Sample Exam Solution

1. (20 pts) Either circle the correct answer or write in a short answer for each of the following.

A (T or F) the real power loss in an ideal capacitor is 0.

True. P = 0, but Q is not 0.

B What does total harmonic current distortion represent? What is considered to be a good numl

Total harmonic distortion, or THD, is a measure of how much harmonic distortion is present, re
to the magnitude of the fundamental component of the current.

$$THD = 100 \cdot \sqrt{\frac{(I_2)^2 + (I_3)^2 + ...}{I_1^{\,2}}}$$

The lower the value of THD the better, ideally it is zero. Harmonic standards such as IEEE 519
upper limits for THD.

C You are given an iron core with a N-turn winding excited by an AC voltage source with a constant voltage magnitude. The peak flux density will (increase or decrease) when the frequen
the voltage is increased.

It will decrease.

$$e(t) = E_m \cdot \cos(\omega \cdot t)$$

$$\phi(t) = \frac{1}{N} \cdot \int E_m \cdot \cos(\omega \cdot t)\, dt = \frac{-1}{N \cdot \omega} \cdot E \cdot \sin(\omega \cdot t)$$

Flux is divided by frequency, so a given voltage corresponds to a smaller flux as frequency increases. Then:

$$B = \frac{\phi(t)}{A}$$

So if flux decreases with frequency, then flux density decreases.

D Suppose a coil of wire is wrapped around a magnetic core. Will the inductance increase or decrease as the mean path length increases.

$$L = \frac{N^2 \cdot \mu_r \cdot \mu_0 \cdot A}{\text{length}}$$

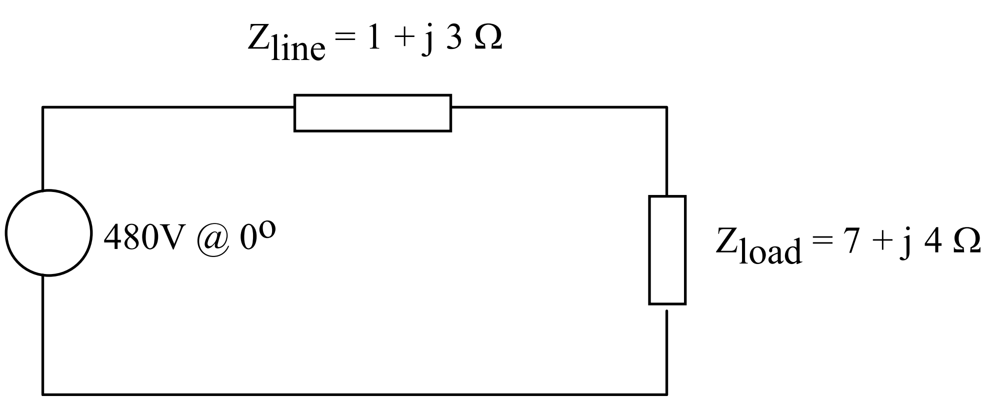
So if the length increases, L will decrease

E. Why does the open circuit test only show excitation losses (losses within the core) and not winding (I^2 R) losses?

The rated terminal voltage is applied to the transformer, but the load current is zero. Application of rated terminal voltage results in normal excitation and therefore normal excitation losses. However, with zero load current, the current through the windings is very small (only the small excitation current), so the i2R losses are very small relative to the excitation losses.

2. (30 pts) Do the following given the 60 Hz circuit shown below:

A Calculate instantaneous power delivered by the source



$VS := 480V$ $\quad$ $Zline := 1ohm + j \cdot 3ohm$ $\quad$ $Zload := 7ohm + j \cdot 4ohm$

$$IS := \frac{VS}{Zline + Zload}$$ $\quad$ $IS = 33.982 - 29.735i\,A$

$|IS| = 45.155\,A$ $\quad$ $arg(IS) = -41.186\,deg$

$v(t) := \sqrt{2}\cdot VS\cdot\cos(2\cdot\pi\cdot 60Hz\cdot t)$ $\qquad \sqrt{2}\cdot VS = 678.823\,V$

