## ECE 320: Lecture 9 <br> Notes

The lecture started with the lab safety quiz. If you missed class, you need to contact your lab TA to take the quiz.

In response to questions about harmonic analysis:

- P and Q are fundamental component quantities in most cases. As discussed earlier, P can be transferred if there voltage and current components at the same frequency that aren't 90 degi out of phase.
- Therefore, if one knows $\mathrm{P}, \mathrm{V}$ and displacement power factor, then you can find the fundame component of the current

$$
\mathrm{P}=\left|\mathrm{V}_{1}\right| \cdot\left|\mathrm{I}_{1}\right| \cdot \cos \left(\phi_{1}\right)
$$

Where $\cos \left(\phi_{1}\right)$ is the displacement factor (or displacement power factor), called "df". $\phi_{1}$ is the angle between the fundamental components of the voltage and current.

$$
\mathrm{Q}=\left|\mathrm{V}_{1}\right| \cdot\left|\mathrm{I}_{1}\right| \cdot \sin \left(\phi_{1}\right)
$$

If there are only fundamental component voltages and currents then:

$$
|\mathrm{S}|=\sqrt{\mathrm{P}^{2}+\mathrm{Q}^{2}}
$$

However, if there are harmonics, then this doesn't hold.

$$
|\mathrm{S}|=\left|\mathrm{V}_{\mathrm{rms}}\right| \cdot\left|\mathrm{I}_{\mathrm{rms}}\right|
$$

The added complex power is at the harmonic frequencies, and is similar to reactive power, since it supports the flow of the harmonic currents. A load that draws harmonic currents, cat modeled as a fundamental component $\mathrm{P}, \mathrm{Q}$ sink and a current source injecting the harmonic components. This current source must see a complete circuit through the power system.

A modified definition of $|\mathrm{S}|$ is:

$$
|\mathrm{S}|=\sqrt{\mathrm{P}^{2}+\mathrm{Q}^{2}+\mathrm{H}^{2}} \quad \begin{aligned}
& \mathrm{H} \text { is harmonic power. Some people also use } \mathrm{D} \\
& \text { instead of } \mathrm{H} .
\end{aligned}
$$

This also modifies the power factor

$$
\mathrm{pf}=\frac{\mathrm{P}}{|\mathrm{~S}|}=\frac{\left(\left|\mathrm{V}_{1}\right| \cdot\left|\mathrm{I}_{1}\right| \cdot \cos \left(\phi_{1}\right)\right)}{\left(\left|\mathrm{V}_{\mathrm{rms}}\right| \cdot\left|\mathrm{I}_{\mathrm{rms}}\right|\right)}
$$

A shortcut equation for the RMS component of current, voltage, or flux waveform when the magnitude of the harmonics are known is (equation 3-28 in Mohan):

$$
\mathrm{F}_{\mathrm{rms}}=\sqrt{\left(\mathrm{F}_{0}\right)^{2}+(1)^{2}+\left(\mathrm{F}_{2}\right)^{2}+\left(\mathrm{F}_{3}\right)^{2}+. .+\left(\mathrm{F}_{\mathrm{n}}\right)^{2}}
$$

## Non-ideal Transformers

Last time discussed ideal transformers.
One of the non-ideal traits is the resistance of the windings. $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ (for a 2 winding transformer). Each winding will have resistance that can be measured, or even computed, given sufficient information.

$$
\mathrm{R}_{\mathrm{dc}}=\frac{\rho \cdot \mathrm{L}}{\mathrm{~A}} \quad \text { Where: } \quad \begin{aligned}
& \rho \text { is the resistivity of the wire. For copper wire } \\
& \rho=1.77 \times 10^{-8} \Omega-\mathrm{m}
\end{aligned}
$$

L is the length of the conductor
A is the cross sectional area of the conductor
Each winding will have a different length (different number of turn and a different cross-sectional area (different current rating for the wire).

However, only the transformer designer will have access to this information.

In addition, this only calculates the DC resistance. In an actual conductor, the resistance wi be higher due to a phenomenon called skin effect. When a dc current flows through a conductor, the current distributes evenly throughout the conductor. When an AC current flows, the current does not flow in the center of the conductor. At 60 Hz , there is only a sm area that carries no current. As the frequency increases, this area increases, until at very hi $\S$ frequencies the current only flows in the outer part of the conductor.

kHz Range

