ECE 320: Lecture 23

Notes

Example from Last Time:

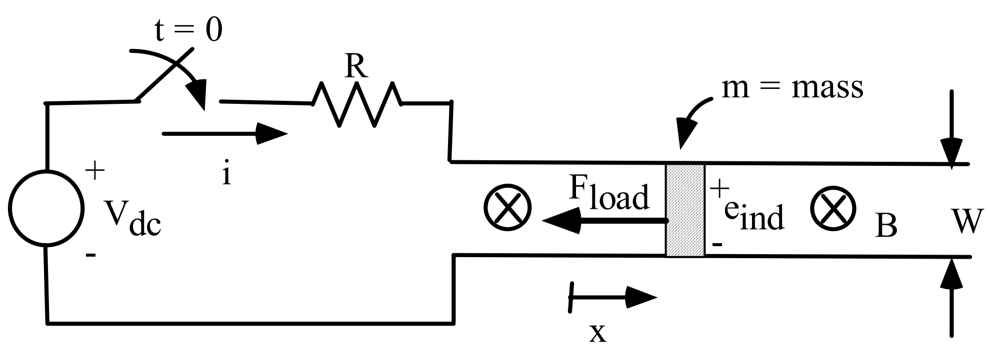
$Vdc := 100V$

$R := 5ohm$ $\quad B := 2T$

$W := 1ft$

Motor Case

Apply a force in the opposition to the direction that the far is moving, called F_{load} on the figure.



Let $\quad Fload := 3N \quad$ Note that $\quad Fmax := B \cdot W \cdot \frac{Vdc}{R} \quad Fmax = 12.19\,N$

Note that is Fmax < Fload it will never start...

Then in steady-state, we need to have force produced electric circuit offset this

$$Iss := \frac{Fload}{B \cdot W} \qquad Iss = 4.92\,A$$

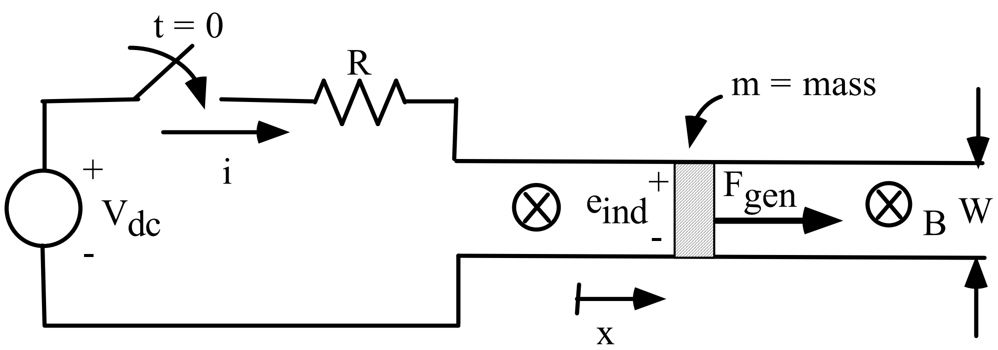
Then: $\quad Ess := Vdc - R \cdot Iss \qquad Ess = 75.39\,V$

$$vel\_ss := \frac{Ess}{W \cdot B} \qquad vel\_ss = 123.68\,\frac{m}{s}$$

No load velocity was:

$$\frac{Vdc}{W \cdot B} = 164.04\,\frac{m}{s}$$

Generator Case



Now suppose: $\quad Fgen := 3N$

$Iss2 := \frac{-Fgen}{B \cdot W}$ $\quad Iss2 = -4.92\,A$ $\quad$ reversed polarity

Then: $\quad Ess2 := Vdc - R \cdot Iss2$ $\quad Ess2 = 124.61\,V$ $\quad$ Larger than Vdc

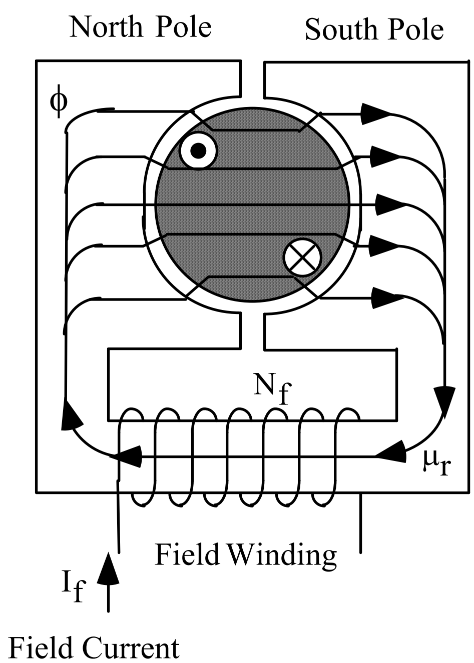
$vel\_ss2 := \frac{Ess2}{W \cdot B}$ $\quad vel\_ss2 = 204.41\,\frac{m}{s}$

