DIRECT AXIS SYNCHRONOUS MACHINE EQUATION MANIPULATION FOR THE PURPOSE OF SIMULATION

Non-Reciprocal Per Unit System \((L_{mdu} = 1 \text{ and } R_{fd} = 1)\)

I. Variables

Inputs: \(v_{fd}\) and \(i_{ds}\)

Output: \(\psi_{ds}\)

Internal: \(i_{fd}, i_{kd}, E_{q}^{f} = \frac{L_{md}}{L_{ffd}} \psi_{fd}\), and \(\psi_{kd}\)

Unknowns: \(i_{fd}, i_{kd}, \psi_{ds}, E_{q}^{f} = \frac{L_{md}}{L_{ffd}} \psi_{fd}\), and \(\psi_{kd}\); i.e., output plus internal variables

Note: There are 5 unknowns. Therefore, we need 5 equations.

II. Voltage Equations:

\[
v_{fd} = \frac{p}{\omega_B} \psi_{fd} + R_{fd} i_{fd} \tag{1}
\]

\[
v_{kd} = 0 = \frac{p}{\omega_B} \psi_{kd} + R_{kd} i_{kd} \tag{2}
\]

III. Flux Linkage Equations:

\[
\psi_{fd} = L_{ffd} i_{fd} + \beta L_{md} (i_{kd} - i_{ds}) = L_{ffd} i_{fd} + \beta L_{md} (i_{fd} + i_{kd} - i_{ds}) \tag{3}
\]

\[
\psi_{kd} = L_{kkd} i_{kd} + L_{md} (i_{fd} - i_{ds}) = L_{kkd} i_{kd} + L_{md} (i_{fd} + i_{kd} - i_{ds}) \tag{4}
\]

\[
\psi_{ds} = -L_d i_{ds} + L_{md} (i_{fd} + i_{kd}) = -L_d i_{ds} + L_{md} (i_{fd} + i_{kd} - i_{ds}) \tag{5}
\]
IV. Solve for the Inputs to the Integrators

Select \( E_q' \) and \( \psi_{kd} \) as the outputs of the integrators.

A. \( E_q' \)

Modify Eq. 1 to get \( E_q' \)

Multiple \( \psi_{fd} \) by 1 in the form of \( \frac{L_{f fd}}{L_{md}} \frac{L_{md}}{L_{ffd}} \)

\[
v_{fd} = \frac{p}{\omega_B} \frac{L_{f fd}}{L_{md}} \frac{L_{md}}{L_{ffd}} \psi_{fd} + R_{f d} l_{f d}
\]

Take the Laplace transform of Eq. 6 and divide by \( R_{f d} \)

\[
\frac{V_{fd}}{R_{f d}} = \frac{V_{fd}}{1} = \frac{s}{\omega_B R_{f d}} E_q' + I_{f d}
\]

where

\[
E_q' = \frac{L_{md}}{L_{ffd}} \psi_{fd}
\]

\[
V_{fd} = s \tau_{do}' E_q' + I_{f d}
\]

where

\[
\tau_{do}' = \frac{L_{f fd}}{\omega_B R_{f d}}
\]

Solve for \( E_q' \)

\[
E_q' = \frac{1}{s \tau_{do}'} (V_{fd} - I_{f d})
\]
B. $\psi_{kd}$

Multiply Eq. 2 by $L''_{do}$ and take the Laplace transform of it.

$$0 = \frac{s}{\omega_B} L''_{do} \Psi_{kd} + R_{kd} L''_{do} I_{kd}$$

(12)

$$L''_{do} = L_{lkd} + \frac{L_{md} L_{lf d}}{\beta}$$

(13)

$$L''_{do} = L_{lkd} + \frac{L_{md} L_{lf d}}{L_{ff d}}$$

$$= L_{lkd} + \frac{L_{md} (L_{lf d} + \beta L_{md}) - \beta L_{md}^2}{L_{ff d}}$$

$$= (L_{lkd} + L_{md}) - \frac{\beta L_{md}^2}{L_{ff d}}$$

$$L''_{do} = L_{kkd} - \frac{\beta L_{md}^2}{L_{ff d}}$$

(14)

Divide Eq. 12 by $R_{kd}$

$$0 = \frac{s}{\omega_B R_{kd}} \Psi_{kd} + L''_{do} I_{kd}$$

(15)

$$0 = s \tau''_{do} \Psi_{kd} + L''_{do} I_{kd}$$

(16)

where

$$\tau''_{do} = \frac{L''_{do}}{\omega_B R_{kd}}$$

(17)

Solve for $\Psi''_{kd}$

$$\Psi_{kd} = -\frac{1}{s \tau''_{do}} L''_{do} I_{kd}$$

(18)
V. Solve for $i_{fd}$ and $-L_{dd}^\prime i_{kd}$

A. $i_{fd}$

Solve Eq. 3 for $L_{ff}i_{fd}$ and take the Laplace Transform of it

$$L_{ff}i_{fd} = \Psi_{fd} + \beta L_{md}(I_{ds} - I_{kd}) \quad (19)$$

Divide by $L_{ff}$

$$i_{fd} = \frac{\Psi_{fd}}{L_{ff}} + \frac{\beta L_{md}}{L_{ff}}(I_{ds} - I_{kd}) \quad (20)$$

