

## ECE 320: Lecture 24

### Notes

Lecture presented by George Hespelt. Notes below are based on his lecture notes

Historically, two of the big breakthroughs in the study of electricity were:

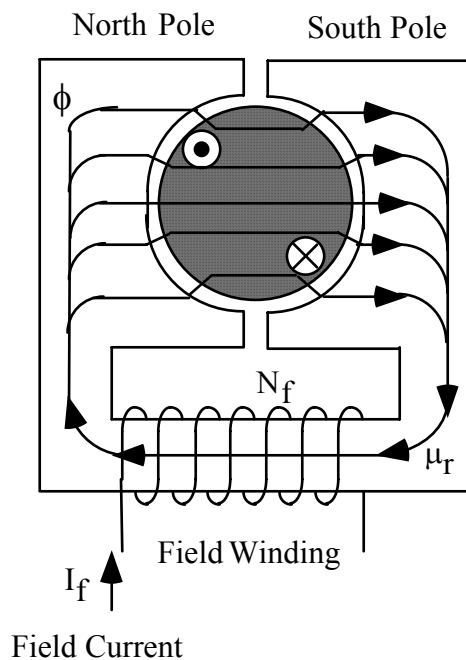
1. When a conductor moves in an area where there is magnetic flux, a voltage is induced on that conductor
2. A force is exerted on a current carrying conductor in an area where there is magnetic flux

This can be expressed by the following two equations:

$$e_{\text{ind}} = (\vec{\text{Vel}} \times \vec{B}) \cdot \vec{\text{Length}}$$

$$\vec{F} = i \cdot (\vec{\text{Length}} \times \vec{B})$$

These can be related to a simple machine:



Note:

1. Flux goes out of the North Pole and in to the South Pole.
2. The flux is perpendicular to the surface of the rotor and the pole faces (to follow the shortest path). The curved pole faces help create this.

Look more closely at the induced voltage equation, first the cross product:

$$\left( \vec{V_{el}} \times \vec{B} \right) = V_{el} \cdot B \cdot \sin(\theta) \cdot \vec{1}$$

- $V_{el}$  is the magnitude of the Velocity vector
- $B$  is the magnitude of the magnetic flux density vector
- $\theta$  is the angle between  $V_{el}$  and  $B$
- and:

$\vec{1}$  is a unit vector perpendicular to the plane of  $\vec{V_{el}}$  and  $\vec{B}$

- Use the right hand rule to get the direction of the unit vector. This is the direction of the resulting cross product
- In this case,  $\theta$  is 90 degrees, so the magnitude of the cross product is:  $V_{el} \cdot B$
- The direction of the unit vector is determined by pointing the fingers of your right hand in the direction of  $V_{el}$ , then curl them in the direction of  $B$ . The direction your thumb points is the direction of the unit vector.
- Then take the dot product of this result with the length vector (define a direction to give yourself a reference frame).
- So, for example:

$$\vec{V_1} \cdot \vec{V_2} = V_1 \cdot V_2 \cdot \cos(\phi) \quad \text{where } \phi \text{ is the angle between } V_1 \text{ and } V_2.$$

- In this case,  $\phi$  will be parallel with the unit vector from the cross product (i.e. the cross product will point down the conductor on the rotor, which is the direction of the length vector).
- So we get:

$$e_{ind} = \left( \vec{V_{el}} \times \vec{B} \right) \cdot \vec{Length} = V_{el} \cdot B \cdot Length$$

- Since there are 2 conductors, this becomes

$$e_{ind} = 2V_{el} \cdot B \cdot Length$$

- Note that the voltage increases for the following three conditions:
  1. Move the conductor faster
  2. Increase magnetic flux density
  3. Increase the length of the conductor interacting with the flux density
- Normally only the first 2 are options once the machine has been built.

Now look at the force equation.

$$F = i \cdot (\overrightarrow{\text{Length}} \times \vec{B})$$

- In the machine diagram above, Length is perpendicular to B
- So the magnitude of the cross product will be Length\*B
- In this equation, current is a scalar, so the direction of the force is the direction of the cross product, which is again from the right hand rule, turning from Length to B
- So we have:

$$F = i \cdot \text{Length} \cdot B \quad \text{tangential to the rotor surface, in counter clockwise direction in the case.}$$

- There are 2 conductors carrying current in opposite directions. The net force is:

$$F_{\text{net}} = 2 \cdot i \cdot \text{Length} \cdot B$$

- Since the conductors are on a cylinder, we can also compute the torque produced.

$$\tau = 2 \cdot i \cdot \text{Length} \cdot B \cdot r \quad \text{where } r \text{ is the radius from the center of cylinder to the conductor}$$

- If the wire is connected to an electric circuit, both equations apply at the same time, and they interact.
- One effect of this is that if the field is suddenly reduced, the machine will turn faster.

### ***Example***

An example was also presented. This is available on the web page as a separate link.