No load velocity was: $\quad \frac{Vdc}{W \cdot B} = 164.04\,\frac{m}{s}$

Moving faster than no-load

Rotating Machine Case:

Rotating machines are far more common than linear machines. Now a loop of wire is placed in a magnetic field, and it is either rotated to produce a voltage or it has current passed through it to produce tangential force, which in turn causes it to rotate on its axis.



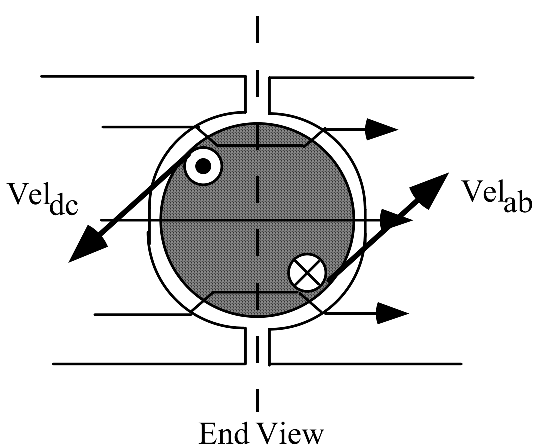
The flux generated by the field winding crosses the air gap perpendicular to the surface (pole face). It cross the air gap and enters the rotor (which is also a ferromagnetic material.

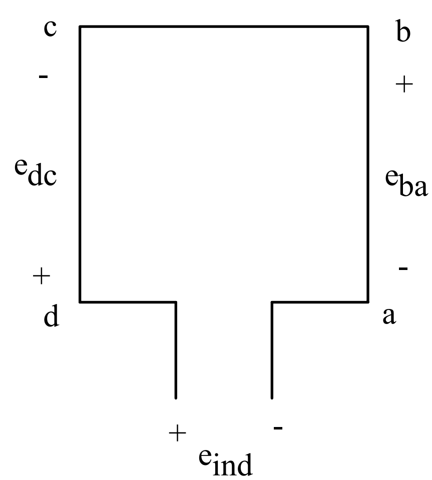
The flux spreads uniformly across the material. If one draws a vertical line where the air gap permits the flux to cross the air gap without entering the rotor, you notice that the flux crossing the air gap has opposite polarities on either side of this line.

We are embedding a loop of wire in the surface of the rotor. We will call this armature (or power) winding.

Case I: Loop rotating in the presence of the magnetic flux

Consider the case where the loop is rotated in a counterclockwise direction. The figure below shows an end view of the loop and a top view. Lable the corners of the loop as shown in the top view figure.





Assuming the flux divides equally, whenever the longitudinal segments (ab and cd) of the loop will see equal flux. The velocity of these segments (ab and cd) is defined as shown, and it tangential to the surface of the rotor at the location of the loop. From Faraday's Law

$$e_{ind} = \frac{d}{dN}\phi = (vel \times B)\cdot len$$

View this on a segment by segment basis:

segment a-b: The flux is perpendicular to the motion of the wire. By the right hand rule, the polarity of the induced voltage will be as shown:

$$e_{ba} = B\cdot len_{ab}\cdot Vel_{ab}$$

segment b-c: Now the field flux is parallel to the length vector, so the cross produce it 0

$$e_{cb} = 0$$

segment c-d: The flux is perpendicular to the motion of the wire. By the right hand rule, the polarity of the induced voltage will be as shown:

$$e_{dc} = B\cdot len_{dc}\cdot Vel_{dc}$$

segment d-a: Now the field flux is parallel to the length vector, so the cross produce it 0

$$e_{da} = B\cdot len_{ab}\cdot Vel_{ab}$$

Assume that the rotor has a uniform size. Then:

$$Vel_{ab} = Vel_{dc} = Vel \qquad \text{and} \qquad len_{ab} = len_{dc} = len$$

Therefore adding the voltages around the loop:

$$e_{ind} = 2\cdot B\cdot len\cdot Vel$$

This is the "armature voltage": $e_a = e_{ind}$

Now express this in terms of the rotational velocity:

$$vel = radius\cdot\omega$$

$$e_a = 2\cdot radius\cdot\omega\cdot B\cdot len$$

Note also that the surface area of the cylindrical rotor is:

$$\text{Area} = A = 2\cdot\pi\cdot\text{radius}\cdot\text{len}$$

The area under each pole is half of this

$$A_p = \frac{A}{2}$$

Then we can express induced voltage in terms of area (and area under each pole).

