ECE 320: Lecture 4

Notes

Single Phase Power Calculations

Single Phase Instantaneous Power:

$$v(t) = Vm \cdot \cos\left(\omega \cdot t + \phi_v\right)$$

$$i(t) = Im \cdot \cos\left(\omega \cdot t + \phi_i\right)$$

$$p(t) = v(t) \cdot i(t)$$

$$p(t) = Vm \cdot Im \cdot \left(\cos\left(\omega t + \phi_v\right) \cdot \cos\left(\omega \cdot t + \phi_i\right)\right)$$

We can use the following Trig

$$2 \cdot \cos(A) \cdot \cos(B) = \cos(A - B) + \cos(A + B)$$

Therefore we see:

$$p(t) = \frac{Vm \cdot Im}{2} \cdot \left(\cos\left(\phi_v - \phi_i\right) + \cos\left(2\omega t + \phi_v + \phi i\ \right)\right.$$

There is a constant term, that is commonly called the Average or Real Power

$$Pave = \frac{Vm \cdot Im}{2} \cdot \left(\cos\left(\phi_v - \phi_i\right)\right)$$

However, we will be more interested in performing these calculations using Phasors

$$\vec{V} = |V| \cdot e^{j \cdot \phi_v} \quad \text{Volts} \qquad \vec{I} = |I| \cdot e^{j \cdot \phi_i} \quad \text{Amperes}$$

Complex Power

$$\vec{S} = \vec{V} \cdot \overrightarrow{\left(\bar{I}\right)} \quad \text{Volt-Amperes}$$

or

$$\vec{S} = |V| \cdot |I| \cdot e^{j \cdot \left(\phi_v - \phi_i\right)} = |V| \cdot |I| \cdot e^{j \cdot \theta}$$

Angle of S is the power factor angle

Apparent Power

$$|S| = |V|\ |I|$$

$$\vec{S} = P + j \cdot Q$$

$$P = |V|\ |I| \cos(\theta)$$

$$Q = |V|\ |I| \sin(\theta)$$

P = Real Power = Average Power   Watts (or kW, MW, etc)

Q = Reactive Power   VAr (Volt-Amperes Reactive)

- Mathematically VA, Watt and VAr are all the same
- Note that all of these S, P, and Q are all defined from RMS averaged quantities, so they have no meaning as instantaneous quantities
- There are many ways to view complex power, and the relationship between real and reactive beyond the mathematical view above.

1. Real power accomplishes work--turns motors, produces heat (either usefully or a losses.

2. Reactive power--does represent the impact of physical currents and voltages and the conductors. It may be imaginary mathematically, but it does describe something that can be measured.

Reactive power is needed to support the transfer of real power. It repesents energy needed to support magnetic fields that in turn allow current to flow through a wire magnetic fields that help the shaft of a motor to turn.

It also supports the electric fields that allow a voltage different different to exist between 2 conductors (or a conductor and ground)

Most loads require real power and lagging (inductive) reactive power to allow them to operate. The real power is produced by generators, that are usually a long distanc away. The generators also produce the reactive power for the loads (and the transmission lines and transformers). As current goes through wires we get losses from current through wire resistance (I^2R losses). It is still makes economic sense in many cases to transfer real power over long distances. Transformers help this

Lagging reactive power demand can be supplied locally for less expense. Most common method is using capacitors. By "producing" reactive power locally the ove current from the source to the load is decreased, decreasing losses, and improving the effective power factor

- Remember KCL and KVL apply to ac circuits.
- One effect is that complex power is conserved. So the total real power generated is equal to real power to loads and losses. Reactive power must also sum to zero.

Examples

Define Units $VA := W$ $VAr := W$

$kVA := kW$ $kVAr := kW$

1) $mV1 := 120V$ $\phi v1 := 10deg$ $V1 := mV1 \cdot e^{j \cdot \phi v1}$

$mI1 := 20A$ $\phi i1 := -30deg$ $I1 := mI1 \cdot e^{j \cdot \phi i1}$

$S1 := V1 \cdot \overline{I1}$ $S1 = 1838.51 + 1542.69i\,VA$

$|S1| = 2400\,VA$ $\theta 1 := arg(S1)$ $\theta 1 = 40\,deg$

$P := Re(S1)$ $P = 1838.51\,W$

$Q := Im(S1)$ $Q = 1542.69\,VAr$

$pf := cos(\theta 1)$

$pf = 0.77$ lagging

2) $mV2 := 480V$ $\phi v2 := 0$ $V2 := mV2 \cdot e^{j \cdot \phi v2}$

$mI2 := 200A$ $\phi i2 := 90deg$ $I2 := mI2 \cdot e^{j \cdot \phi i2}$

$S2 := V2 \cdot \overline{I2}$ $S2 = -96i\,kVA$

$|S2| = 96\,kVA$ $\theta 2 := arg(S2)$ $\theta 2 = -90\,deg$

$P2 := Re(S2)$ $P2 = -3.68 \times 10^{-11}\,W$

$Q2 := Im(S2)$ $Q2 = -96\,kVAr$

$pf\_2 := cos(\theta 2)$

$pf\_2 = 0$ leading (This is a capacitor)

2a) $mV2 := 480V$ $\phi v2 := 0$ $V2 := mV2 \cdot e^{j \cdot \phi v2}$

$mI2 := 200A$ $\phi i2 := -90deg$ $I2 := mI2 \cdot e^{j \cdot \phi i2}$

$S2 := V2 \cdot \overline{I2}$ $S2 = 96i\,kVA$

$|S2| = 96\,kVA$ $\quad \theta 2 := \arg(S2)$ $\quad \theta 2 = 90\,deg$

$P2 := Re(S2)$ $\quad P2 = -3.68 \times 10^{-11}\,W$

$Q2 := Im(S2)$ $\quad Q2 = 96\,kVAr$

$pf\_2 := \cos(\theta 2)$

$pf\_2 = 0$ lagging (This is an inductor)

3) $mV3 := 200V$ $\quad \phi v3 := 10deg$ $\quad V3 := mV3 \cdot e^{j \cdot \phi v3}$

$mI3 := 10A$ $\quad \phi i3 := 10deg$ $\quad I3 := mI3 \cdot e^{j \cdot \phi i3}$

$S3 := V3 \cdot \overline{I3}$ $\quad S3 = 2\,kVA$

$|S3| = 2\,kVA$ $\quad \theta 3 := \arg(S3)$ $\quad \theta 3 = 0\,deg$

$P3 := Re(S3)$ $\quad P3 = 2000\,W$

$Q3 := Im(S3)$ $\quad Q3 = 0\,kVAr$

$pf\_3 := \cos(\theta 3)$

$pf\_3 = 1$ Unity Power Factor