I. Draw the winding function, $N_{FuPUD}(\alpha)$, for a full pitch uniformly distributed coil.

Determine the equations for $N_{FuPUD}(\alpha)$ over the non-horizontal regions.

$$\frac{\pi}{2} - \frac{\beta}{2} \le \alpha < \frac{\pi}{2} + \frac{\beta}{2}$$

$$m = \frac{-N}{\beta} \qquad 0 = \frac{-N}{\beta}\cdot\frac{\pi}{2} + b_1 \qquad \Rightarrow \qquad b_1 = \frac{N\cdot\pi}{2\cdot\beta}$$

$$N_{FuPUD}(\alpha) = \frac{-N}{\beta}\cdot\alpha + \frac{N\cdot\pi}{2\cdot\beta}$$

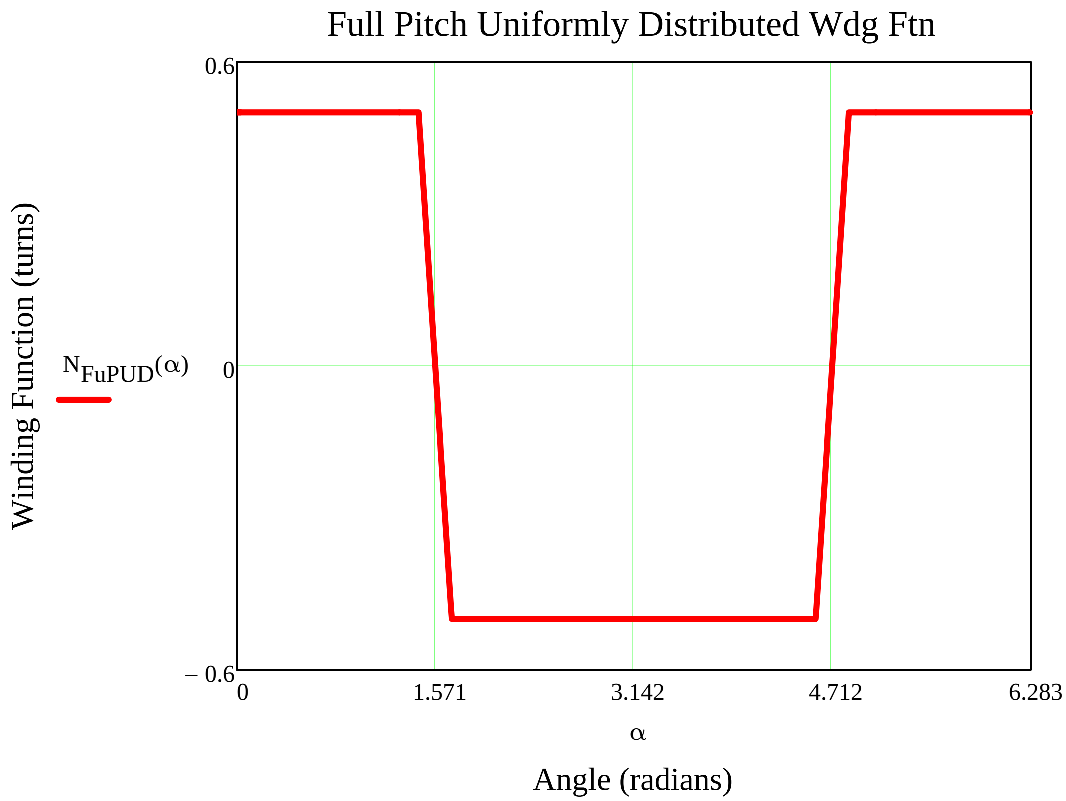
$$\frac{3\pi}{2} - \frac{\beta}{2} \le \alpha < \frac{3\cdot\pi}{2} + \frac{\beta}{2}$$

$$m = \frac{N}{\beta} \qquad 0 = \frac{N}{\beta}\cdot\frac{3\pi}{2} + b_2 \qquad \Rightarrow \qquad b_2 = \frac{-3\cdot N\cdot\pi}{2\cdot\beta}$$

$$N_{FuPUD}(\alpha) = \frac{N}{\beta}\cdot\alpha - \frac{3\cdot N\cdot\pi}{2\cdot\beta}$$

