Example 12.3

All values are in pu unless noted otherwise.

\[
\begin{aligned}
X_{du} &:= 1.81 & X_{qu} &:= 1.76 & X_{dp} &:= 0.3 & X_L &:= 0.16 \\
R_a &:= 0.003 & T_{dop} &:= 8.0 \text{-sec} & H &:= 3.5 & K_D &:= 0.0 \\
A_{sat} &:= 0.031 & B_{sat} &:= 6.93 & \Psi_{TI} &:= 0.8 & L_{dp} &:= X_{dp} \\
L_{mdu} &:= 1.65 & L_{mqu} &:= 1.60 & L_L &:= X_L & X_S &:= 0.65 \\
R_{fd} &:= 0.0006 & L_{Lfd} &:= 0.153
\end{aligned}
\]

**Steady State**

Ignore amortisseur windings.

Given: \(P := 0.9\) \(Q := 0.3\) \(V_{as} := 1.0 \cdot e^{j36 \text{ deg}}\)

\[
\begin{aligned}
S &:= P + jQ & S &= 0.9 + 0.3j \\
\phi &:= \text{arg}(S) & \phi &= 18.4349 \text{ deg} \\
I_{as} &:= \left( \frac{S}{V_{as}} \right) & |I_{as}| &= 0.9487 & \text{arg}(I_{as}) &= 17.5651 \text{ deg} \\
\alpha &:= \text{arg}(V_{as}) & \alpha &= 36 \text{ deg} \\
\Psi_{at0} &:= \left| V_{as} + I_{as} \left( R_a + j X_L \right) \right| \\
\Psi_{I0} &:= A_{sat} e^{B_{sat} \left( \Psi_{at0} - \Psi_{TI} \right)} & \Psi_{I0} &= 0.1884 \\
K_{sd} &:= \frac{\Psi_{at0}}{\Psi_{at0} + \Psi_{I0}} & K_{sd} &= 0.8491 \\
K_{sq} &:= K_{sd} & K_{sq} &= 0.8491
\end{aligned}
\]
\[ L_{md} := K_{sd} L_{mdu} \]
\[ L_{mq} := K_{sq} L_{mqu} \]
\[ L_d := L_{md} + L_L \]
\[ L_q := L_{mq} + L_L \]
\[ X_{md} := L_{md} \]
\[ X_q := L_q \]
\[ X_d := L_d \]
\[ E_B := V_{as} - j X_s I_{as} \]
\[ E_{qas} := V_{as} + (R_a + j X_q) I_{as} \]
\[ I_{ad0} := |I_{as}| \cdot \sin(\delta_i + \phi) e^{j \theta_{r0}} \]
\[ I_{aq0} := |I_{as}| \cdot \cos(\delta_i + \phi) e^{j \left( \theta_{r0} + \frac{\pi}{2} \right)} \]
\[ E_{as0} := E_{qas} + j (X_d - X_q) I_{ad0} \]
\[ i_{fdphasor0} := \frac{E_{as0}}{j X_{md}} \]

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Example 12.3

Power System Stability
Complex Vector Variables

\[ V_{dq0} := V_{as} e^{-j \theta_0} \]
\[ V_{d0} := \text{Re}(V_{dq0}) \]
\[ V_{q0} := \text{Im}(V_{dq0}) \]
\[ I_{dq0} := I_{as} e^{-j \theta_0} \]
\[ I_{d0} := \text{Re}(I_{dq0}) \]
\[ I_{q0} := \text{Im}(I_{dq0}) \]
\[ E_{dq0} := E_{as0} e^{-j \theta_0} \]
\[ E_{d0} := \text{Re}(E_{dq0}) \]
\[ E_{q0} := \text{Im}(E_{dq0}) \]
\[ I_{fd0} := \frac{E_{dq0}}{j \cdot X_{md}} \]
\[ E_{fd0} := I_{fd0} L_{mdu} \]
\[ V_{fd0} := R_{fd} I_{fd0} \]
\[ T_{e0} := P + \left( |I_{dq0}|^2 \cdot R_a \right) \]

\[ |V_{dq0}| = 1 \]
\[ \text{arg}(V_{dq0}) = 46.8745 \text{ deg} \]
\[ V_{d0} = 0.6836 \]
\[ V_{q0} = 0.7299 \]
\[ |I_{dq0}| = 0.9487 \]
\[ \text{arg}(I_{dq0}) = 28.4396 \text{ deg} \]
\[ I_{d0} = 0.8342 \]
\[ I_{q0} = 0.4518 \]
\[ |E_{dq0}| = 2.0335 \]
\[ \text{arg}(E_{dq0}) = 90 \text{ deg} \]
\[ E_{d0} = 0 \]
\[ E_{q0} = 2.0335 \]
\[ |I_{fd0}| = 1.4514 \]
\[ E_{fd0} = 2.3947 \]
\[ V_{fd0} = 8.7081 \times 10^{-4} \]
\[ T_{e0} = 0.9027 \]
Example 12.3

\[
\Psi_{md0} := L_{md} (I_{d0} - I_{q0}) \quad \psi_{md0} = 0.8647
\]

\[
\Psi_{mq0} := -L_{mq} I_{q0} \quad \psi_{mq0} = -0.6138
\]

\[
\delta_0 := \delta_i + \alpha \quad \delta_0 = 79.13 \text{ deg}
\]

