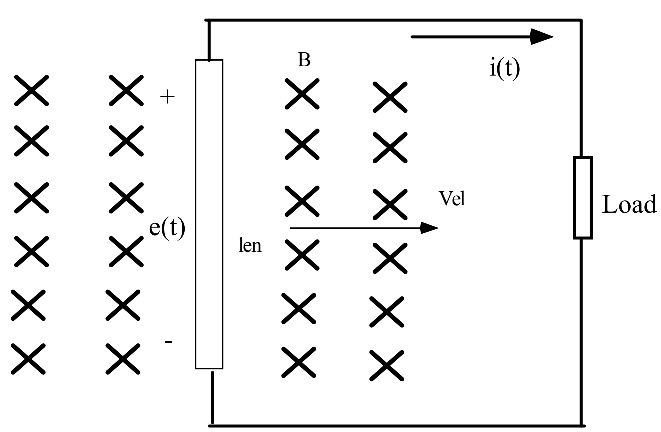
i = Magnitude of the current in the wire
len = length of the wire
B = magnetic flux density vector

This simplifies to:

$F = i \cdot len \cdot B \cdot \sin(\theta)$ where θ is the angle between wire (think of as a vector in the direction of current flow) and magnetic flux vector

Right hand rule determine direction of force.

Also, if a conductor moves through a magnetic field, a voltage will be induced on it in a manner to the voltage induced by a changing flux (in this case the flux is changing due to motion of the v

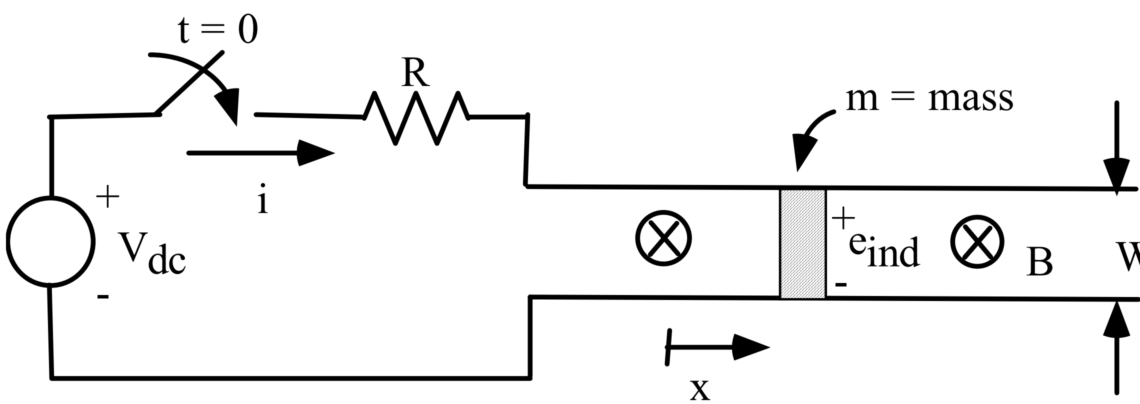


$e_{ind} = (vel \times B) \bullet len$

- Note that if the wire is connected to a load, then a current will flow.

- Also notice that the current will in the opposite direction from the current that was used to calculate force above, so the current will serve to slow down the piece of wire. However, if induced current produced a force to accelerate the wire, we would have a perpetual motion machine...

Example:




$$vel = \frac{d}{dt}x \qquad vel_0 = vel(0) = 0\frac{m}{s}$$

$$f = i \cdot B \cdot W \qquad (\theta=90)$$

also

$$f = m \cdot a = m \cdot \frac{d}{dt}vel = m \cdot \frac{d^2}{dt^2}x$$

therefore from these 2 equations: $i \cdot B \cdot W = m \cdot \frac{d}{dt}vel$ (1)

$$e_{ind} = vel \cdot B \cdot W = B \cdot W \cdot \left(\frac{d}{dt}x\right)$$

Initially: $eind = 0V$

$$V_{dc} = R \cdot i + e_{ind} \qquad \text{(KVL)}$$

So initially: $i(0) = \dfrac{V_{dc}}{R}$

and: $f(0) = \dfrac{V_{dc}}{R} \cdot B \cdot W$

Then we can rewrite the KVL equation as:

$$V_{dc} = R \cdot i + vel \cdot B \cdot W$$

or in terms of velocity:

$$vel = \frac{(Vdc - R \cdot i)}{W \cdot B}$$

Then we can substitute velocity into equation (1)

$$i \cdot B \cdot W = m \cdot \frac{d}{dt}\left[\frac{\left(V_{dc} - R \cdot i\right)}{W \cdot B}\right]$$

But $V_{dc} = \text{constant}$ so:

$$i \cdot B \cdot W = \frac{-m \cdot R}{W \cdot B} \cdot \frac{d}{dt}(i)$$

Rewrite as:

$$\frac{m \cdot R}{W^2 \cdot B^2} \cdot \frac{d}{dt}(i) + i = 0 \qquad \text{set constant:} \qquad \tau = \frac{m \cdot R}{W^2 \cdot B^2}$$

$$\tau \cdot \frac{d}{dt}(i) + i = 0$$

Solution: $i(t) = i_p + i_c$ but $i_p = 0$

$$i_c = K \cdot e^{\frac{-t}{\tau}} \qquad \text{where} \qquad K = i(0) = \frac{V_{dc}}{R}$$

Therefore:

$$i(t) = \frac{V_{dc}}{R} \cdot e^{\frac{-t}{\tau}}$$

Time constant depends on mass, resistance spacing between rails, and magnetic flux density

Note that this will decay to zero with time....

Similarly, force as a function of time is:

$$f(t) = B \cdot W \cdot \frac{V_{dc}}{R} \cdot e^{\frac{-t}{\tau}}$$

This will also decay to zero...