For graphing purposes only: $N := 1.0$ $\quad \beta := \frac{\pi}{12}$

$$N_{FuPUD}(\alpha) := \begin{cases} \frac{N}{2} & \text{if } 0 \le \alpha < \frac{\pi}{2} - \frac{\beta}{2} \\ \frac{N}{\beta}\cdot\frac{\pi}{2} - \frac{N}{\beta}\cdot\alpha & \text{if } \frac{\pi}{2} - \frac{\beta}{2} \le \alpha < \frac{\pi}{2} + \frac{\beta}{2} \\ \frac{-N}{2} & \text{if } \frac{\pi}{2} + \frac{\beta}{2} \le \alpha < \frac{3\pi}{2} - \frac{\beta}{2} \\ \frac{N}{\beta}\cdot\alpha - \frac{N}{\beta}\cdot\frac{3\cdot\pi}{2} & \text{if } \frac{3\cdot\pi}{2} - \frac{\beta}{2} \le \alpha < \frac{3\cdot\pi}{2} + \frac{\beta}{2} \\ \frac{N}{2} & \text{if } \frac{3\cdot\pi}{2} + \frac{\beta}{2} \le \alpha < 2\cdot\pi \end{cases}$$



II. Determine the winding series for a full pitch uniformly distributed coil; i.e., determine the Fourier series coefficients of the winding function.

Symmetry Conditions of N(α)

a) Even, N(-α) = N(α) => b_h = 0 for all h

b) Negative half-wave, N($\alpha+\pi$) = -N(α) => $a_h = b_h$ = 0 for all even h

a & b together => quarter wave symmetry and

$$a_h = \frac{4}{\pi} \cdot \int_0^{\frac{\pi}{2}} N_{FuPUD}(\alpha) \cdot \cos(h\alpha)\, d\alpha$$

$$a_h = \frac{4}{\pi}\cdot\int_0^{\frac{\pi-\beta}{2}} \frac{N}{2}\cdot\cos(h\alpha)\,d\alpha + \frac{4}{\pi}\cdot\int_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} \left(\frac{N}{\beta}\cdot\frac{\pi}{2} - \frac{N}{\beta}\cdot\alpha\right)\cdot\cos(h\alpha)\,d\alpha$$

$$a_h = \frac{4}{\pi}\cdot\int_0^{\frac{\pi-\beta}{2}} \frac{N}{2}\cdot\cos(h\alpha)\,d\alpha + \frac{4}{\pi}\cdot\int_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} \frac{N}{\beta}\cdot\frac{\pi}{2}\cdot\cos(h\alpha)\,d\alpha + \frac{4}{\pi}\cdot\int_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} \frac{-N}{\beta}\cdot\alpha\cdot\cos(h\alpha)\,d\alpha$$

$$a_h = \frac{2\cdot N}{\pi}\cdot\int_0^{\frac{\pi-\beta}{2}} \cos(h\alpha)\,d\alpha + \frac{2\cdot N}{\beta}\cdot\int_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} \cos(h\alpha)\,d\alpha + \frac{-4\cdot N}{\pi\cdot\beta}\cdot\int_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} \alpha\cdot\cos(h\alpha)\,d\alpha$$

$$a_h = \frac{2\cdot N}{\pi}\cdot\frac{1}{h}\cdot\sin(h\cdot\alpha)\cdot\Bigg|_0^{\frac{\pi-\beta}{2}} + \frac{2\cdot N}{\beta}\cdot\frac{1}{h}\cdot\sin(h\cdot\alpha)\cdot\Bigg|_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} \ldots$$
$$+ \frac{-4\cdot N}{\pi\cdot\beta}\cdot\frac{1}{h^2}\cdot\cos(h\cdot\alpha)\cdot\Bigg|_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}} - \frac{4\cdot N}{\pi\cdot\beta}\cdot\frac{1}{h}\cdot\alpha\sin(h\cdot\alpha)\cdot\Bigg|_{\frac{\pi-\beta}{2}}^{\frac{\pi}{2}}$$