$i(t) := \sqrt{2}\cdot IS\cdot\cos(2\cdot\pi\cdot 60Hz\cdot t - 41.186deg)$ $\qquad \sqrt{2}\cdot|IS| = 63.858\,A$

$2\cdot|VS|\cdot|IS| = 43.348\,kW$

$p(t) = v(t)\cdot i(t) = \frac{43.48}{2}kW\cdot[\cos[0 - (-41.186deg)] - \cos(4\cdot\pi\cdot 60Hz\cdot t - 41.186deg)]$

$PAVE := \frac{2\,||VS||\,|IS|}{2}\cos(41.186deg)$ $\qquad PAVE = 16.311\,kW$

As a check, solve with phasors

$Re(VS\cdot\overline{IS}) = 16.312\,kW$

B Compute the power factor of the load

$PFangle := \arg(Zload)$ $\qquad PFangle = 29.745\,deg$

$PF := \cos(PFangle)$ $\qquad PF = 0.868$ lagging

C Determine the per phase capacitance needed to make the effective power factor of the load capacitor bank unity.

First find Vload

$Vload := VS - IS\cdot Zline$ $\qquad Vload = 356.814 - 72.212i\,V$

$|Vload| = 364.048\,V$ $\qquad \arg(Vload) = -11.441\,deg$

$kVA := kW$ $\qquad kVAR := kW$

We want Qcap equal and opposite to the imaginary part of Qload

$Sload := (|IS|)^2 Zload$ $\qquad Sload = 14.273 + 8.156i\,kVA$

$Qcap := -Im(Sload)$ $\quad Qcap = -8.156\,kVAR$

$Xcap := \frac{(|Vload|)^2}{|Qcap|}$ $\quad Xcap = 16.25\,\Omega$

$Cap := \frac{1}{2\cdot\pi\cdot 60Hz\cdot Xcap}$ $\quad Cap = 163.236\,\mu F$

$Zeq := \left(\frac{1}{Zload} + \frac{1}{-j\cdot Xcap}\right)^{-1}$ $\quad Zeq = 9.286\,\Omega$ purely real

3. (30pts) A 1300:460V, 50 kVA single phase transformer supplies a rated kVA load at 0.8 PF lagging at 440V. The impedances referred to the high voltage side are:

$R1 := 0.5ohm$ $\quad R2p := 0.5ohm$ $\quad N1 := 1300$

$X1 := 2.0ohm$ $\quad X2p := 2.0ohm$ $\quad N2 := 460$

$Xm := 400ohm$ $\quad Rc := 1200ohm$

A. Determine the transformer voltage regulation for this load.

$Vload3 := 440V$ set angle at 0

$MagIload := \frac{50kVA}{Vload3}$ $\quad MagIload = 113.636\,A$

$angIload := -acos(0.8)$ $\quad angIload = -36.87\,deg$

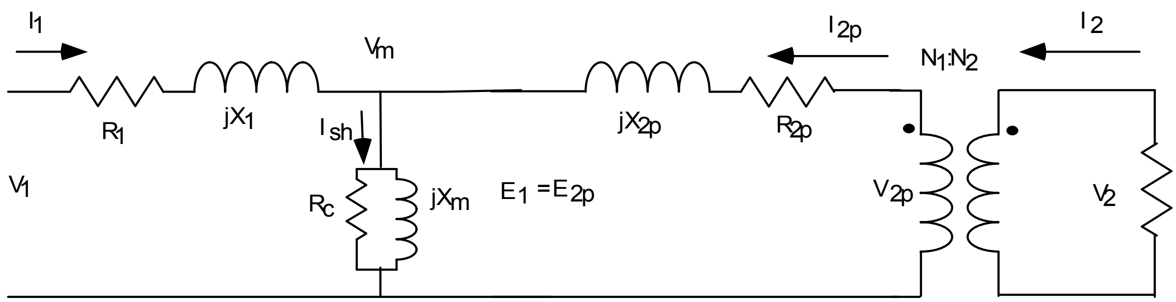
$Iload := MagIload\cdot e^{j\cdot angIload}$

