Open Stator Circuit Voltage Build-up

I. Define the Situation

A synchronous generator is rotating at aconstant angular velocity, ω.

The rotor angle, θ_r, is given by the following. $\theta_r = \omega \cdot t + \theta_{r0}$

The three stator windings are open-circuited; i.e., $i_a = i_b = i_c = 0$

A DC voltage is applied to the field voltage at time equal to zero; i.e.,

$v_f = V_{fdc}$ for time greater or equal to zero

Initial field current is zero.

II. Determine $i_f(t)$.

$$V_{fdc} = R_{fd} \cdot i_f(t) + \frac{d}{dt}\psi_f(t) \qquad (1)$$

$$\psi_f(t) = L_{ff} \cdot i_f(t) \qquad (2)$$

Substitute Eq. 2 in to Eq. 1.

$$V_{fdc} = R_{fd} \cdot i_f(t) + \frac{d}{dt}\left(L_{ff} \cdot i_f(t)\right) \qquad (3)$$

L_{ff} is a constant w.r.t. time.

$$V_{fdc} = R_{fd} \cdot i_f(t) + L_{ff} \cdot \frac{d}{dt} i_f(t) \qquad (4)$$

The solution of Eq. 4 is given below.

$$i_f(t) = \frac{V_{fdc}}{R_f} \cdot \left(1 - e^{\frac{-t}{\tau_f}}\right) \qquad (5)$$

where $\tau_f = \frac{L_{ff}}{R_f}$

III. Determine $v_a(t)$.

$$v_a(t) = \frac{d}{dt}\psi_a(t) \qquad (6)$$

$$\psi_a(t) = L_{af}(t)\cdot i_f(t) \qquad (7)$$

$$L_{af}(t) = L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0}) \qquad (8)$$

Substitute Eq. 8 in to Eq. 7.

$$\psi_a(t) = L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\cdot i_f(t) \qquad (9)$$

Substitute Eq. 9 in to Eq. 6.

$$v_a(t) = \frac{d}{dt}\left(L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\cdot i_f(t)\right) \qquad (10)$$

Using the product rule of differentiating yields:

$$v_a(t) = i_f(t)\cdot\frac{d}{dt}\left(L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\right) \ldots$$
$$+ L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\cdot\left(\frac{d}{dt}i_f(t)\right) \qquad (11)$$

$$v_a(t) = -i_f(t)\cdot\omega\cdot L_{sf}\cdot\sin(\omega\cdot t + \theta_{r0}) \ldots$$
$$+ L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\cdot\frac{d}{dt}\left[\frac{V_{fdc}}{R_f}\cdot\left(1 - e^{\frac{-t}{\tau_f}}\right)\right] \qquad (12)$$

$$v_a(t) = -\left[\frac{V_{fdc}}{R_f}\cdot\left(1 - e^{\frac{-t}{\tau_f}}\right)\right]\cdot\omega\cdot L_{sf}\cdot\sin(\omega\cdot t + \theta_{r0}) \ldots$$
$$+ L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\cdot\frac{V_{fdc}}{\tau_f\cdot R_f}\cdot e^{\frac{-t}{\tau_f}} \qquad (13)$$

$$v_a(t) = -\frac{V_{fdc}}{R_f}\cdot\left(1 - e^{\frac{-t}{\tau_f}}\right)\cdot\omega\cdot L_{sf}\cdot\sin(\omega\cdot t + \theta_{r0}) \;...$$

$$+ L_{sf}\cdot\cos(\omega\cdot t + \theta_{r0})\cdot\frac{V_{fdc}}{L_f}\cdot e^{\frac{-t}{\tau_f}} \qquad (14)$$

$$v_a(t) = -\frac{V_{fdc}}{R_f}\cdot\left(1 - e^{\frac{-t}{\tau_f}}\right)\cdot\omega\cdot L_{sf}\cdot\sin(\omega\cdot t + \theta_{r0}) \;...$$

$$+ V_{fdc}\cdot\frac{L_{sf}}{L_f}\cdot e^{\frac{-t}{\tau_f}}\cdot\cos(\omega\cdot t + \theta_{r0}) \qquad (15)$$

or

$$v_a(t) = -\frac{V_{fdc}}{R_f}\cdot\omega\cdot L_{sf}\cdot\sin(\omega\cdot t + \theta_{r0}) \;...$$

$$+ V_{fdc}\cdot e^{\frac{-t}{\tau_f}}\cdot\left(\frac{\omega\cdot L_{sf}}{R_f}\cdot\sin(\omega\cdot t + \theta_{r0}) + \frac{L_{sf}}{L_f}\cdot\cos(\omega\cdot t + \theta_{r0})\right) \qquad (16)$$

IV. Graph to Visualize

$V_{fdc} := 140\cdot V$ $\quad R_f := 0.20\cdot\Omega$ $\quad L_f := 1.4\cdot H$ $\quad L_{sf} := 0.0743\cdot H$ $\quad \omega := 377\cdot\frac{rad}{sec}$

$\frac{V_{fdc}}{R_f} = 700\ A$ $\quad \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{V_{fdc}}{R_f}\cdot\omega\cdot L_{sf} = 24.015\ kV$ $\quad$ A 24 kV machine.