Induced voltage is:

$$e_{ind} = V_{dc} - R \cdot i = V_{dc} - V_{dc} \cdot e^{\frac{-t}{\tau}}$$

so this will eventually equal the dc voltage

Now look at the velocity:

$$vel = \frac{e_{ind}}{W \cdot B} = \frac{V_{dc} \cdot \left(1 - e^{\frac{-t}{\tau}}\right)}{W \cdot B}$$

In the absence of drag it will reach a terminal velocity

Now put in some numbers and plot the waveforms:

$Vdc := 100V$ $\qquad$ $m := 1kg$

$R := 5ohm$ $\qquad$ $B := 2T$

$W := 1ft$

$$\tau := \frac{m \cdot R}{W^2 \cdot B^2} \qquad \tau = 13.45\,s$$

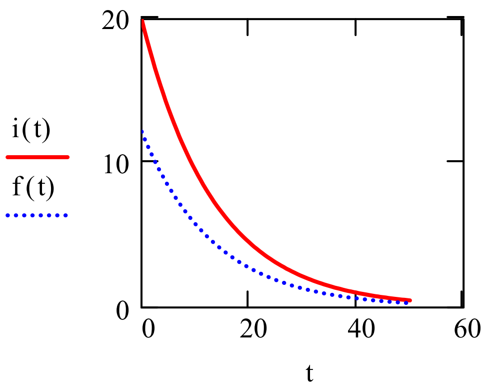
$t := 0sec, 0.5sec .. 50sec$

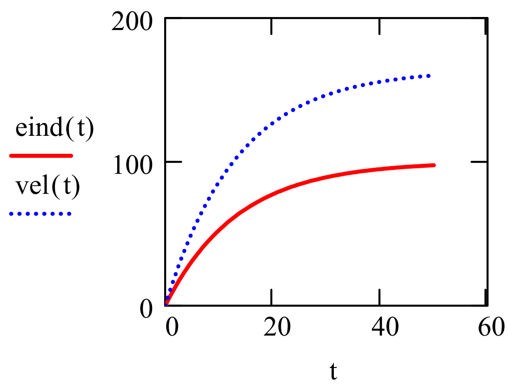
$$i(t) := \frac{Vdc}{R} \cdot e^{\frac{-t}{\tau}}$$

$$eind(t) := Vdc - Vdc \cdot e^{\frac{-t}{\tau}}$$

$$f(t) := B \cdot W \cdot \frac{Vdc}{R} \cdot e^{\frac{-t}{\tau}}$$

$$vel(t) := \frac{Vdc \cdot \left(1 - e^{\frac{-t}{\tau}}\right)}{W \cdot B}$$



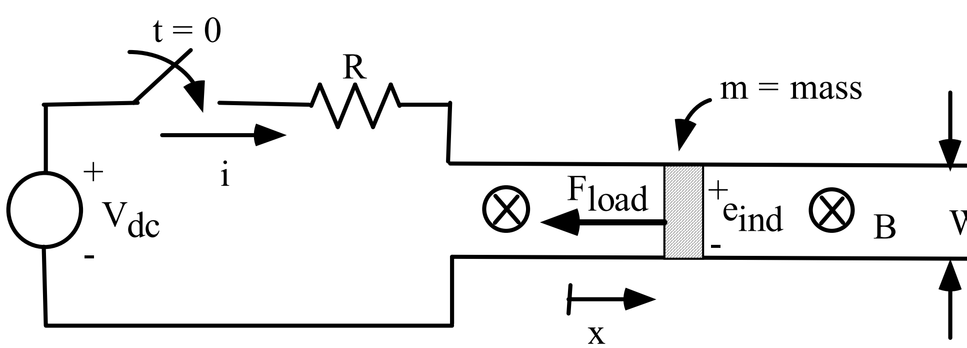




- Note that the above cases are all "unloaded." There is no external force trying to slow the bar down or trying to pull it along. That is why the current and force go to zero once the bar has reached a steady-state velocity.

- In reality, there will always be external forces. We can divide these into 2 additional cases.

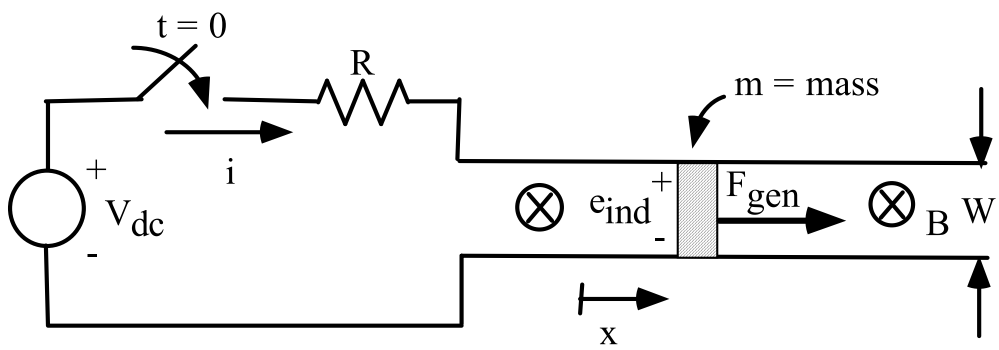
### *Motor Case*

Apply a force in the opposition to the direction that the far is moving, called $F_{load}$ on the figure.



- To supply this steady-state force, there needs to be a steady-state current in positive direction
- Eind < Vdc
- Steady-state velocity slower than no-load speed.

### *Generator Case*




- To compensator this steady-state force, there needs to be a steady-state current in negative direction.
- Eind > Vdc
- Steady-state velocity faster than no-load speed.