# DIRECT AXIS SYNCHRONOUS MACHINE EQUATION MANIPULATION FOR THE PURPOSE OF SIMULATION

Non-Reciprocal Per Unit System ($L_{mdu} = 1$ and $R_{fd} = 1$)

## I. Variables

Inputs: $v_{fd}$ and $i_{ds}$

Output: $\psi_{ds}$

Internal: $i_{fd}$, $i_{kd}$, $E'_q = \frac{L_{md}}{L_{ffd}}\psi_{fd}$, and $\psi_{kd}$

Unknowns: $i_{fd}$, $i_{kd}$, $\psi_{ds}$, $E'_q = \frac{L_{md}}{L_{ffd}}\psi_{fd}$, and $\psi_{kd}$; i.e., output plus internal variables

Note: There are 5 unknows. Therefore, we need 5 equations.

## II. Voltage Equations:

$$v_{fd} = \frac{p}{\omega_B}\psi_{fd} + R_{fd}i_{fd} \tag{1}$$

$$v_{kd} = 0 = \frac{p}{\omega_B}\psi_{kd} + R_{kd}i_{kd} \tag{2}$$

## III. Flux Linkage Equations:

$$\psi_{fd} = L_{ffd}i_{fd} + \beta L_{md}(i_{kd} - i_{ds}) = L_{lfd}i_{fd} + \beta L_{md}(i_{fd} + i_{kd} - i_{ds}) \tag{3}$$

$$\psi_{kd} = L_{kkd}i_{kd} + L_{md}(i_{fd} - i_{ds}) = L_{lkd}i_{kd} + L_{md}(i_{fd} + i_{kd} - i_{ds}) \tag{4}$$

$$\psi_{ds} = -L_d i_{ds} + L_{md}(i_{fd} + i_{kd}) = -L_l i_{ds} + L_{md}(i_{fd} + i_{kd} - i_{ds}) \tag{5}$$

## IV. Solve for the Inputs to the Integrators

Select $E'_q$ and $\psi_{kd}$ as the ouputs of the integrators.

**A.** $E'_q$

Modify **Eq. 1** to get $E'_q$

Multiple $\psi_{fd}$ by 1 in the form of $\frac{L_{ffd}}{L_{md}}\frac{L_{md}}{L_{ffd}}$

$$v_{fd} = \frac{p}{\omega_B}\frac{L_{ffd}}{L_{md}}\frac{L_{md}}{L_{ffd}}\psi_{fd} + R_{fd} i_{fd} \tag{6}$$

Take the Laplace transform of Eq. 6 and divide by $R_{fd}$

$$\frac{V_{fd}}{R_{fd}} = \frac{V_{fd}}{1} = s\frac{L_{ffd}}{\omega_B R_{fd}}E'_q + I_{fd} \tag{7}$$

where

$$E'_q \equiv \frac{L_{md}}{L_{ffd}}\Psi_{fd} \tag{8}$$

$$V_{fd} = s\,\tau'_{do}E'_q + I_{fd} \tag{9}$$

where

$$\tau'_{do} \equiv \frac{L_{ffd}}{\omega_B R_{fd}} \tag{10}$$

Solve for $E'_q$

$$E'_q = \frac{1}{s\,\tau'_{do}}(V_{fd} - I_{fd}) \tag{11}$$

**B.** $\psi_{kd}$

Multiply **Eq. 2** by $L''_{do}$ and take the Laplace transform of it.

$$0 = \frac{s}{\omega_B} L''_{do} \Psi_{kd} + R_{kd} L''_{do} I_{kd} \tag{12}$$

$$\begin{aligned}
L''_{do} &\equiv L_{lkd} + \frac{L_{md}\frac{L_{lfd}}{\beta}}{\frac{L_{ffd}}{\beta}} \\
L''_{do} &= L_{lkd} + \frac{L_{md} L_{lfd}}{L_{ffd}} \qquad (13) \\
&= L_{lkd} + \frac{L_{md} L_{lfd} + (L_{md}\beta L_{md} - \beta L_{md}^2)}{L_{ffd}} \\
&= L_{lkd} + \frac{L_{md}(L_{lfd} + \beta L_{md}) - \beta L_{md}^2}{L_{ffd}} \\
&= (L_{lkd} + L_{md}) - \frac{\beta L_{md}^2}{L_{ffd}} \\
L''_{do} &= L_{kkd} - \frac{\beta L_{md}^2}{L_{ffd}} \qquad (14)
\end{aligned}$$

Divide Eq. 12 by $R_{kd}$

$$0 = s \frac{L''_{do}}{\omega_B R_{kd}} \Psi_{kd} + L''_{do} I_{kd} \tag{15}$$

$$0 = s \tau''_{do} \Psi_{kd} + L''_{do} I_{kd} \tag{16}$$

where

$$\tau''_{do} \equiv \frac{L''_{do}}{\omega_B R_{kd}} \tag{17}$$

Solve for $\Psi''_{kd}$

$$\Psi_{kd} = -\frac{1}{s\tau''_{do}} L''_{do} I_{kd} \tag{18}$$

## V. Solve for $i_{fd}$ and $-L''_{do} i_{kd}$

**A.** $I_{fd}$

Solve **Eq. 3** for $L_{ffd} i_{fd}$ and take the Laplace Transform of it

$$L_{ffd} I_{fd} = \Psi_{fd} + \beta L_{md}(I_{ds} - I_{kd}) \tag{19}$$

Divide by $L_{ffd}$

$$I_{fd} = \frac{\Psi_{fd}}{L_{ffd}} + \frac{\beta L_{md}}{L_{ffd}}(I_{ds} - I_{kd}) \tag{20}$$