Multiple $\Psi_{fd}$ by 1 in the form of $\frac{L_{md}}{L_{md}}$

$$i_{fd} = \frac{L_{md}}{L_{md}} \frac{\Psi_{fd}}{L_{ff}} + \frac{\beta L_{md}}{L_{ff}}(I_{ds} - I_{kd}) \quad (21)$$

$$i_{fd} = \frac{E_{q}^t}{L_{md}} + \frac{\beta L_{md}}{L_{ff}}(I_{ds} - I_{kd}) \quad (22)$$

$$\begin{align*}
(L_d - L_d') &= L_d - (L_d - \frac{\beta L_{md}^2}{L_{ff}^2}) \\
(L_d - L_d') &= \frac{\beta L_{md}^2}{L_{ff}^2} \\
\frac{L_d - L_d'}{L_{md}} &= \frac{\beta L_{md}}{L_{ff}^2} \\
(L_{du} - L_{du}') &= \frac{\beta L_{md}}{L_{ff}^2} \\
\end{align*} \quad (23)$$

$$i_{fd} = \frac{E_{q}^t}{L_{md}} + (L_{du} - L_{du}') (I_{ds} - I_{kd}) \quad (24)$$
B. \(-L_{do}'' i_{kd}\)

Copy Eq. 4 here for easy reference.

\[
\begin{align*}
\psi_{kd} &= L_{kkd} i_{kd} + L_{md} (i_{fd} - i_{ds}) \\
\psi_{kd} &= L_{kkd} i_{kd} + L_{md} i_{fd} - L_{md} i_{ds}
\end{align*}
\]  

(25)

Multiply Eq. 22 by \(L_{md}\) and write it below.

\[
L_{md} i_{fd} = E_q' + \frac{\beta L_{md}^2}{L_{ffd}} (I_{ds} - I_{kd})
\]  

(26)

Substitute Eq. 26 into Eq. 25 and take the Laplace Transform of it.

\[
\psi_{kd} = L_{kkd} I_{kd} + E_q' + \frac{\beta L_{md}^2}{L_{ffd}} (I_{ds} - I_{kd}) - L_{md} I_{ds}
\]  

(27)

Write Eq. 27 using the distributive property of multiplication.

\[
\psi_{kd} = L_{kkd} I_{kd} + E_q' + \frac{\beta L_{md}^2}{L_{ffd}} I_{ds} - \frac{\beta L_{md}^2}{L_{ffd}} I_{kd} - L_{md} I_{ds}
\]  

(28)

Write Eq. 27 with terms grouped by current.

\[
\psi_{kd} = E_q' - (L_{md} - \frac{\beta L_{md}^2}{L_{ffd}}) I_{ds} + (L_{kkd} - \frac{\beta L_{md}^2}{L_{ffd}}) I_{kd}
\]  

(29)

Copy Eq. 14 here for easy reference.

\[
L_{do}'' = L_{kkd} - \frac{\beta L_{md}^2}{L_{ffd}}
\]  

(30)

Investigate \(L_{md} - \frac{\beta L_{md}^2}{L_{ffd}}\).
\[ L'_d = L_d - \frac{\beta L_{md}^2}{L_{ffd}} \]
\[ = L_{md} + L_l - \frac{\beta L_{md}^2}{L_{ffd}} \]
\[ L'_d - L_l = L_{md} - \frac{\beta L_{md}^2}{L_{ffd}} \]
\[ = \frac{L_{md}(\beta L_{md} + L_{lf} - \beta L_{md}^2)}{L_{ffd}} \]
\[ = \frac{L_{lf} L_{md}}{L_{ffd}} \quad (32) \]

Equation 29 can be rewritten using preceding equations for inductances.

\[ \Psi_{kd} = E'_q - (L'_d - L_l)I_{ds} + L''_d I_{kd} \quad (33) \]

Solving Eq. 28 for \(-L''_{do} I_{kd}\)

\[ -L''_{do} I_{kd} = E'_q - (L'_d - L_l)I_{ds} - \Psi_{kd} \quad (34) \]

C. \(\psi_{ds}\)

**Equation 5** can be used to solve for \(\Psi_{ds}\)

\[ \Psi_{ds} = -L_l I_{ds} + L_{md}(I_{fd} + I_{kd} - I_{ds}) \quad (35) \]

**Summary**

\[ E'_q = \frac{1}{s \sqrt{\chi_{do}}} (V_{fd} - I_{fd}) \quad (36) \]

\[ \Psi_{kd} = \frac{1}{s \sqrt{\chi_{do}}} L''_{do} I_{kd} \quad (37) \]
\[ I_{fd} = \frac{E_q'}{L_{md}} + (L_{du} - L_{du}') (I_{ds} - I_{kd}) \]  \hspace{1cm} (38)

\[ -L_{do}' I_{kd} = E_q' - (L_d' - L_l) I_{ds} - \Psi_{kd} \]  \hspace{1cm} (39)

\[ \Psi_{ds} = -L_d I_{ds} + L_{md} (I_{fd} + I_{kd} - I_{ds}) \]  \hspace{1cm} (40)