EE/CompE 243
Examples (from Hw #1 and #2 in Spring 99

1. Simplify each of the expression using one of the theorems

2.4(a) A'B'C + (A'B'C)'
Let X = A'B'C
X + X' = 1 Theorem (5)
Solution: 1

2.4(b) A(B+C'D) + B + C'D
=A(B+C'D) + (B + C'D)
Let X = B+C'D
Let Y = A
XY + X = X Theorem (10)
Solution:
B + C'D

2. Multiply to obtain a Sum of Products

2.6(c) (A + B'C + D')(B'C + D' + E)(A + E')(AD + E')
= (AB'C + AD' + AE + B'C + B'CD' + B'CE + B'CD' + D' + D'E)
(AD + AE' + ADE' + E')
= (B'C + AD' + AE + B'CD' + B'CE + D')(AD + E')
= (B'C + D' + AE + B'CE)(AD + E')
= (B'C + D' + AE)(AD + E')
= AB'CD + B'CE' + D'E' + ADE

Solution:
AB'CD + B'CE' + D'E' + ADE

2.6(d) (A' + BE')(BE' + C + D)(E + C')
= (A'BE' + A'C + A'D + BE' + BCE' + BDE')(E + C')
= A'CE + A'DE + A'BC'E' + A'C'D + BC'E' + BC'DE'
= BC'E' + A'CE + A'DE + A'C'D

Solution:
BC'E' + A'CE + A'DE + A'C'D

3. Factor each of the expression to obtain a Product of Sums

2.9(b) WX' + WY'Z' + WYZ
= W(X' + Y'Z' + YZ)
Let X = X' + Y'Z'
YZ = YZ

Second Distributive Law

X + YZ = (X + Y)(X + Z)
W[(X' + Y'Z') + Y][(X' + Y'Z') + Z]

Second Distributive Law

W[(Y + X' + Z')(Y + X' + Y')][(Z + X' + Z')(Z + X' + Y')]
= W(X' + Y + Z')(X' + Y' + Z)

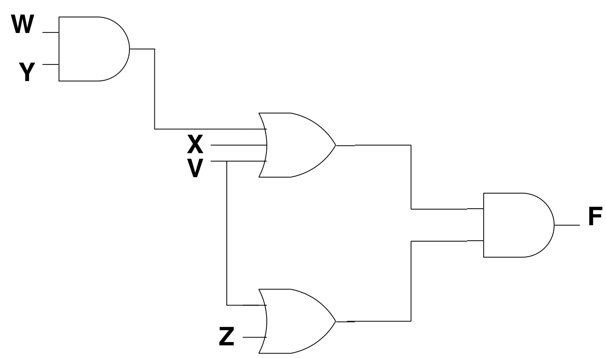
Solution:
W(X' + Y + Z')(X' + Y' + Z)

2.9(e) AC'D + C'D' + A'D'
= C'(AD + D') + A'D'
= C'(D + D')(A + D') + A'D' 2nd Distributive Law
= C'(A + D') + A'D'
= [C'(A + D') + A'][C'(A + D') + D'] 2nd Distributive Law
= (A' + C')(A' + A + D')(C' + D')(A + D' + D') 2nd Distributive Law
= (A + D')(C' + D')(A' + C')

Solution:
(A + D')(C' + D')(A' + C')

4. Realize the function and draw the network using two OR gates and two AND gates

2.13 F= (V + W + X)(V + X + Y)(V + Z)
= (V + VX + VY + WV + WX + WY + VX + X + XY)(V + Z)
= (V + X + WY)(V + Z)



5. Finding the complement of each expression

3.4(a) (a'b + 1)(cd + e') + f(g' + 0) + h
= [(a'b + 1)(cd + e') + f(g' + 0) + h]'
= [(a'b + 1)' + (cd + e')'][f' + (g' + 0)'h']
= [(a + b')·0 + (c' + d')e][f' + (g·1)]h'

Solution:
[(a + b')·0 + (c' + d')e][f' + (g·1)]h'

3.4(b) [a'b'(c + d')(c' + d) + ab(c'd + cd')]'
= [(a'b')' + (c + d')' + (c' + d)'][(ab)' + (c'd')'(cd')']
= [a + b + (c'd) + cd'][a' + b' + (c + d')(c' + d)]

Solution:
[a + b + (c'd) + cd'][a' + b' + (c + d')(c' + d)]

6. Finding the dual of each expression

3.6(a) (a'b +1)(cd + e') + f(g' + 0) + h

The dual is found by replacing AND by OR, OR by AND, 0 by 1 and 1 by 0

Solution:
[(a' + b)0 + (c + d)e'][f + (g'·1)]h

3.6(b) = a'b'(c + d')(c' +d) + ab(c'd + cd')

Solution:
[a' + b' + cd' + c'd][a + b + (c' + d)(c + d')]

7. Multiplying to obtain a sum of four terms

3.10(a) (A' + B' + C')(A + C + D')(A + B)(A' + D)(A' + C + D)
= (A'C + A'D' + AB' + B'C + B'D' + AC' + C'D')(AD + A'B + BD)(A' + C + D)
= (A'C + A'D' + AB' + B'C + B'D' + AC' + C'D')
(ADC + AD + A'B + A'BC + A'BD + A'BD + BCD + BD')
= (A'C + A'D' + AB' + B'C + B'D' + AC' + C'D')(AD + A'B + BD)
= A'BC + A'BCD + A'BD' + AB'D + AB'CD + AC'D + ABC'D + A'BC'D'
= AB'D + AC'D + A'BC + A'BD'