$\tau_f := \frac{L_f}{R_f}$ $\quad \tau_f = 7\ s$ $\quad \theta_{r0} := \frac{-\pi}{2}$

$$v_{a1}(t) := -\left[\frac{V_{fdc}}{R_f}\cdot\left(1 - e^{\frac{-t}{\tau_f}}\right)\right]\cdot\omega\cdot L_{sf}\cdot\sin\left(\omega\cdot t + \theta_{r0}\right)$$

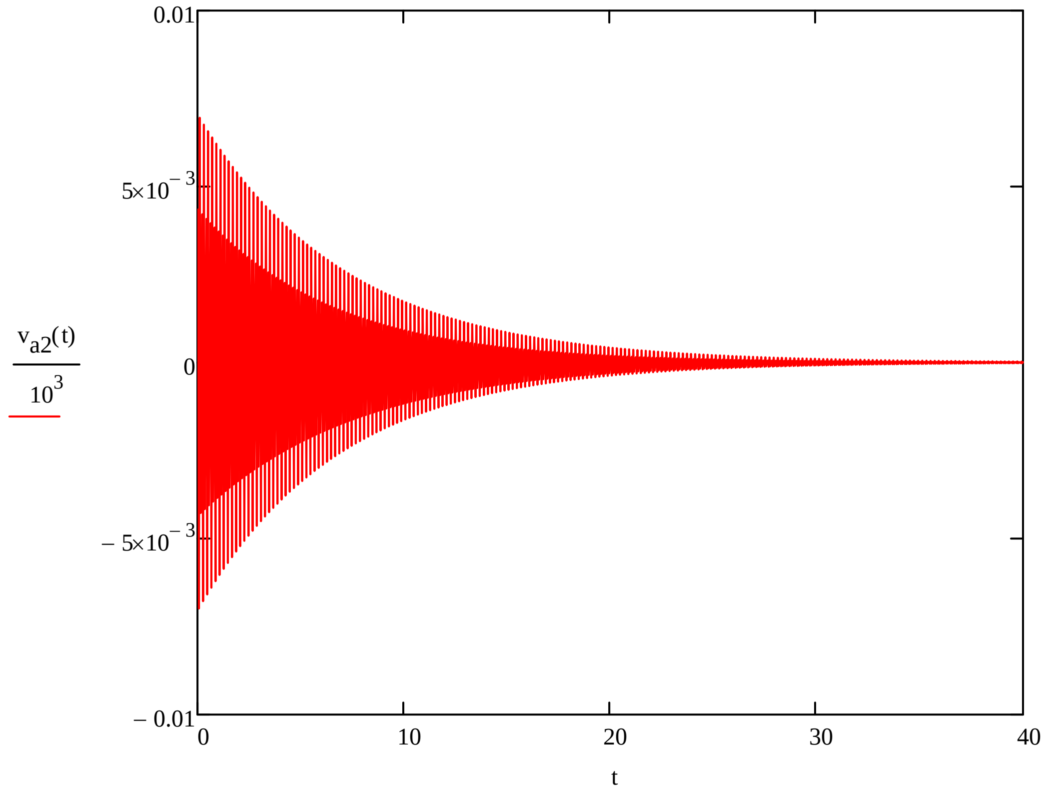
$$\frac{V_{fdc}}{R_f}\cdot\omega\cdot L_{sf} = 19.608\,\text{kV}$$

