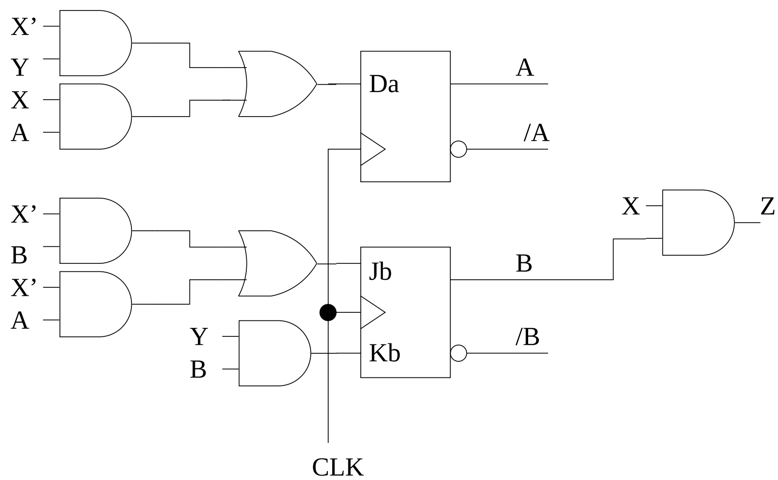
COE/EE 243 **Sample Exam #5 Solution** **Nov 29, 2000**

1. (12 pts) A sequential circuit has 2 rising edge triggered flip-flops (outputs A and B), two inputs (X and Y) and one output Z. The logic expressions for this circuit are:

$$
\begin{aligned}
D_a &= X' \cdot Y + X \cdot A \\
J_b &= X' \cdot B + X' \cdot A \\
K_b &= Y \cdot B \\
Z &= X \cdot B
\end{aligned}
$$

A Sketch a circuit diagram



B Construct a transition table

First, construct the flip-flop excitation table:

	D_a				J_b K_b				Z			
A B	XY=00	01	10	11	XY=00	01	10	11	XY=00	01	10	11
0 0	0	1	0	0	00	00	00	00	0	0	0	0
0 1	0	1	0	0	10	11	00	01	0	0	1	1
1 0	0	1	1	1	10	10	00	00	0	0	0	0
1 1	0	1	1	1	10	11	10	11	0	0	1	1

Now apply the next-state equations for the two types of flip-flops.

For the D flip-flop, $Q+ = D_a$.

For the JK flip-flop, $Q+ = J_b \cdot B' + K_b' \cdot B$

The resulting transition table is as shown:

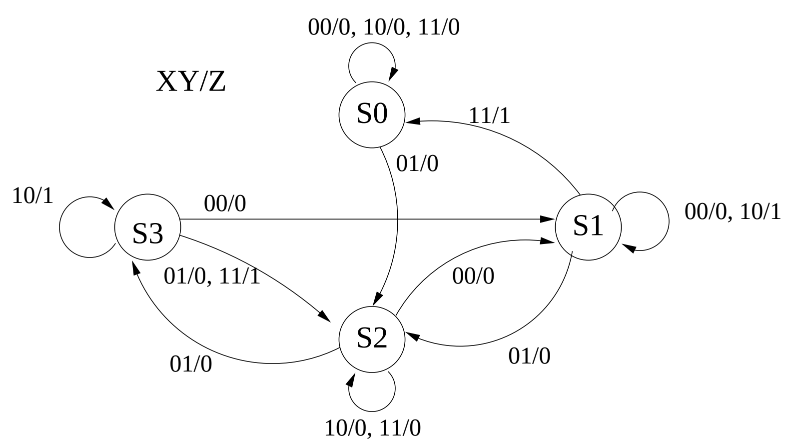
	$A+B+$				Z			
A B	XY=00	01	10	11	XY=00	01	10	11
0 0	00	10	00	00	0	0	0	0
0 1	01	10	01	00	0	0	1	1
1 0	01	11	10	10	0	0	0	0
1 1	01	10	11	10	0	0	1	1

C Construct a state diagram

Assign states: $S0 = 00$, $S1 = 01$, $S2 = 10$, $S3 = 11$ and then make a state table

Current	Next State				Z			
State	XY=00	01	10	11	XY=00	01	10	11
S0	S0	S2	S0	S0	0	0	0	0
S1	S1	S2	S1	S0	0	0	1	1
S2	S1	S3	S2	S2	0	0	0	0
S3	S1	S2	S3	S2	0	0	1	1

The resulting state diagram is shown below.



2. (6 pts) Suppose a Moore machine has three flip-flops, two inputs, and five outputs. Answer the following.

A What is the maximum and minimum number of states in the state diagram?

Maximum number is $2^{numflip-flops} = 8$. The minimum number is also 8, since 3 flip-flops will create 8 distinct states whether they are used or not.

B What are the maximum and minimum numbers of transition arrows starting at a particular state?

The maximum number is $2^{numinputs} = 4$ in this case. The minimum is 1 if all for input conditions lead to the same next state.

C What are the maximum and minimum numbers of transition arrows ending at a particular state?

The maximum number is $2^{numflip-flops} * 2^{numinputs} = 32$. The minimum is 0.

D What are minimum and maximum number of output patterns that can appear?

The minimum number is 1 if all of the states have the same output pattern for each input (the output pattern is the set of 0's and 1's for the 5 outputs for a given input combination).

The maximum number that can be exist for a given state machine (and shown on a state table) will be 8 (the number of states).

E Are the outputs synchronous or asynchronous?

