1 Basis of Switching Algebra

1.1 Axioms

Axioms are minimal set of definitions assumed to be true

\[
\begin{align*}
X &= 0 \text{ if } X \leftrightarrow 1 & X &= 1 \text{ if } X \leftrightarrow 0 \\
\text{If } X = 0 \text{ then } X' &= 1 & \text{If } X = 1 \text{ then } X' &= 0 \\
0 \cdot 0 &= 0 & 1 + 1 &= 1 \\
1 \cdot 1 &= 1 & 0 + 0 &= 0 \\
0 \cdot 1 &= 0 & 1 + 0 &= 0 + 1 = 1
\end{align*}
\]

The above five axioms completely define switching algebra. All other facts about the system can be proved starting with the axioms.

1.2 Single-Variable Theorems

\[
\begin{align*}
X + 0 &= X & X \cdot (1) &= X \\
X + 1 &= 1 & X \cdot (0) &= 0 \\
X + X &= X & X \cdot X &= X \\
(X')' &= X & \text{Involution} \\
X + X' &= 1 & X \cdot X' &= 0
\end{align*}
\]

These theorems are easily proved by showing that they are valid for both possible values of X. Any boolean expression can be substituted for the variable X in the above theorems.

1.3 Two and Three Variable Theorems

\[
\begin{align*}
X \cdot Y &= Y \cdot X & X + Y &= Y + X \\
X \cdot (Y \cdot Z) &= (X \cdot Y) \cdot Z & X + (Y + Z) &= (X + Y) + Z
\end{align*}
\]

From the above we see that any number of boolean variables ANDed together will have a value of “1” if and only if every variable has a value of “1”. Likewise, any number of boolean variables ORed together will have a value of “0” if and only if every variable has a value of “0”.

\[
\begin{align*}
X \cdot (Y + Z) &= X \cdot Y + X \cdot Z & X + Y \cdot Z &= (X + Y) \cdot (X + Z) \\
X + X \cdot Y &= X & X \cdot (X + Y) &= X \\
X \cdot Y + X' \cdot Y = X & (X + Y) \cdot (X + Y') = X \\
X \cdot Y + X' \cdot Z + Y \cdot Z &= X \cdot Y + X' \cdot Z \\
(X + Y) \cdot (X' + Z) \cdot (Y + Z) &= (X + Y) \cdot (X' + Z) \\
(X + Y') \cdot Y &= X \cdot Y & X \cdot Y' + Y &= X + Y \\
(X + Y) \cdot (Y' + Y) &= (X + Y) \cdot 1
\end{align*}
\]

Any boolean expression can be substituted for X and Y in the above theorems.