DIRECT AXIS SYNCHRONOUS MACHINE EQUATION MANIPULATION FOR THE PURPOSE OF SIMULATION

Non-Reciprocal Per Unit System ($L_{md}=1$ and $R_{fd}=1$)

I. Variables

Inputs: $v_{fd}$ and $i_{ds}$

Output: $\psi_{ds}$

Internal: $i_{fd}$, $i_{kd}$, $E'_q = \frac{L_{md}}{L_{ffd}} \psi_{fd}$, and $\psi_{kd}$

Unknowns: $i_{fd}$, $i_{kd}$, $\psi_{ds}$, $E'_q = \frac{L_{md}}{L_{ffd}} \psi_{fd}$, and $\psi_{kd}$; i.e., output plus internal variables

Note: There are 5 unknowns. Therefore, we need 5 equations.

II. Voltage Equations:

\[
v_{fd} = \frac{p}{\omega_B} \psi_{fd} + R_{fd} i_{fd} \tag{1}
\]

\[
v_{kd} = 0 = \frac{p}{\omega_B} \psi_{kd} + R_{kd} i_{kd} \tag{2}
\]

III. Flux Linkage Equations:

\[
\psi_{fd} = L_{ffd} i_{fd} + \beta L_{md} (i_{kd} - i_{ds}) = L_{ffd} i_{fd} + \beta L_{md} (i_{fd} + i_{kd} - i_{ds}) \tag{3}
\]

\[
\psi_{kd} = L_{kkd} i_{kd} + L_{md} (i_{fd} - i_{ds}) = L_{kkd} i_{kd} + L_{md} (i_{fd} + i_{kd} - i_{ds}) \tag{4}
\]

\[
\psi_{ds} = -L_d i_{ds} + L_{md} (i_{fd} + i_{kd}) = -L_d i_{ds} + L_{md} (i_{fd} + i_{kd} - i_{ds}) \tag{5}
\]
IV. Solve for the Inputs to the Integrators

Select $E'_q$ and $\psi_{kd}$ as the outputs of the integrators.

A. $E'_q$

Modify Eq. 1 to get $E'_q$

Multiple $\psi_{fd}$ by 1 in the form of $\frac{L_{ffd} L_{md}}{L_{md} L_{ffd}}$

$$v_{fd} = \frac{p}{\omega_B} \frac{L_{ffd} L_{md}}{L_{md} L_{ffd}} \psi_{fd} + R_{fd} i_{fd}$$

Take the Laplace transform of Eq. 6 and divide by $R_{fd}$

$$\frac{V_{fd}}{R_{fd}} = \frac{V_{fd}}{1} = s \frac{L_{ffd}}{\omega_B R_{fd}} E'_q + I_{fd}$$

where

$$E'_q \equiv \frac{L_{md}}{L_{ffd}} \psi_{fd}$$

$$V_{fd} = s \tau_{do}' E'_q + I_{fd}$$

where

$$\tau_{do}' = \frac{L_{ffd}}{\omega_B R_{fd}}$$

Solve for $E'_q$

$$E'_q = \frac{1}{s \tau_{do}'} (V_{fd} - I_{fd})$$
B. $\psi_{kd}$

Multiply Eq. 2 by $L''_{do}$ and take the Laplace transform of it.

$$0 = \frac{s}{\omega_B} L''_{do} \psi_{kd} + R_{kd} L''_{do} I_{kd}$$  \hfill (12)

\[
L''_{do} \equiv L_{lkd} + \frac{L_{md} L_{fd}}{\beta} \frac{L_{fd}}{L_{ffd}}
\]

\[
L''_{do} = L_{lkd} + \frac{L_{md} L_{fd}}{L_{ffd}} + \frac{(L_{md} \beta L_{md} - \beta L^2_{md})}{L_{ffd}}
\]

\[
= L_{lkd} + L_{md} \left( L_{fd} + \beta L_{md} - \frac{\beta L^2_{md}}{L_{ffd}} \right)
\]

\[
= L_{lkd} + L_{md} - \frac{\beta L^2_{md}}{L_{ffd}}
\]

\[
L''_{do} = L_{kkd} - \frac{\beta L^2_{md}}{L_{ffd}}
\]  \hfill (13)

Divide Eq. 12 by $R_{kd}$

$$0 = \frac{s}{\omega_B R_{kd}} \psi_{kd} + \frac{L''_{do}}{R_{kd}} I_{kd}$$  \hfill (15)

$$0 = s \tau''_{do} \psi_{kd} + L''_{do} I_{kd}$$  \hfill (16)

where

$$\tau''_{do} \equiv \frac{L''_{do}}{\omega_B R_{kd}}$$  \hfill (17)

Solve for $\Psi''_{kd}$

$$\Psi_{kd} = -\frac{1}{s \tau''_{do}} L''_{do} I_{kd}$$  \hfill (18)
V. Solve for $i_{fd}$ and $-L''_{do}i_{kd}$

A. $I_{fd}$

Solve Eq. 3 for $L_{ffd}i_{fd}$ and take the Laplace Transform of it

$$L_{ffd}i_{fd} = \Psi_{fd} + \beta L_{md}(I_{ds} - I_{kd})$$  \hspace{1cm} (19)