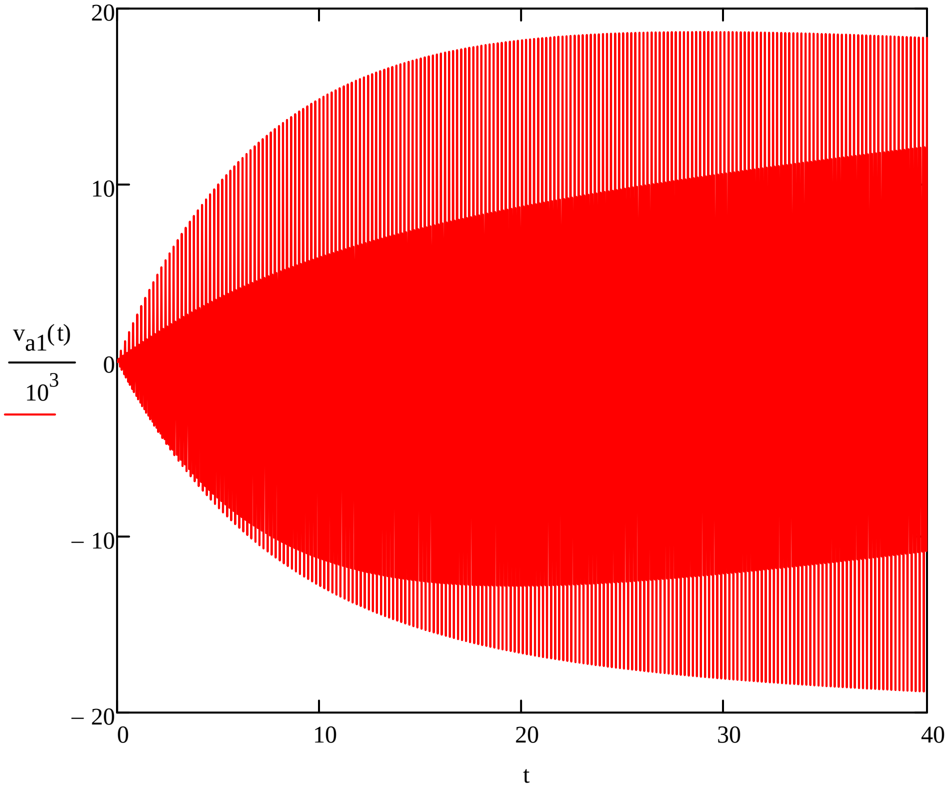
$$v_{a2}(t) := V_{fdc}\cdot\frac{L_{sf}}{L_f}\cdot e^{\frac{-t}{\tau_f}}\cdot\cos\left(\omega\cdot t + \theta_{r0}\right)$$

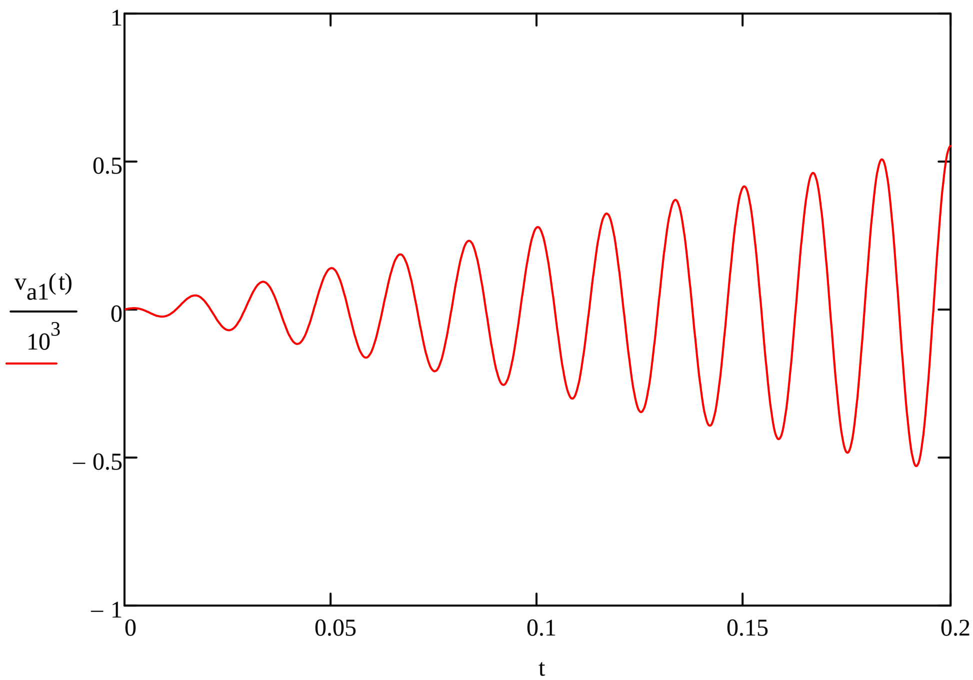
$$V_{fdc}\cdot\frac{L_{sf}}{L_f} = 7.43\,\text{V}$$