$$e_a = \frac{4}{\pi}\cdot A\cdot B\cdot\omega = \frac{2}{\pi}\cdot A_p\cdot B\cdot\omega$$

Now recall the definition of flux crossing a pole face:

$$\phi = B\cdot Ap$$

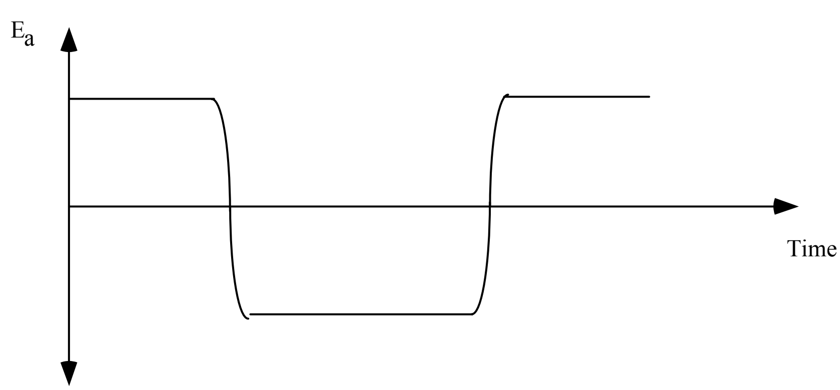
So finally, we can express the voltage in terms of the flux:

$$\boxed{e_a = \frac{2}{\pi}\cdot\phi\cdot\omega}$$

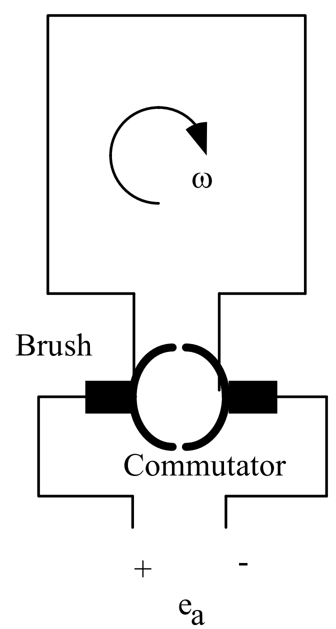
Notice that the voltage depends on the following:

1. Flux
2. Speed
3. Machine characteristic (the $2/\pi$ term will change somewhat when there are more turns in the armature winding).

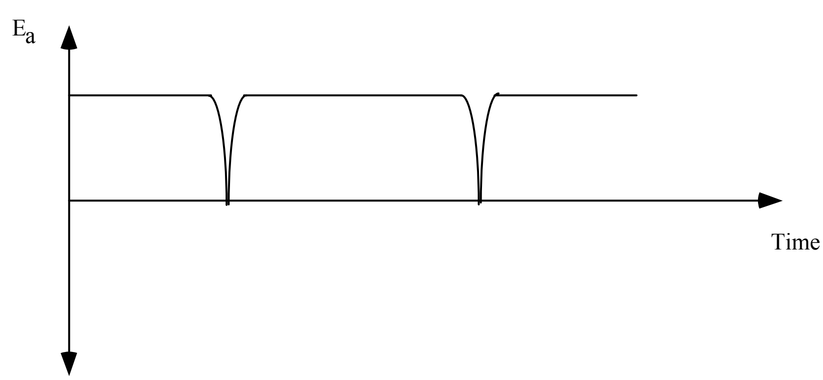
When the loop passes the vertical line dividing the pole faces the voltage will reverse polarity and the sign of the induced voltage will reverse, resulting in a voltage waveform as shown below.



