1. ( 6 pts ) Short answer (3 points each)
(a) When designing for a PLA, one is often required to find multiple sum-or-products expressions for each output. Why is this so?
Both the AND and the OR stages of the PLA are programmable. The ability to program OR stage allows the outputs of individual AND gates to be shared with multiple OR gates. This use of common terms allows for a possible minimal solution for the multiple output function where none of the individual outputs has been minimized fully.
(b) However, when designing for a PAL, one is only required to find the minimal sum-of-products expressions for each output. Why is this so?
The AND stage of the PAL is programmable, so it is useful to simplify each expression down to the minimal sum-of-products. However, the OR stage is fixed, and it is not possible to share outputs of the AND stage with multiple OR gates, therefore, it is not useful to find common terms to minimize the number of gates or inputs.
2. (6 pts) Realize the three output function shown below using a 3-to-8 decoder and the appropriate logic gates.
Multiply out and expand each function, so all of its terms include $a, b, c$. Notice and use common terms.

$$
\begin{aligned}
f_{1}(a, b, c) & =a b+b^{\prime} c \\
& =a b c^{\prime}+a b c+a^{\prime} b^{\prime} c+a b^{\prime} c \\
f_{2}(a, b, c) & =\left(a+b^{\prime}+c\right)\left(a^{\prime}+b\right) \\
& =a b+a^{\prime} b^{\prime}+a^{\prime} c+b c \\
& =a b c^{\prime}+a b c+a^{\prime} b^{\prime} c^{\prime}+a^{\prime} b^{\prime} c+a^{\prime} b c
\end{aligned}
$$


3. (6 pts) Use a PLA to realize a 4-to-1 multiplexer.

Expression for a 4-to-1 MUX is: $F=A^{\prime} B^{\prime} I_{0}+A^{\prime} B I_{1}+A B^{\prime} I_{2}+A B I_{3}$ The PLA implementation is shown below.

f
4. (6 pts) Implement $f(A, B, C, D)=A C^{\prime} D^{\prime}+B^{\prime} D$ using a 4-to-1 multiplexer. Choose the appropriate control inputs.

Notice that D is common to both terms in $f$. Therefore D should be one of the control inputs. Since the first term has both $A$ and $C$, choose one of them as the other input, so you don't need a combination of both as an input. Your choices of control inputs are therefore: $A$ and $D$ or the set $C$ and $D$.

If you choose, expand the second term in $f$ to add the dependence of the second term on $A$, resulting in: $f(A, B, C, D)=A C^{\prime} D^{\prime}+\left(A^{\prime}+A\right) B^{\prime} D$ which simplifies to: $f(A, B, C, D)=A C^{\prime} D^{\prime}+A^{\prime} B^{\prime} D+A B^{\prime} D$ Similarly, if $C$ had been chosen as the second control input, then expand expression out to:
$f(A, B, C, D)=A C^{\prime} D^{\prime}+B^{\prime} C^{\prime} D+B^{\prime} C D$
These would be implemented as follows:

5. (8 pts) (a) Simplify the following multiple output function for implementation with a PAL, and (b) implement it using the AND-OR based PAL shown below.

$$
\begin{aligned}
& F 1(A, B, C, D)=\sum_{m}(0,2,7,10)+\sum_{d}(12,15) \\
& F 2(A, B, C, D)=\sum_{m}(2,4,5)+\sum_{d}(6,7,8,10)
\end{aligned}
$$


$\mathrm{F} 1=\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{D}^{\prime}+\mathrm{B}^{\prime} \mathrm{CD}^{\prime}+\mathrm{BCD}$

$\mathrm{F} 2=\mathrm{A}^{\prime} \mathrm{B}+\mathrm{B}^{\prime} \mathrm{CD}^{\prime}$