$$a_h = \frac{2 \cdot N}{\pi \cdot h} \cdot \sin\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$
$$+ \frac{-2 \cdot N}{\pi \cdot h} \cdot \sin(0) \ldots$$
This term equals zero for all h.
$$+ \frac{2 \cdot N}{\beta \cdot h} \cdot \sin\left(\frac{h \cdot \pi}{2}\right) \ldots$$
$$+ \frac{-2 \cdot N}{\beta \cdot h} \cdot \sin\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$
$$+ \frac{-4 \cdot N}{\pi \cdot \beta \cdot h^2} \cdot \cos\left(h \cdot \frac{\pi}{2}\right) \ldots$$
This term equals zero for all h.
$$+ \frac{4 \cdot N}{\pi \cdot \beta \cdot h^2} \cdot \cos\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$
$$+ \frac{-4 \cdot N}{\pi \cdot \beta} \cdot \frac{1}{h} \cdot \frac{\pi}{2} \sin\left(h \cdot \frac{\pi}{2}\right) \ldots$$
$$+ \frac{4 \cdot N}{\pi \cdot \beta} \cdot \frac{1}{h} \cdot \left(\frac{\pi}{2} - \frac{\beta}{2}\right) \sin\left(h \cdot \frac{\pi - \beta}{2}\right)$$

$$a_h = \frac{2 \cdot N}{\pi \cdot h} \cdot \sin\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$ Term 1 — Term 1 + Term 7 = 0
$$+ \frac{2 \cdot N}{\beta \cdot h} \cdot \sin\left(\frac{h \cdot \pi}{2}\right) \ldots$$ Term 2 — Term 2 + Term 5 = 0
$$+ \frac{-2 \cdot N}{\beta \cdot h} \cdot \sin\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$ Term 3 — Term 3 + Term 6 = 0
$$+ \frac{4 \cdot N}{\pi \cdot \beta \cdot h^2} \cdot \cos\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$ Term 4
$$+ \frac{-2 \cdot N}{\beta \cdot h} \cdot \sin\left(h \cdot \frac{\pi}{2}\right) \ldots$$ Term 5
$$+ \frac{2 \cdot N}{\beta \cdot h} \cdot \sin\left(h \cdot \frac{\pi - \beta}{2}\right) \ldots$$ Term 6
$$+ \frac{-2 \cdot N}{\pi \cdot h} \cdot \sin\left(h \cdot \frac{\pi - \beta}{2}\right)$$ Term 7

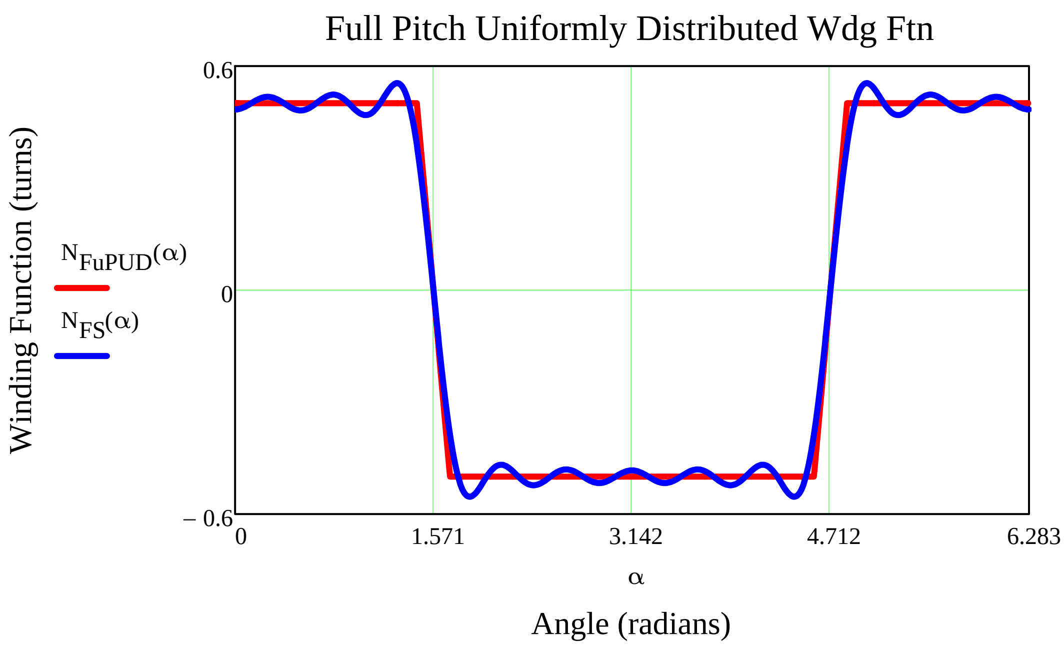
$$a_h = \frac{4 \cdot N}{\pi \cdot \beta \cdot h^2} \cdot \cos\left(h \cdot \frac{\pi}{2} - h \cdot \frac{\beta}{2}\right) = \frac{4 \cdot N}{\pi \cdot \beta \cdot h^2} \cdot \left(\cos\left(h \cdot \frac{\pi}{2}\right) \cdot \cos\left(h \cdot \frac{\beta}{2}\right) + \sin\left(h \cdot \frac{\pi}{2}\right) \cdot \sin\left(h \cdot \frac{\beta}{2}\right)\right)$$