Divide by $L_{ffd}$

$$I_{fd} = \frac{\Psi_{fd}}{L_{ffd}} + \frac{\beta L_{md}}{L_{ffd}}(I_{ds} - I_{kd})$$  \hspace{1cm} (20)

Multiple $\Psi_{fd}$ by 1 in the form of $\frac{L_{md}}{L_{md}}$

$$I_{fd} = \frac{L_{md}}{L_{md}} \frac{\Psi_{fd}}{L_{ffd}} + \frac{\beta L_{md}}{L_{ffd}}(I_{ds} - I_{kd})$$  \hspace{1cm} (21)

$$I_{fd} = \frac{E'_{qd}}{L_{md}} + \frac{\beta L_{md}}{L_{ffd}}(I_{ds} - I_{kd})$$  \hspace{1cm} (22)

$$(L_d - L'_d) = L_d - (L_d - \frac{\beta L_{md}^2}{L_{ffd}})$$

$$(L_d - L'_d) = \frac{\beta L_{md}^2}{L_{ffd}}$$

$$\frac{L_d - L'_d}{L_{md}} = \frac{\beta L_{md}}{L_{ffd}}$$

$$(L_{du} - L_{d_{du}}') = \frac{\beta L_{md}}{L_{ffd}}$$  \hspace{1cm} (23)

$$I_{fd} = \frac{E'_{qd}}{L_{md}} + (L_{du} - L_{d_{du}}')(I_{ds} - I_{kd})$$  \hspace{1cm} (24)
B. \(-L''_{do}i_{kd}\)

Copy Eq. 4 here for easy reference.

\[
\begin{align*}
\psi_{kd} &= L_{kkd}i_{kd} + L_{md}(i_{fd} - i_{ds}) \\
\psi_{kd} &= L_{kkd}i_{kd} + L_{md}i_{fd} - L_{md}i_{ds}
\end{align*}
\] (25)

Multiply Eq. 22 by \(L_{md}\) and write it below.

\[
L_{md}I_{fd} = E'_q + \frac{\beta L^2_{md}}{L_{ffd}}(I_{ds} - I_{kd})
\] (26)

Substitute Eq. 26 into Eq. 25 and take the Laplace Transform of it.

\[
\Psi_{kd} = L_{kkd}I_{kd} + E'_q + \frac{\beta L^2_{md}}{L_{ffd}}(I_{ds} - I_{kd}) - L_{md}I_{ds}
\] (27)

Write Eq. 27 using the distributive property of multiplication.

\[
\Psi_{kd} = L_{kkd}I_{kd} + E'_q + \frac{\beta L^2_{md}}{L_{ffd}}I_{ds} - \frac{\beta L^2_{md}}{L_{ffd}}I_{kd} - L_{md}I_{ds}
\] (28)

Write Eq. 27 with terms grouped by current.

\[
\Psi_{kd} = E'_q - (L_{md} - \frac{\beta L^2_{md}}{L_{ffd}})I_{ds} + (L_{kkd} - \frac{\beta L^2_{md}}{L_{ffd}})I_{kd}
\] (29)

Copy Eq. 14 here for easy reference.

\[
L''_{do} = L_{kkd} - \frac{\beta L^2_{md}}{L_{ffd}}
\] (30)

Investigate \(L_{md} - \frac{\beta L^2_{md}}{L_{ffd}}\).
\[
L_d' \equiv L_d - \frac{\beta L_{md}^2}{L_{ffd}} \\
= L_{md} + L_l - \frac{\beta L_{md}^2}{L_{ffd}} \\
L_d' - L_l = L_{md} - \frac{\beta L_{md}^2}{L_{ffd}} \\
= \frac{L_{md}(\beta L_{md} + L_{lf}d) - \beta L_{md}^2}{L_{ffd}} \\
= \frac{L_{lf}d L_{md}}{L_{ffd}} \tag{32}
\]

Equation 29 can be rewritten using preceding equations for inductances.

\[
\Psi_{kd} = E_q' - (L_d' - L_l)I_{ds} + L_{do}''I_{kd} \tag{33}
\]

Solving Eq. 28 for \(-L_{do}''I_{kd}\)

\[
-L_{do}''I_{kd} = E_q' - (L_d' - L_l)I_{ds} - \Psi_{kd} \tag{34}
\]

C. \(\psi_{ds}\)

Equation 5 can be used to solve for \(\Psi_{ds}\)

\[
\Psi_{ds} = -L_l I_{ds} + L_{md}(I_{fd} + I_{kd} - I_{ds}) \tag{35}
\]

Summary

\[
E_q' = \frac{1}{s \tau_{do}'}(V_{fd} - I_{fd}) \tag{36}
\]

\[
\Psi_{kd} = -\frac{1}{s \tau_{do}''}L_{do}''I_{kd} \tag{37}
\]
\[ I_{fd} = \frac{E_q'}{L_{md}} + (L_{du} - L_{du}') (I_{ds} - I_{kd}) \]  \hspace{1cm} (38)

\[ -L_{do}' I_{kd} = E_q' - (L_d' - L_l) I_{ds} - \Psi_{kd} \]  \hspace{1cm} (39)

\[ \Psi_{ds} = -L_d I_{ds} + L_{md} (I_{fd} + I_{kd} - I_{ds}) \]  \hspace{1cm} (40)