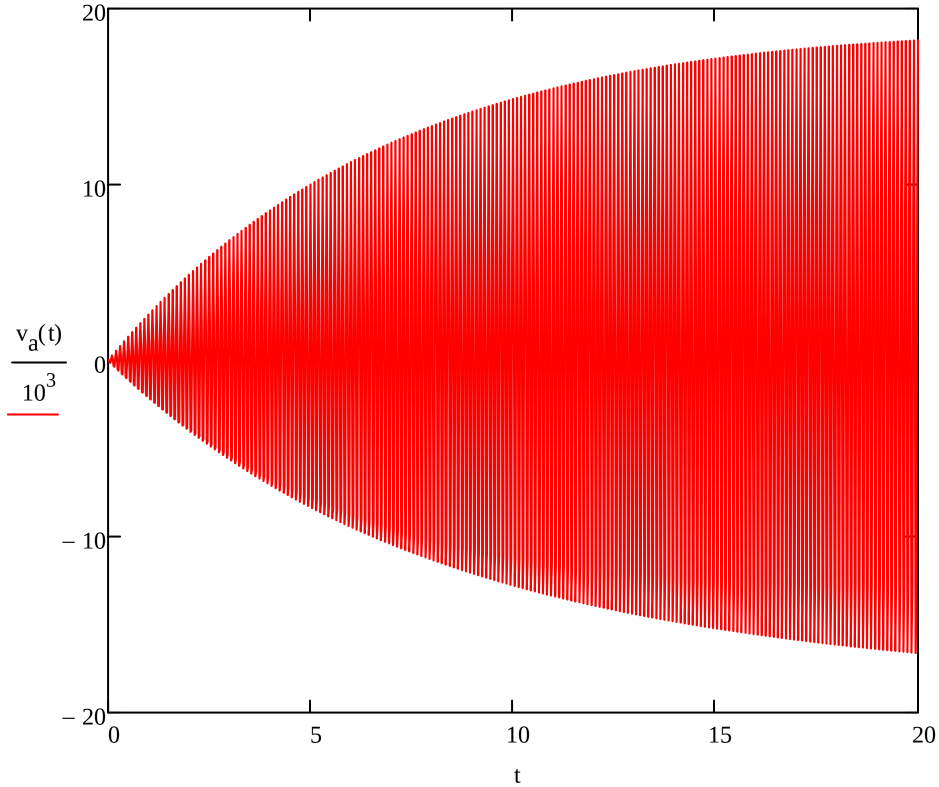
The transformer voltage term is negligible compared to the speed voltage term.

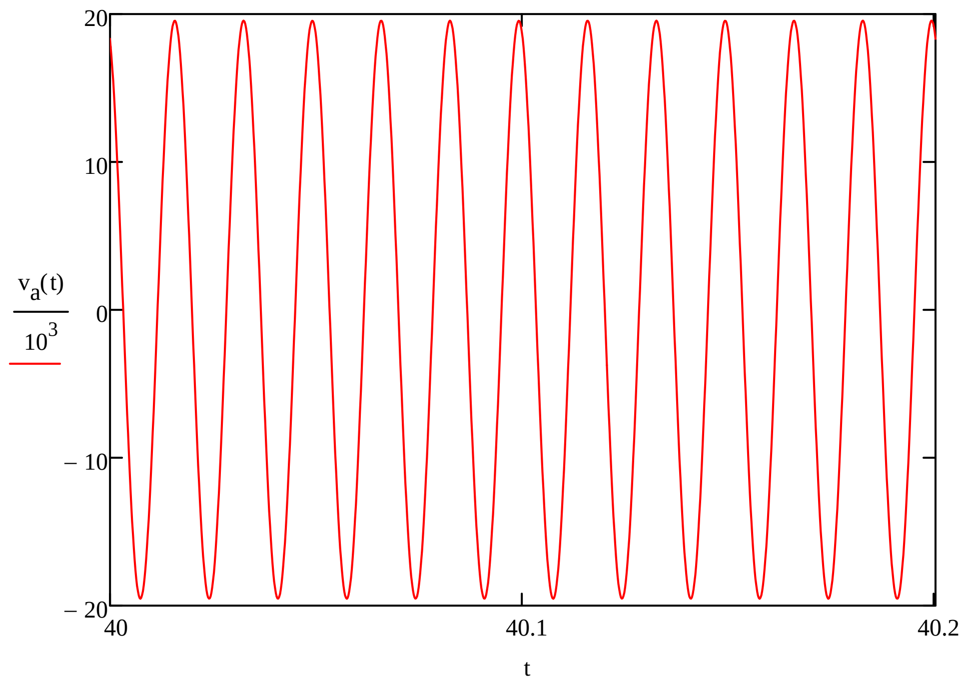
$$v_a(t) := v_{a1}(t) + v_{a2}(t)$$




20
10
0
− 10
− 20
0
10
20
30
40
t
$\frac{v_{a1}(t)}{10^3}$


1
0.5
0
− 0.5
− 1
0
0.05
0.1
0.15
0.2
t
$\frac{v_{a1}(t)}{10^3}$


20
10
0
– 10
– 20
$\frac{v_a(t)}{10^3}$
0
5
10
15
20
t


20
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$\frac{v_a(t)}{10^3}$
40
40.1
40.2
t