$$\cos\left(h \cdot \frac{\pi}{2}\right) = 0$$ for all h

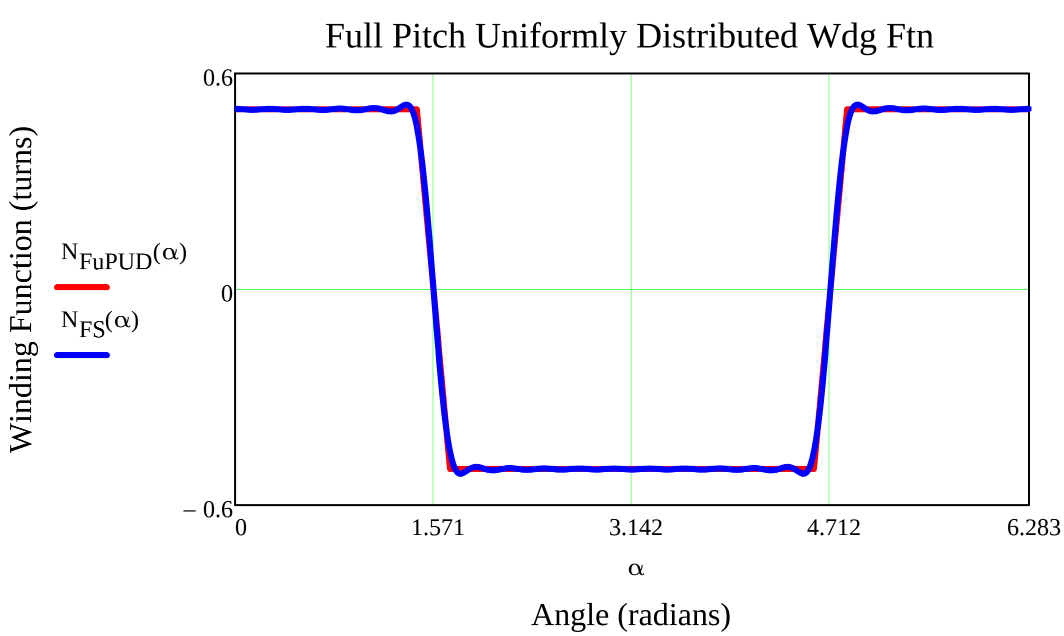
$$a(h) = \frac{4 \cdot N}{\pi \cdot \beta \cdot h^2} \cdot \sin\left(h \cdot \frac{\pi}{2}\right) \cdot \sin\left(h \cdot \frac{\beta}{2}\right) \qquad a(h) = \frac{2 \cdot N}{\pi \cdot h} \cdot \sin\left(h \cdot \frac{\pi}{2}\right) \cdot \frac{2}{\beta \cdot h} \cdot \sin\left(h \cdot \frac{\beta}{2}\right)$$

$$a(h) := \frac{2 \cdot N}{\pi \cdot h} \cdot \sin\left(h \cdot \frac{\pi}{2}\right) \cdot \mathrm{sinc}\left(h \cdot \frac{\beta}{2}\right)$$