Solution:
AB'D + AC'D + A'BC + A'BD'

8. 3.21(a) Show that $x \oplus y = (x \equiv y)'$

$$
\begin{aligned}
x \oplus y &= x'y + xy' \\
&= (x'y + x)(x'y + y') && \text{Since } X + YZ = (X+Y)(X+Z) \\
&= (x' + x)(y + x)(x' + y')(y + y') \\
&= (x + y)(x' + y') \\
&= (xy + x'y')' \\
&= (x \equiv y)'
\end{aligned}
$$

Solution:

$$x \oplus y = (x \equiv y)'$$

3.21(b) Realize $a'b'c' + a'bc + ab'c + abc'$ using only two input equivalence gates

$$a'b'c' + a'bc + ab'c + abc'$$
$$c'(a'b' + ab) + c(a'b + ab')$$
$$c'(a \equiv b) + c(a \oplus b)$$
$$c'(a \equiv b) + c(a \equiv b)' \qquad \text{Since } X \oplus Y = (X \equiv Y)'$$
$$c \oplus (a \equiv b)$$

Solution:

$$[(a \equiv b) \equiv c]'$$

9. Express f as a minterm and maxterm expansion using m and M notation.
$f(a,b,c) = a'(b + c')$

$a'(b + c') = a'b + a'c'$

a	b	c	f	f'
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

(a) Expressing f as a minterm expansion
$f = \sum m(0,2,3)$
Form a summation of the input combinations when $f = 1$

(b) Expressing f as a maxterm expansion
$f = \prod M(1,4,5,6,7)$
Form a product of the input combinations when $f = 0$

(c) Expressing f' as a minterm expansion
$f' = \sum m(1,4,5,6,7)$
Form a summation of the input combinations when $f' = 1$

(d) Expressing f' as a maxterm expansion
$f' = \prod M(0,2,3)$
Form a product of the input combinations when $f' = 0$

10. Express each of the following as minterm and maxterm expansions using algebraic and m (or M) notation.

(a) $f(a,b,c) = (b+c)(a'+b+c')(a+c)$

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$f = \sum m(1,3,6,7)$$

$$= a'b'c + a'bc + abc' + abc$$
$$= a'c + ab$$

$$f = \prod M(0,2,4,5)$$

$$= (a+b+c)(a+b'+c)(a'+b+c)(a'+b+c')$$
$$= (a+c)(a'+b)$$

(b) $f(x,y,z) = \sum m(0,3,4,7)$

a	b	c	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$f = \sum m(0,3,4,7)$$
$$= x'y'c' + x'yz + xy'z' + xyz$$

$$= y'z' + yz$$
$$= y \equiv z$$

$$f = \prod M(1,2,5,6)$$
$$= (x+y+z')(x+y'+z)(x'+y+z')(x'+y'+z)$$
$$= (y+z')(y'+z)$$
$$= y \equiv z$$

11. Express the minterm and maxterm expansions for the following: (use m and M notation). (a) $f(a,b,c,d) = c(a'+b)+b'd$

a	b	c	d	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

$$f = \sum m(1,2,3,6,7,9,11,14,15)$$
$$f = a'b'c'd' + a'b'cd' + a'b'cd + a'bcd' + a'bcd + ab'c'd + ab'cd + abcd' + abcd$$
$$= a'b'c'd' + a'b'c + a'bc + ab'd + abc$$
$$= a'b'c'd' + a'c + ab'd + abc$$
$$= b'd(a'c' + a) + a'c + abc$$

$$f = \prod M(0,4,5,8,10,12,13)$$
$$f = (a+b+c+d)(a+b'+c+d)(a+b'+c+d')(a'+b+c+d)$$
$$(a'+b+c'+d)(a'+b'+c+d)(a'+b'+c+d)$$
$$= (a+c+d)(b'+c+d')(a'+b+d)(a'+b'+c+d)$$

(b) $g(w,x,y,z) = xy'z + w'xy' + yz' + w'x'z$

w	x	y	z	g
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$g = \sum m(1,2,3,4,5,6,10,13,14)$$

$$g = w'x'y'z' + w'x'yz' + w'x'yz + w'xyz + w'xy'z + w'xyz' + wx'yz' + wxy'z + wxyz'$$

$$g = \prod M(0,7,8,9,11,12,15)$$

$$\begin{aligned} g &= (w+x+y+z)(w+x'+y'+z')(w'+x+y+z)(w'+x+y+z')(w'+x+y'+z') \\ &\quad (w'+x'+y+z)(w'+x'+y'+z') \\ &= (x+y+z)(x'+y'+z')(w'+x+z')(w'+x'+y+z) \end{aligned}$$

12. Express f from the table below, as a minterm expansion (use m) and as a maxterm expansion (use M).

a	b	c	d	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

$$f = \sum m(0,2,4,5,7,8,9,11,12,14)$$

$$\begin{aligned} f &= a'b'c'd' + a'b'cd' + a'bc'd' + a'bc'd + a'bcd + ab'c'd' + ab'c'd \\ &\qquad + ab'cd + abc'd' + abcd' \\ &= a'b'd' + a'bc' + ab'c' + abd' + a'bcd + ab'cd \end{aligned}$$

$$f = \prod M(1,3,6,10,13,15)$$

$$\begin{aligned} f &= (a+b+c+d')(a+b+c'+d')(a+b'+c'+d)(a'+b+c'+d) \\ &\qquad (a'+b'+c+d')(a'+b'+c'+d') \\ &= (a+b+d')(a+b'+c'+d)(a'+b+c'+d)(a'+b'+d') \end{aligned}$$