1 Calculating Instantaneous Power

$$p(t) = v(t)i(t)$$

Where

$$v(t) = V_{dc} + \sum_{m=1}^{\infty} V_{pk,m}\cos(m\omega t + \alpha_m)$$

and

$$i(t) = I_{dc} + \sum_{n=1}^{\infty} i_{pk,n}\cos(n\omega t + \beta_n)$$

The average power can then be calculated from:

$$P_{ave} = \frac{1}{T}\int_t^{t+T} p(t)dt$$

$$\begin{aligned} = & \frac{1}{T}\int_t^{t+T} V_{dc}I_{dc}dt + \\ & \frac{1}{T}\int_t^{t+T} V_{dc}\sum_{n=1}^{\infty} I_{pk,n}\cos(n\omega t + \beta_n)dt + \\ & \frac{1}{T}\int_t^{t+T} I_{dc}\sum_{m=1}^{\infty} V_{pk,m}\cos(m\omega t + \alpha_m)dt + \\ & \frac{1}{T}\int_t^{t+T} \left(\sum_{m=1}^{\infty} V_{pk,m}\cos(m\omega t + \alpha_m)\right)\left(\sum_{n=1}^{\infty} I_{pk,n}\cos(n\omega t + \beta_n)\right)dt \end{aligned}$$

$$\begin{aligned} = & V_{dc}I_{dc} + \\ & \frac{V_{dc}}{T}\sum_{n=1}^{\infty}\int_t^{t+T} I_{pk,n}\cos(n\omega t + \alpha_n)dt + \\ & \frac{I_{dc}}{T}\sum_{m=1}^{\infty}\int_t^{t+T} V_{pk,m}\cos(m\omega t + \beta_m)dt + \\ & \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{T}\int_t^{t+T} V_{pk,m}\cos(m\omega t + \alpha_m)I_{pk,n}\cos(n\omega t + \beta_n)dt \end{aligned}$$

We can use the results of section 1.1 on the next page to simplify the equation for P_{ave} as follows:

$$P_{ave} = V_{dc}I_{dc} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{T}\int_{t}^{t+T} V_{pk,m}\cos(m\omega t + \alpha_m)I_{pk,n}\cos(n\omega t + \beta_n)dt$$

Then we can simplify the product term by using the results of section 1.2 on page 4:

$$P_{ave} = V_{dc}I_{dc} + \sum_{m=1}^{\infty}\frac{1}{T}\int_{t}^{t+T} V_{pk,m}\cos(m\omega t + \alpha_m)I_{pk,m}\cos(m\omega t + \beta_m)dt$$

So :

$$P_{ave} = V_{dc}I_{dc} + \sum_{m=1}^{\infty} V_{rms,m}I_{rms,m}\cos(\alpha_m - \beta_m)$$

1.1 Integration of Sinusoid Over One Period

$$\text{Assume } k \neq 0$$

$$\text{and } T = \frac{2\pi}{\omega}$$

$$\begin{aligned}
\int_t^{t+T} \cos(k\omega t + \psi)dt &= \frac{\sin(k\omega t + \psi)}{k\omega} \Big|_t^{t+T} \\
&= \frac{\sin(k\omega(t+T) + \psi) - \sin(k\omega t + \psi)}{k\omega} \\
&= \frac{\sin(k\omega t + k\omega T + \psi) - \sin(k\omega t + \psi)}{k\omega} \\
&= \frac{\sin(k\omega t + k2\pi + \psi) - \sin(k\omega t + \psi)}{k\omega} \\
&= \frac{\sin(k\omega t + \psi) - \sin(k\omega t + \psi)}{k\omega} \\
&= \mathbf{0}
\end{aligned}$$

$$\int_t^{t+T} \cos(k\omega t + \psi) = \mathbf{0}$$

1.2 Integrals of Products of Sinusoids

$$
\begin{aligned}
P'_{ave} &= \frac{1}{T}\int_t^{t+T} V_{pk,m}\cos(m\omega t+\alpha_m) I_{pk,n}\cos(n\omega t+\beta_n)dt \\
&= \frac{V_{pk,m}I_{pk,n}}{2T}\int_t^{t+T}\cos\left[(m-n)\omega t+\alpha_m-\beta_n\right]dt + \\
&\quad \frac{V_{pk,m}I_{pk,n}}{2T}\int_t^{t+T}\cos\left[(m+n)\omega t+\alpha_m+\beta_n\right]dt \\
&= \frac{V_{rms,m}I_{rms,n}}{T}\int_t^{t+T}\cos\left[(m-n)\omega t+\alpha_n-\beta_n\right]dt + \\
&\quad \frac{V_{rms,m}I_{rms,n}}{T}\int_t^{t+T}\cos\left[(m+n)\omega t+\alpha_m+\beta_n\right]dt
\end{aligned}
$$

Case I. $m \neq n$:
From section 1.1 $P'_{ave} = 0.0$

Case II. $m = n$:

$$
\begin{aligned}
P'_{ave} &= \frac{V_{rms,m}I_{rms,m}}{T}\int_t^{t+T}\cos(\alpha_m-\beta_m)+\cos(2m\omega t+\alpha_m+\beta_m)dt \\
&= \frac{V_{rms,m}I_{rms,m}}{T}\cos(\alpha_m-\beta_m)\int_t^{t+T}dt \\
&= V_{rms,m}I_{rms,m}\cos(\alpha_m-\beta_m)
\end{aligned}
$$