$h := 1, 3 .. 11$ $N_{FS}(\alpha) := \sum_h (a(h) \cdot \cos(h \cdot \alpha))$



$h := 1, 3 .. 21$ $N_{FS}(\alpha) := \sum_h (a(h) \cdot \cos(h \cdot \alpha))$



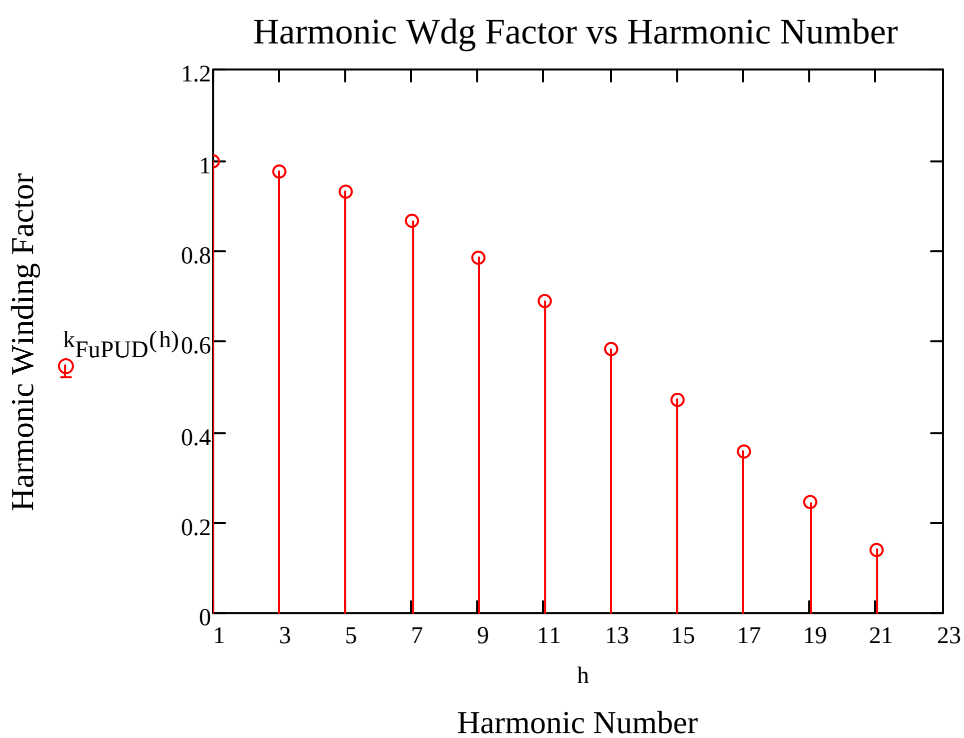
III. Determine the winding factor for a full pitch uniformly distributed coil; i.e., the distribution factor.

$$a_{FuPUD}(h) := \frac{2\cdot N}{\pi\cdot h}\cdot\sin\left(h\cdot\frac{\pi}{2}\right)\cdot\text{sinc}\left(h\cdot\frac{\beta}{2}\right)$$ for a full pitch uniformly distributed coil

$$a_{FuPC}(h) := \frac{2\cdot N}{\pi\cdot h}\cdot\sin\left(h\cdot\frac{\pi}{2}\right)$$ for a full pitch uniformly distributed coil

$$k_{FuPUD}(h) = \frac{a_{FuPUD}(h)}{a_{FuPC}(h)} = \frac{\frac{2\cdot N}{\pi\cdot h}\cdot\sin\left(h\cdot\frac{\pi}{2}\right)\cdot\text{sinc}\left(h\cdot\frac{\beta}{2}\right)}{\frac{2\cdot N}{\pi\cdot h}\cdot\sin\left(h\cdot\frac{\pi}{2}\right)}$$

$$k_{FuPUD}(h) := \text{sinc}\left(h\cdot\frac{\beta}{2}\right)$$



h =	$a_{FuPUD}(h)$ =	$a_{FuPC}(h)$ =	$k_{FuPUD}(h)$ =
1	0.635	0.637	0.997
3	-0.207	-0.212	0.974
5	0.118	0.127	0.93
7	-0.079	-0.091	0.866
9	0.055	0.071	0.784
11	-0.04	-0.058	0.689
13	0.029	0.049	0.583
15	-0.02	-0.042	0.471
17	0.013	0.037	0.357
19	$-8.201 \cdot 10^{-3}$	-0.034	0.245
21	$4.22 \cdot 10^{-3}$	0.03	0.139