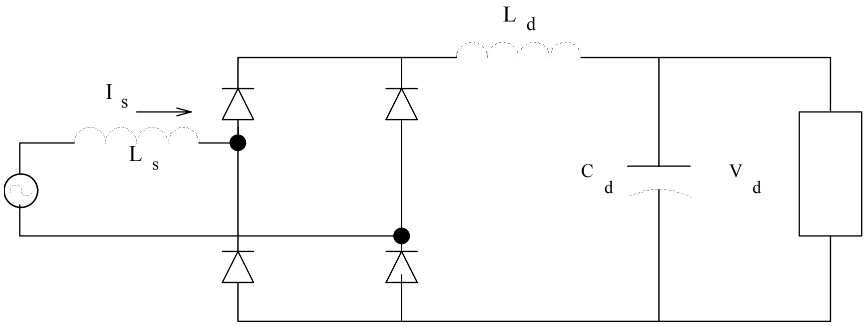
Sample Exam Solution

Problem 1: You are given a single phase diode rectifier, as shown below. Do the following:



$Vd := 310V$ $\quad$ $Xs := 0.4ohm$ $\quad$ at 400 Hz

$$Vspk := 360V \qquad Vs := \frac{Vspk}{\sqrt{2}} \qquad \omega := 2\cdot\pi\cdot 400Hz$$

$$Ls := \frac{Xs}{\omega} \qquad Ls = 0.2\,mH$$

A. Assume that $L_d = 0$ and C_d is large. Plot the ac current versus time for one 400 Hz cycle. Determine the angles at which conduction begins in degrees. Also determine the peak dc current.

Since the capacitor voltage is fixed at 310 V, there will be no decrease in Vd due to the voltage drop across Ls, and since no current flows prior to diode turn on in this case, we can find θb from Vd = Vs*sin(θb). Note that we are already given the peak voltage

$$\theta b := \operatorname{asin}\left(\frac{Vd}{\sqrt{2}\cdot Vs}\right) \qquad \theta b = 59.4\,deg$$

Note: This exam was originally assigned as a take home exam, so solving for θ_f in order to plot the waveform was not a time issue. In the final in this class you won't need to do so.

Now solve for θf, using the equation for the current.

$\theta f1 := 150deg$

Given

$$\left[\sqrt{2}\cdot Vs\cdot\left(\cos(\theta b)-\cos(\theta f1)\right)-Vd\cdot(\theta f1-\theta b)\right] = 0$$

$\theta f := Find(\theta f1)$ $\qquad$ $\theta f = 152\,deg$

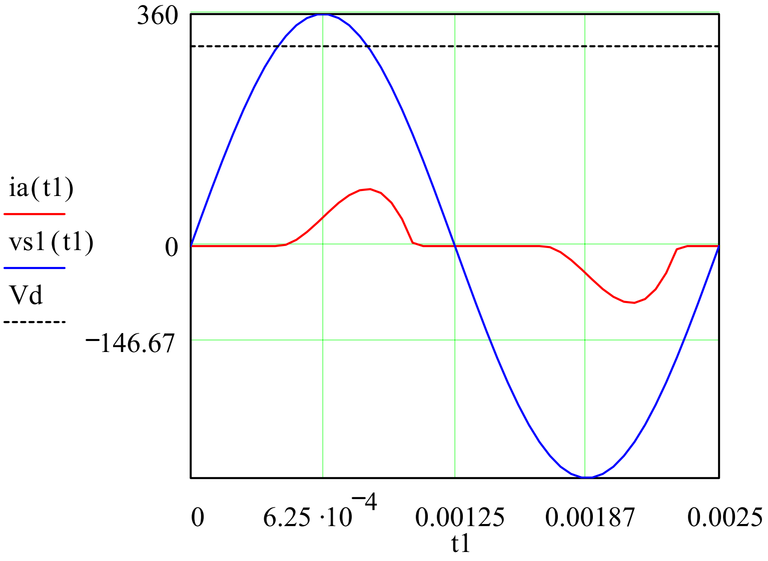
$\theta b2 := \theta b + \pi$ $\qquad$ $\theta b2 = 239.4\,deg$

$\theta f2 := \theta f + \pi$ $\qquad$ $\theta f2 = 332\,deg$

Expression for current (conditional so it doesn't reverse)

$t1 := 0, 0.00005sec\,..\,0.0025sec$ $\qquad$ $vs1(t1) := \sqrt{2}\cdot Vs\cdot\sin(\omega\cdot t1)$

$$ia(t1) := \begin{array}{|l} \left[\dfrac{1}{\omega\cdot(Ls)}\right]\cdot\left[\sqrt{2}\cdot Vs\cdot\left(\cos(\theta b)-\cos(\omega\cdot t1)\right)-Vd\cdot(\omega\cdot t1-\theta b)\right] \text{ if } \theta b \le \omega\cdot t1 \le \theta f \\ \left[\dfrac{1}{\omega\cdot(Ls)}\right]\cdot\left[\sqrt{2}\cdot Vs\cdot\left(\cos(\theta b2)-\cos(\omega\cdot t1)\right)+Vd\cdot(\omega\cdot t1-\theta b2)\right] \text{ if } \theta b2 \le \omega\cdot t1 \le \theta f2 \\ 0 \text{ otherwise} \end{array}$$



Find angle of peak current and peak current:

$$\theta p := \frac{\pi}{2} + \left(\frac{\pi}{2} - \theta b\right) \qquad \boxed{\theta p = 120.6\,deg}$$

Convert to seconds:

$$tpeak := \frac{\theta p}{\omega} \qquad tpeak = 8.4 \times 10^{-4}\,s$$

$$\boxed{ia(tpeak) = 88.5\,A}$$

(b) In many cases it is desirable to have continuous conduction. Describe potential benefits to having continuous conduction. How would the circuit need to be modified from the conditions of part A to achieve continuous conduction? Would the voltages need to change?