### Steady State

**Small Signal**

\[
K_{sinc} := \frac{1}{1 + B_{sat} A_{sat} e^{B_{sat} (\psi_{at0} - \psi_{TI})}} \quad K_{sinc} = 0.4337
\]

\[
K_{sqinc} := K_{sinc}
\]

\[
R_E := 0
\]

\[
R_T := R_a + R_E \quad R_T = 3 \times 10^{-3}
\]

\[
X_E := 0.65
\]

\[
L_{mdi} := K_{sinc} L_{mdu} \quad L_{mdi} = 0.7156
\]

\[
L_{mqi} := K_{sqinc} L_{mqu} \quad L_{mqi} = 0.694
\]

\[
L_{di} := L_{mdi} + L_L \quad L_{di} = 0.8756
\]

\[
L_{qi} := L_{mqi} + L_L \quad L_{qi} = 0.854
\]

\[
X_{mdi} := L_{mdi}
\]

\[
X_{qi} := L_{qi} \quad X_{di} := L_{di}
\]

\[
X_{Tq} := X_E + X_{qi} \quad X_{Tq} = 1.504
\]

\[
X_{Td} := X_E + X_{mdpi} + X_L \quad X_{Td} = 0.9361
\]

\[
D := R_T^2 + X_{Tq} X_{Td}
\]
\[ m_1 := \frac{E_B \left( X_{Tq} \sin(\delta_0) - R_T \cos(\delta_0) \right)}{D} \quad m_1 = 1.0436 \]

\[ n_1 := \frac{E_B \left( R_T \sin(\delta_0) + X_{Td} \cos(\delta_0) \right)}{D} \quad n_1 = 0.1269 \]

\[ m_2 := \frac{X_{Tq}}{D} \left( \frac{L_{mdi}}{L_{mdi} + L_{Lfd}} \right) \quad m_2 = 0.8801 \]

\[ n_2 := \frac{R_T}{D} \left( \frac{L_{mdi}}{L_{mdi} + L_{Lfd}} \right) \quad n_2 = 0.0018 \]

\[ K_1 := n_1 \left( \Psi_{md0} + L_{mqi} I_{d0} \right) - m_1 \left( \Psi_{mq0} + L_{mdpi} I_{q0} \right) \quad K_1 = 0.7643 \]

\[ K_2 := n_2 \left( \Psi_{md0} + L_{mqi} I_{d0} \right) - m_2 \left( \Psi_{mq0} + L_{mdpi} I_{q0} \right) + \frac{L_{mdpi}}{L_{Lfd}} I_{q0} \quad K_2 = 0.8649 \]

\[ a_{1,1} := \frac{-K_D}{2H} \quad a_{1,2} := \frac{-K_1}{2H} \quad a_{1,3} := \frac{-K_2}{2H} \]

\[ a_{2,1} := 2 \cdot 60 \pi \quad a_{2,2} := 0 \quad a_{2,3} := 0 \]

\[ a_{3,1} := 0 \quad a_{3,2} := \frac{-2 \cdot 60 \pi \cdot R_{fd}}{L_{Lfd}} \cdot m_1 \cdot L_{mdi} \quad a_{3,3} := \frac{-2 \cdot 60 \pi \cdot R_{fd}}{L_{Lfd}} \left( 1 - \frac{L_{mdpi}}{L_{Lfd}} + m_2 \cdot L_{mdpi} \right) \]

\[
\begin{bmatrix}
0 & -0.1092 & -0.1236 \\
376.9911 & 0 & 0 \\
0 & -0.1945 & -0.4244
\end{bmatrix}
\]

\[
\begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{pmatrix} = \text{eigenvals}(a)
\]

\[ \lambda_1 = -0.1099 - 6.4115i \quad \lambda_2 = -0.1099 + 6.4115i \quad \lambda_3 = -0.2046 \]

\[
\text{RMM}_1 := -j \cdot \text{eigenvvec}(a, \lambda_1) \quad \text{RMM}_2 := -j \cdot \text{eigenvvec}(a, \lambda_2) \quad \text{RMM}_3 := \text{eigenvvec}(a, \lambda_3)
\]
\[ RMM := \text{augment}(RMM_2, RMM_1, RMM_3) \]

\[ RMM = \begin{pmatrix} 
0.01698 - 0.00087i & -0.01699 - 0.00064i & 0.00041 - 1.76255i 
-10^{-7} 
-0.06836 - 0.99706i & 0.055 - 0.99788i & -0.74898 + 0.00032i 
0.03027 - 0.00059i & -0.03028 - 0.00018i & 0.6626 - 0.00029i 
\end{pmatrix} \]

\[ LMM := RMM^{-1} \]

\[ LMM = \begin{pmatrix} 
29.4322 + 1.0072i & -0.0257 + 0.5003i & -0.0471 + 0.5649i 
-29.443 + 0.6128i & 0.019 + 0.5006i & 0.0395 + 0.5654i 
-2.6911 - 1.167i \times 10^{-3} & 1.4604 \times 10^{-3} + 6.333i \times 10^{-7} & 1.5125 + 6.5589i \times 10^{-4} 
\end{pmatrix} \]

\[
k := 1..3 \quad i := 1..3
\]

\[ sP_{k,i} := RMM_{k,i} \cdot LMM_{i,k} \]

\[ \begin{pmatrix} 
0.5006 & 0.5006 & 0.0011 
0.5006 & 0.5006 & 0.0011 
0.0172 & 0.0172 & 1.0022 
\end{pmatrix} \]  \[ \rightarrow \]

\[ \begin{pmatrix} 
-0.9802 & 0.9802 & -180 
-0.9802 & 0.9802 & 180 
93.6542 & -93.6542 & 0 
\end{pmatrix} \]  \[ \text{deg} \]