Multiple $\Psi_{fd}$ by 1 in the form of $\frac{L_{md}}{L_{md}}$

$$I_{fd} = \frac{L_{md}}{L_{md}} \frac{\Psi_{fd}}{L_{ffd}} + \frac{\beta L_{md}}{L_{ffd}}(I_{ds} - I_{kd}) \tag{21}$$

$$I_{fd} = \frac{E'_q}{L_{md}} + \frac{\beta L_{md}}{L_{ffd}}(I_{ds} - I_{kd}) \tag{22}$$

$$\begin{aligned}
(L_d - L'_d) &= L_d - (L_d - \frac{\beta L_{md}^2}{L_{ffd}}) \\
(L_d - L'_d) &= \frac{\beta L_{md}^2}{L_{ffd}} \\
\frac{L_d - L'_d}{L_{md}} &= \frac{\beta L_{md}}{L_{ffd}} \\
(L_{du} - L'_{du}) &= \frac{\beta L_{md}}{L_{ffd}} \qquad (23)
\end{aligned}$$

$$I_{fd} = \frac{E'_q}{L_{md}} + (L_{du} - L'_{du})(I_{ds} - I_{kd}) \tag{24}$$

**B.** $-L''_{do} i_{kd}$

Copy **Eq. 4** here for easy reference.

$$\begin{aligned} \psi_{kd} &= L_{kkd} i_{kd} + L_{md}(i_{fd} - i_{ds}) \\ \psi_{kd} &= L_{kkd} i_{kd} + L_{md} i_{fd} - L_{md} i_{ds} \end{aligned} \tag{25}$$

Multiply Eq. 22 by $L_{md}$ and write it below.

$$L_{md} I_{fd} = E'_q + \frac{\beta L_{md}^2}{L_{ffd}}(I_{ds} - I_{kd}) \tag{26}$$

Substitute Eq. 26 into Eq. 25 and take the Laplace Transform of it.

$$\Psi_{kd} = L_{kkd} I_{kd} + E'_q + \frac{\beta L_{md}^2}{L_{ffd}}(I_{ds} - I_{kd}) - L_{md} I_{ds} \tag{27}$$

Write Eq. 27 using the distributive property of multiplication.

$$\Psi_{kd} = L_{kkd} I_{kd} + E'_q + \frac{\beta L_{md}^2}{L_{ffd}} I_{ds} - \frac{\beta L_{md}^2}{L_{ffd}} I_{kd} - L_{md} I_{ds} \tag{28}$$

Write Eq. 27 with terms grouped by current.

$$\Psi_{kd} = E'_q - (L_{md} - \frac{\beta L_{md}^2}{L_{ffd}}) I_{ds} + (L_{kkd} - \frac{\beta L_{md}^2}{L_{ffd}}) I_{kd} \tag{29}$$

Copy Eq. 14 here for easy reference.

$$L''_{do} = L_{kkd} - \frac{\beta L_{md}^2}{L_{ffd}} \tag{30}$$

Investigate $L_{md} - \frac{\beta L_{md}^2}{L_{ffd}}$.

$$\begin{aligned}
L'_d &\equiv L_d - \frac{\beta L_{md}^2}{L_{ffd}} \\
&= L_{md} + L_l - \frac{\beta L_{md}^2}{L_{ffd}} \\
L'_d - L_l &= L_{md} - \frac{\beta L_{md}^2}{L_{ffd}} \qquad (31) \\
&= \frac{L_{md}(\beta L_{md} + L_{lfd}) - \beta L_{md}^2}{L_{ffd}} \\
&= \frac{L_{lfd} L_{md}}{L_{ffd}} \qquad (32)
\end{aligned}$$

Equation 29 can be rewritten using preceding equations for inductances.

$$\Psi_{kd} = E'_q - (L'_d - L_l) I_{ds} + L''_{do} I_{kd} \qquad (33)$$

Solving Eq. 28 for $-L''_{do} I_{kd}$

$$-L''_{do} I_{kd} = E'_q - (L'_d - L_l) I_{ds} - \Psi_{kd} \qquad (34)$$

**C.** $\psi_{ds}$

**Equation 5** can be used to solve for $\Psi_{ds}$

$$\Psi_{ds} = -L_l I_{ds} + L_{md}(I_{fd} + I_{kd} - I_{ds}) \qquad (35)$$

**Summary**

$$E'_q = \frac{1}{s\,\tau'_{do}}(V_{fd} - I_{fd}) \qquad (36)$$

$$\Psi_{kd} = -\frac{1}{s\tau''_{do}} L''_{do} I_{kd} \qquad (37)$$

$$I_{fd} = \frac{E_q'}{L_{md}} + (L_{du} - L_{du}')(I_{ds} - I_{kd}) \tag{38}$$

$$-L_{do}'' I_{kd} = E_q' - (L_d' - L_l)I_{ds} - \Psi_{kd} \tag{39}$$

$$\Psi_{ds} = -L_l I_{ds} + L_{md}(I_{fd} + I_{kd} - I_{ds}) \tag{40}$$