- Since we are hoping for a dc machine, this is not desirable. We can fix this by adding a slotted ring to the terminals. The slots are cut so they align with where the voltage reverses polarity. This is called a commutator. The sections between the gaps are called commutator segments. See the figure below.
- In addition, we need a sliding contact since the rotor is turning to make sure that any wires we connect don't just twist. These are usually pieces of carbon called "brushes". In a machine these have springs to push them against the ring.

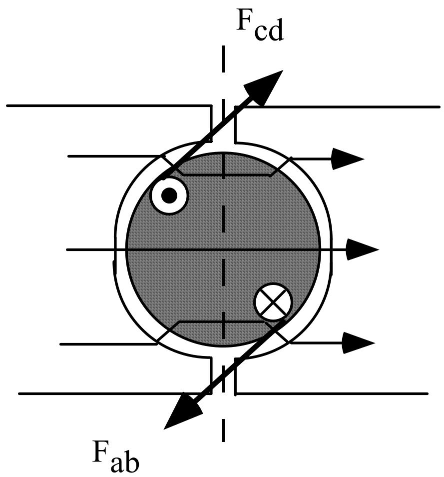


- Now connect the rings to a fixed electrical circuit and measure that voltage. Now the circuit connection to the commutator reverses at the same point where the voltage reverses, resulting in the voltage waveform below, which is a much better approximation of DC.



Case II: Run current through a stationary loop in the magnetic field.

Now the loop is stationary, but current is run into it in the polarity as shown below. Note that the commutator will allow the current polarity to reverse at the same point the voltage polarity would reverse.



Now the current interacting with the magnetic flux density (or flux) produces a force on the wire. Since the current in segments ab and cd are in opposite directions, two forces are produced that will try to rotate the wire loop.

$$F = i \cdot (\text{len} \times B)$$

Again, evaluate segment by segment:

Segment ab: B is perpendicular to length. Force direction by the right hand rule.

$$F_{ab} = i_{ab} \cdot \text{len}_{ab} \cdot B$$

Segment cd: B is perpendicular to length

$$F_{cd} = i_{cd} \cdot \text{len}_{cd} \cdot B$$

Segments bc and da: B is parallel to length vector so:

$$F_{bc} = F_{da} = 0$$

Since its a loop, the currents are equal (call this the armature current, i_a), and as before, the length of the parallel segments are equal.

$$F_{net} = 2 \cdot i \cdot \text{len} \cdot B$$

These forces will serve to force the rotor to turn in the counterclockwise direction. We can view the forces as producing torque.

$$\tau = F_{net} \cdot radius$$

Therefore we can express torque as:

$$\tau = 2radius \cdot i_a \cdot len \cdot B$$

As with the voltage, define the area of the rotor aligned with the pole face as:

$$A_p = \pi \cdot radius \cdot len$$

Then we can express torque in terms of area (and area under each pole).

$$\tau = \frac{2}{\pi} \cdot A_p \cdot i_a \cdot B$$

Now recall the definition of flux crossing a pole face:

$$\phi = B \cdot Ap$$

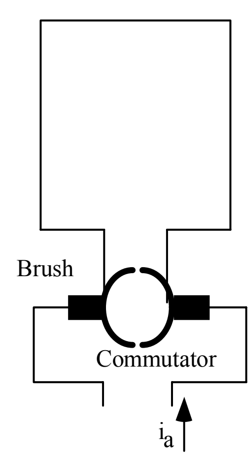
So finally, we can express the voltage in terms of the flux:

$$\boxed{\tau = \frac{2}{\pi} \cdot \phi \cdot i_a}$$

Notice that the torque depends on the following:

1. Flux
2. Current
3. Machine characteristic (the $2/\pi$ term will change somewhat when there are more turns in the armature winding).

- This will cause the machine to rotate.
- Note that if the current doesn't reverse every half rotation, the torque will reverse when the effective direction of the flux reverse when the rotor passes the center line, the torque will reverse and halt the rotation
- However the commutator will prevent this, since it will force the current to effectively reverse direction as it passes from one segment to the next.



Once the machine starts to spin, an induced voltage will be produced, just like the speed voltage seen in the linear machine. This will serve to limit the current, and if there is no load, the rotating loop (rotating machine) will reach a steady-state, no-load, speed.

One can relate the dc power in to the armature winding to the torque (and the mechanical power) by the following (the proof is assigned as a homework problem).

$$P_a = E_a \cdot I_a = \tau \cdot \omega = P_{mech}$$

Note: this holds when torque is in N-m and ω is in rad/sec