1. If it is continuous conduction, the peak current will be lower (if the average current is the same). Also the harmonic content of the waveform may improve somewhat.

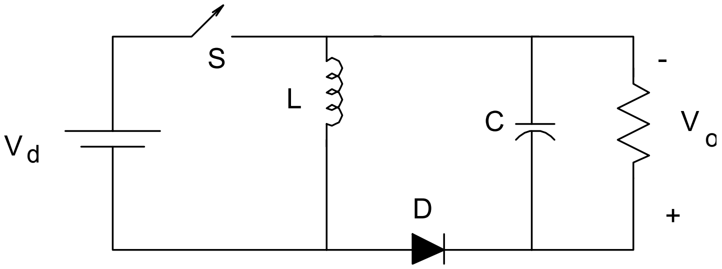
2. We would want to add enough inductance on the dc side of the circuit to bring it to continuous conduction. Adding additional inductance on the ac side can help somewhat too, but the dc side inductance has the larger impact.

3. There would be commutation overlap due to the ac side inductance, so the average dc voltage wou be somewhat lower.

Problem 2: For the converter shown below, determine the following:

(a) What type of converter is it?

Buck-Boost



(b) Duty cycle

$Vd := 25V$ $\quad L := 0.2mH$ $\quad Rout := 3.25ohm$

$Vo := 18V$ $\quad fs := 5kHz$ $\quad Ts := \frac{1}{fs}$

Solve the equation: $\frac{Vo}{Vd} = \frac{D1}{1 - D1}$ for D:

$D := \frac{Vo}{Vo + Vd}$ $\quad \boxed{D = 0.4}$

Is it in continuous conduction?

$Iob := \frac{Vo \cdot (1 - D)^2}{fs \cdot 2L}$ $\quad Iob = 3\,A$

$Io := \frac{Vo}{Rout}$ $\quad Io = 5.5\,A$ $\quad$ Since Io > Iob, continuous conduction

So the result for D above is correct.

c. We want to limit ΔV_o to 0.5% of the output voltage. Determine the necessary capacitance.

$$\Delta Vo = \frac{Io \cdot D \cdot Ts}{C}$$

$\Delta Vo := 0.5\% \cdot Vo$ $\quad \Delta Vo = 0.1\,V$

Therefore we can solve for C:

$C := \frac{Io \cdot D \cdot Ts}{\Delta Vo}$ $\quad \boxed{C = 5152.1\,\mu F}$

What would happen if a smaller capacitor was used?

If the size of the capacitor is decreased, the peak to peak ripple voltage would increase, and the current to the load would also show more ripple.

Problem 3

A. You are given a boost converter. You measure the input voltage to be 15 V from a 100W sourc and have an output current of 4 A. Assuming no power losses determine: input current, output voltage, duty ratio. The filter inductance is 100 μH and the switching frequency is 10 kHz.

$Vd := 15V$ $\quad L := 100\mu H$

$Pin := 100W$ $\quad fs := 10kHz$

$Iout := 4A$

$Pout := Pin$ $\quad Vout := \frac{Pout}{Iout}$ $\quad \boxed{Vout = 25\,V}$

$Iin := \frac{Pin}{Vd}$ $\quad \boxed{Iin = 6.7\,A}$

$D := \frac{Vout - Vd}{Vout}$ $\quad \boxed{D = 0.4}$

Check continuous:

$Iob := \frac{Vout}{2 \cdot L \cdot fs} \cdot D \cdot (1 - D)^2$ $\quad Iob = 1.8\,A$ This is smaller than the load current, so it is in continuous conduction.

B Given a diode rectifier with a large dc filter capacitor, how does the average dc voltage normall relate to the RMS ac voltage if there is now ac side inductance.

$Vdo = \frac{2 \cdot \sqrt{2}}{\pi} \cdot Vs$ $\quad$ where $\quad v_s(t) = \sqrt{2} \cdot Vs \cdot \sin(\omega \cdot t)$

$\frac{2 \cdot \sqrt{2}}{\pi} = 0.9$

or simplifying:

$Vdo = 0.9 \cdot Vs$

If the ac side inductance is called Ls:

$$Vd = Vdo - \frac{\omega \cdot Ls}{\pi} \cdot Idave$$

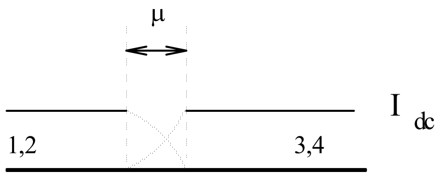
If Ls is 0, then:

$$Vd = 0.9 \cdot Vs$$

Problem 4: Answer the following questions.

(a) What does the commutation overlap angle represent?

The commutation overlap angle (μ or *u*) represents the impact of the ac side inductance on the transition (or commutation) from one pair of diode to the next in the cycle. If the circuit is in contiuous conduction, the current through the ac side inductor will not be zero when the trans occurs. As a result, the current in the inductor will need to go to zero. At the same time the current in the other inductor will be ringing up.



(b) What must be present in the circuit for commutation overlap to occur?

AC side inductance and enough dc side inductance to bring the circuit nearly to continuous conduction.

(c) Will there be commutation overlap if the diode current goes to zero before $\theta = 180$deg? Explain

No. Commutation overlap only occurs when there current through the diode needs to be forced to zero (along with the inductor current) at the voltage zero crossing.

(d) How does commutation overlap impact the dc voltage? Explain

The dc voltage is reduced as follows:

$$Vdo = 0.9 \cdot Vs$$

$$Vd = Vdo - \frac{\omega \cdot Ls}{\pi} \cdot Idave$$