$I2 := -Iload$

Refer across ideal transformer

$V2P := \frac{N1}{N2}\cdot Vload3$ $\quad V2P = 1.243\,kV$

$I2P := \frac{N2}{N1}\cdot I2$ $\quad I2P = -32.168 + 24.126i\,A$ $\quad |I2P| = 40.21\,A$



$\text{Vm} := \text{V2P} - \text{I2P}\cdot(\text{R2p} + j\cdot\text{X2p})$ $\quad \text{Vm} = 1.308\times10^3 + 52.273i\ \text{V}$

$|\text{Vm}| = 1.309\ \text{kV} \quad \arg(\text{Vm}) = 2.289\ \text{deg}$

$\text{Ish} := \dfrac{\text{Vm}}{\text{Rc}} + \dfrac{\text{Vm}}{j\cdot\text{Xm}}$ $\quad \text{Ish} = 1.221 - 3.226i\ \text{A}$

Node equation for I1

$\text{I1} := -\text{I2P} + \text{Ish}$ $\quad \text{I1} = 33.388 - 27.352i\ \text{A}$

$|\text{I1}| = 43.161\ \text{A} \quad \arg(\text{I1}) = -39.324\ \text{deg}$

$\text{V1} := \text{Vm} + \text{I1}\cdot(\text{R1} + j\cdot\text{X1})$ $\quad \text{V1} = 1.379\times10^3 + 105.374i\ \text{V}$

$|\text{V1}| = 1.383\ \text{kV} \quad \arg(\text{V1}) = 4.369\ \text{deg}$

$\text{VR} := \dfrac{|\text{V1}| - |\text{V2P}|}{|\text{V2P}|}$ $\quad \text{VR} = 11.239\,\%$

B. Determine the transformer efficiency

$\text{Pin} := \text{Re}\left(\text{V1}\cdot\overline{\text{I1}}\right)$ $\quad \text{Pin} = 43.167\ \text{kW}$

$\text{Pout} := -\text{Re}\left(\text{Vload3}\cdot\overline{\text{I2}}\right)$ $\quad \text{Pout} = 40\ \text{kW}$

$\eta := \dfrac{\text{Pout}}{\text{Pin}}$ $\quad \eta = 92.662\,\%$

Losses

$$\text{Ploss} := (|I1|)^2 \cdot R1 + (|I2P|)^2 \cdot R2p + \frac{(|Vm|)^2}{Rc}$$

$$\text{Ploss} = 3.167\ \text{kW}$$

4. (25 pts) The current waveforms drawn by a power supply fed by a sinusoidal voltage sourc have the following harmonic components (in RMS Amperes):

$I1 := 100A$ $I3 := 50A$ $I5 := 20A$ $I7 := 14A$ $I9 := 11A$ $I11 := 9A$

A Calculate true RMS current. Compare this to the fundamental component RMS value and comment.

$$\text{Irms} := \sqrt{I1^2 + I3^2 + I5^2 + I7^2 + I9^2 + I11^2}$$

$$\text{Irms} = 115.317\ \text{A}$$

True RMS current is 15% larger

B Assuming the displacement power factor is 0.9 lagging, compute the true power factor.

True power factor is:

$$pf = \frac{P}{|S|} = \frac{\left(|V_1| \cdot |I_1| \cdot \cos(\phi_1)\right)}{\left(|V_{rms}| \cdot |I_{rms}|\right)} = \frac{V_1}{V_{rms}} \cdot \frac{I_1}{I_{rms}} \cdot \text{displacement\_factor}$$

Perfect sinusoidal voltage:

$$pf := 1 \cdot \frac{I1}{\text{Irms}} \cdot 0.9 \qquad pf = 0.78 \quad \text{lagging}$$

lowered due to harmonics

C Compute total harmonic distortion in the current

$$\text{THD} := \sqrt{\frac{(I3)^2 + (I5)^2 + (I7)^2 + (I9)^2 + (I11)^2}{I1^2}} \qquad \text{THD} = 57.428\,\%$$

ber?

elative