1 Basis of Switching Algebra

1.1 Axioms

Axioms are minimal set of definitions assumed to be true

$$X = 0 \text{ if } X <> 1 \qquad X = 1 \text{ if } X <> 0 \tag{1}$$
$$\text{If } X = 0 \text{ then } X' = 1 \quad \text{If } X = 1 \text{ then } X' = 0 \tag{2}$$
$$0 \cdot 0 = 0 \qquad 1 + 1 = 1 \tag{3}$$
$$1 \cdot 1 = 1 \qquad 0 + 0 = 0 \tag{4}$$
$$0 \cdot 1 = 1 \cdot 0 = 0 \qquad 1 + 0 = 0 + 1 = 1 \tag{5}$$

The above five axioms completely define switching algebra. All other facts about the system can be proved starting with the axioms.

1.2 Single-Variable Theorems

$X + 0 = X$	$X \cdot (1) = X$	(6)	Identities
$X + 1 = 1$	$X \cdot (0) = 0$	(7)	Null elements
$X + X = X$	$X \cdot X = X$	(8)	Idempotency
$(X')' = X$		(9)	Involution
$X + X' = 1$	$X \cdot X' = 0$	(10)	Complements

These theorems are easily proved by showing that they are valid for both possible values of X. Any boolean expression can be substituted for the variable X in the above theorems.

1.3 Two and Three Variable Theorems

$X \cdot Y = Y \cdot X$	$X + Y = Y + X$	(11)	Commutativity
$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$	$X + (Y + Z) = (X + Y) + Z$	(12)	Associativity

From the above we see that any number of boolean variables ANDed together will have a value of “1” if and only if every variable has a value of “1”. Likewise, any number of boolean variables ORed together will have a value of “0” if and only if every variable has a value of “0”.

$X \cdot (Y + Z) = X \cdot Y + X \cdot Z$	$X + Y \cdot Z = (X + Y) \cdot (X + Z)$	(13)	Distributivity
$X + X \cdot Y = X$	$X \cdot (X + Y) = X$	(14)	Covering
$X \cdot Y + X \cdot Y' = X$	$(X + Y) \cdot (X + Y') = X$	(15)	Combining
$X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$		(16)	Consensus
$(X + Y) \cdot (X' + Z) \cdot (Y + Z) = (X + Y) \cdot (X' + Z)$		(16)’	Consensus

$$(X + Y') \cdot Y = X \cdot Y \qquad X \cdot Y' + Y = X + Y$$
$$(X + Y) \cdot (Y' + Y) = (X + Y) \cdot 1$$

Any boolean expression can be substituted for X and Y in the above theorems.