Homework 5
Machine Windings

Do the following problems from the textbook:

1. **Questions B-1 and B-7 on page 657.**

   **B-1. Why are distributed windings used instead of concentrated windings in ac machine stators?**

   Distributing the windings allows us to get more turns into the machine stator within the available space.

   **B-7. What are slot harmonics? How can they be reduced?**

   Regular variations in the reluctance and magnetic flux along the stator’s surface cause harmonic components in the voltage, current, and torque waveforms. These variations are called slot harmonics.

   They may be reduced through the use of fractional slot windings and skewing the slots.

2. **Problem B-3 on page 658.**

   A three phase four pole synchronous machine has 96 stator slots. The slots contain a double layer winding (two coils per slot) with four turns per coil. The coil pitch is 19/24.

   a. Find the slot and coil pitch in electrical degrees.

   \[
   \text{Degrees}_\text{Mechanical} := \text{deg} \quad \text{Degrees}_\text{Electrical} := \text{deg}
   \]

   \[
   \text{CoilPitch} := \frac{19}{24} \quad \rho := \text{CoilPitch} \cdot 180 \cdot \text{deg} = 142.5 \cdot \text{Degrees}_\text{Electrical}
   \]

   \[
   \text{Poles} := 4 \quad \text{slots} := 96
   \]

   \[
   \text{SlotPitch} := \frac{360 \cdot \text{Degrees}_\text{Mechanical}}{\text{slots}} = 3.75 \cdot \text{Degrees}_\text{Mechanical}
   \]

   \[
   \gamma := \text{SlotPitch} \cdot \frac{\text{Poles}}{2} = 7.5 \cdot \text{Degrees}_\text{Electrical}
   \]
b. Find the pitch, distribution, and winding factors for this machine.

\[ \rho = 142.5 \text{ Degrees Electrical} \quad n := 8 \]

\[ k_p := \sin \left( \frac{\rho}{2} \right) = 0.947 \quad k_d := \frac{\sin \left( \frac{n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{\gamma}{2} \right)} = 0.956 \quad k_w := k_p \cdot k_d = 0.905 \]

c. How will this winding suppress third, fifth, seventh, ninth, and eleventh harmonics? Be sure to consider the effects of both coil pitch and winding distribution in your answer.

Generated voltage is

\[ E = 4.44 \cdot k_d \cdot k_p \cdot N \cdot \phi \cdot f \]

We will evaluate the effect of pitch and distribution factors on the \(|E|\). Calculate the pitch and distribution factors at each of the required harmonics.

\[ h := 3 \]

\[ k_p := \sin \left( \frac{h \cdot \rho}{2} \right) = -0.556 \quad k_d := \frac{\sin \left( \frac{h \cdot n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{h \cdot \gamma}{2} \right)} = 0.641 \quad k_w := k_p \cdot k_d = -0.356 \]

\[ h := 5 \]

\[ k_p := \sin \left( \frac{h \cdot \rho}{2} \right) = -0.065 \quad k_d := \frac{\sin \left( \frac{h \cdot n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{h \cdot \gamma}{2} \right)} = 0.194 \quad k_w := k_p \cdot k_d = -0.013 \]

\[ h := 7 \]

\[ k_p := \sin \left( \frac{h \cdot \rho}{2} \right) = 0.659 \quad k_d := \frac{\sin \left( \frac{h \cdot n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{h \cdot \gamma}{2} \right)} = -0.141 \quad k_w := k_p \cdot k_d = -0.093 \]

\[ h := 9 \]

\[ k_p := \sin \left( \frac{h \cdot \rho}{2} \right) = -0.981 \quad k_d := \frac{\sin \left( \frac{h \cdot n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{h \cdot \gamma}{2} \right)} = -0.225 \quad k_w := k_p \cdot k_d = 0.221 \]

\[ h := 11 \]

\[ k_p := \sin \left( \frac{h \cdot \rho}{2} \right) = 0.897 \quad k_d := \frac{\sin \left( \frac{h \cdot n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{h \cdot \gamma}{2} \right)} = -0.095 \quad k_w := k_p \cdot k_d = -0.085 \]

This does fairly well in suppressing 5, 7, and 11 (<10% of nominal). It does less well on 3 and 9, but a three phase system by nature eliminates triplen harmonics.
3. Problem B-4 on page 658.

A three phase four pole winding of the double layer type is to be installed on a 48 slot stator. The pitch of the stator windings is 5 / 6 and there are 10 turns per coil in the windings. All coils in each phase are connected in series and the three phases are connected in \( \Delta \). The flux per pole in the machine is 0.054 Wb and the speed of rotation of the magnetic field is 1800 RPM.

a. What is the pitch factor of this winding?

\[
\begin{align*}
\text{poles} & := 4 \quad \text{phases} := 3 \quad \text{slots} := 48 \quad \text{pitch} := \frac{5}{6} \quad \text{turns} := 10 \quad \varphi := 0.054 \cdot \text{Wb} \quad \text{speed} := 1800 \cdot \text{RPM} \\
\rho & := \text{pitch} \cdot \frac{360 \cdot \text{deg}}{2} = 150 \cdot \text{deg} \\
k_p & := \sin \left( \frac{\rho}{2} \right) = 0.966
\end{align*}
\]

b. What is the distribution factor of this winding?

\[
\begin{align*}
\gamma & := \frac{\text{slots}}{\text{poles} \cdot \text{phases}} = 4 \\
\gamma & := \frac{360 \cdot \text{deg}}{\text{slots} \cdot 2} = 15 \cdot \text{Degrees_Electrical} \\
k_d & := \frac{\sin \left( \frac{n \cdot \gamma}{2} \right)}{n \cdot \sin \left( \frac{\gamma}{2} \right)} = 0.958
\end{align*}
\]

c. What is the frequency of the voltage produced in this winding?

\[
f_e := \text{speed} \cdot \frac{\text{poles}}{2} = 60 \cdot \text{Hz}
\]

d. What are the resulting phase and terminal voltages of this stator?

\[
N_p := n \cdot \text{turns} = 40
\]

\[
E_G := \sqrt{2} \cdot \pi \cdot N_p \cdot k_p \cdot k_d \cdot \varphi \cdot f_e = 532.63 \text{ V}
\]

For a four-pole machine with pole pairs in series,

\[
V_T := \frac{\text{poles}}{2} \cdot E_G = 1.065 \cdot \text{kV}
\]
4. **Problem B-5 on page 658.**

A three phase Y connected six pole synchronous generator has six slots per pole on its stator winding. The winding itself is a chorded (fractional pitch) double layer winding with eight turns per coil. The distribution factor $k_d=0.956$ and the pitch factor $k_p=0.981$. The flux in the generator is $0.02$ Wb per pole and the speed of rotation is 1200 RPM. What is the line voltage produced by this generator at these conditions?

```plaintext
phases := 3  poles := 6  n := 6  turns := 8  k_d := 0.956  k_p := 0.981  \phi := 0.02 \cdot \text{Wb}  speed := 1200 \cdot \text{min}^{-1}

slots := n \cdot \text{poles} = 36  n := \frac{\text{slots}}{\text{phases}} = 12  N_p := n \cdot \text{turns} = 96  f_e := \text{speed} \cdot \frac{\text{poles}}{2} = 60 \cdot \text{Hz}

E_G := \sqrt{2} \cdot \pi \cdot N_p \cdot k_p \cdot k_d \cdot \phi \cdot f_e = 480.003 \text{ V}

For a Y connected winding,

V_T := \sqrt{3} \cdot E_G = 831.39 \text{ V}
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