Since its a Moore machine, the outputs are synchronous (they can change with the clock)

F Which of the above will change for a Moore Machine? (give the letter and the new answer)

Part **D** will change. The minimum number will stay the same. The maximum number of states is the smaller of $2^{numflip-flops} * 2^{numinputs}$ or $2^{numoutputs}$. In this case, both are 32.

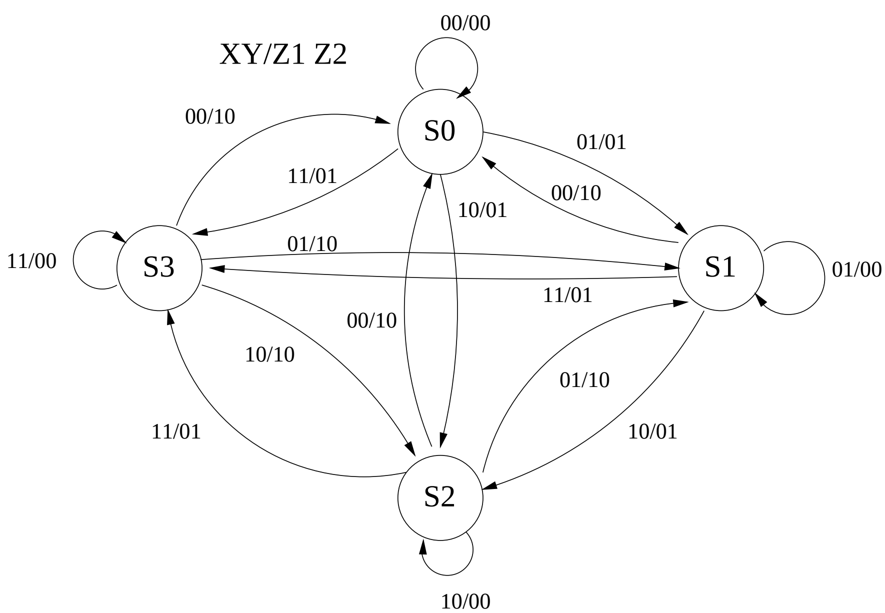
Part **E** will change to asynchronous since the outputs can change when the inputs change, and the inputs aren't necessarily synchronized with the clock.

3. (14 pts) Draw the state diagram for a Mealy state machine with two inputs (X and Y) and two outputs (Z1 and Z2). The two inputs represent a two bit binary number (N). If the present value of N is greater than the previous value of N then Z1=0 and Z2=1. And if the present value of N is less than the previous of N then Z1=1 and Z2=0. Otherwise Z1=Z2=0.

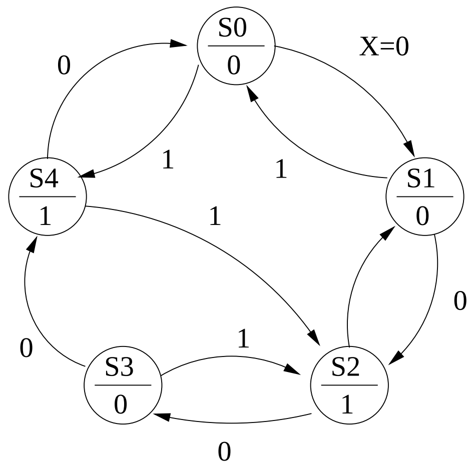
   One option is to assign states (flip-flop outputs A and B) as:

State	X	Y
S0	0	0
S1	0	1
S2	1	0
S3	1	1

   Note that this is not the only solution.



4. (18 pts) Complete the design for the state machine described in the state diagram below.



A. Write out the state table

Present State	Next State X=0	Next State X=1	Z
S0	S1	S4	0
S1	S2	S0	0
S2	S3	S1	1
S3	S4	S2	0
S4	S0	S2	1

B. Assign states using a simple binary order (S0 = ABC = 000) and assign the unused states to go to State S2 as their next state if X=1 and S1 if X=0. The write out the transition table.

Set the outputs for the unused states as don't care conditions

A	B	C	A+B+C+ X=0	A+B+C+ X=1	Z
0	0	0	001	100	0
0	0	1	010	000	0
0	1	0	011	001	1
0	1	1	100	010	0
1	0	0	000	010	1
1	0	1	001	010	X
1	1	0	001	010	X
1	1	1	001	010	X

C. Write out the flip-flop input excitation table assuming JK flip-flops are used

Since we have JK flip-flops, we know $Q+ = J \cdot Q' + K' \cdot Q$ and we can create a flip-flop excitation table as follows.

Q	Q+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

	J_aK_a		J_bK_b		J_cK_c		
ABC	X=0	X=1	X=0	X=1	X=0	X=1	Z
000	0X	1X	0X	0X	1X	0X	0
001	0X	0X	1X	0X	X1	X1	0
010	0X	0X	X0	X1	1X	1X	1
011	1X	0X	X1	X0	X1	X1	0
100	X1	X1	0X	1X	0X	0X	1
101	X1	X1	0X	1X	X0	X1	X
110	X1	X1	X1	X0	1X	0X	X
111	X1	X1	X1	X0	X0	X1	X

D. Sketch the circuit diagram

Using K-maps to find minimal expressions for the J and K inputs for each flip-flop and for Z we get the following:

$$\begin{aligned} J_a &= X \cdot B' \cdot C' + X' \cdot B \cdot C \\ K_a &= 1 \\ \\ J_b &= X \cdot A + X' \cdot A' \cdot C \\ K_b &= X' \cdot A + X' \cdot C + X \cdot A' \cdot C' \\ \\ J_c &= X' \cdot A' + X' \cdot B + A' \cdot B \\ K_c &= X + A' \\ \\ Z &= A + B \cdot C' \end{aligned} \tag{1}$$