Transformer equivalent circuit:

- Where Rp, Rs (or R1 and R2) are winding resistance.
- Xp and Xs (or X1 and X2) are leakage reactance (inductance). Function of magnetic material, geometry of the core.
- Xm is magnetizing reactance. This is a non-linear inductance, that varies with the voltage applied across the inductance. Function of magnetic material properties.
- Rc represents core losses.
- The transformer in the middle is an ideal transformer where the following relationships hold:

\[
\frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \text{or} \quad \frac{E_1}{N_1} = \frac{E_2}{N_2}
\]

and

\[
N_1 \cdot I_1 = -N_2 \cdot I_2 \quad \text{or} \quad N_1 \cdot I_2 + N_2 \cdot I_2 = 0 \quad \text{or} \quad \frac{I_1}{I_2} = -\frac{N_2}{N_1}
\]

If we treat I2 as flowing out of the polarity mark, then the negative sign goes away.

Combining these:

\[
S_1 = V_1 \cdot I_1
\]

\[
S_2 = V_2 \cdot I_2 = \left[ \left( \frac{N_2}{N_1} \right) \cdot V_1 \right] \cdot \left[ \left( \frac{-N_1}{N_2} \right) \cdot I_1 \right] = -V_1 \cdot I_1
\]

- Note that Ip is not equal to I1, since there are currents in the magnetizing and core loss branch. The magnitude of this current is generally 2-5% of the rated load current.
• The rated load current is based on the Volt-Ampere rating of the transformer and the rated voltages (primary and secondary). These are generally given on the transformer nameplate.

• Impedances can also be referred across the ideal transformer. This can make analysis easier, and also make it easier to determine parameters through testing.

For example, consider the secondary circuit with a load in place:

\[ I_2 = \frac{E_2}{R_s + jX_{ss} + R_{load}} \]

\[ Z_{eq} = \frac{E_2}{I_2} \]

We can express this in terms of the primary quantities

\[ Z_{eq} = \frac{N_2}{N_1} \cdot \frac{E_1}{I_1} = \left( \frac{N_2}{N_1} \right)^2 \cdot \frac{E_1}{I_1} \quad \text{The negative sign is left off for now.} \]

We can use this to refer the secondary impedances across the windings as shown. We can refer only the winding resistance and leakage inductance, or we could also refer the load too.
Where:
\[ E_2^1 = \left( \frac{N_1}{N_2} \right) E_2 \quad \quad \quad I_2^1 = \left( \frac{N_2}{N_1} \right) I_2 \]
\[ X_{s1}^1 = \left( \frac{N_2}{N_1} \right)^2 X_s \quad \quad \quad R_{s1}^1 = \left( \frac{N_2}{N_1} \right)^2 R_s \]

**Determination of Transformer Parameters from Test Data**

The nameplate for a transformer doesn't always include the values for parameters in the equivalent circuit. They can be determined through 2 tests.

1. **Short Circuit Test:**
   - Short circuit the secondary.
   - Apply a small voltage to the primary (it is easy to produce large currents)
   - Measure: \( V_{sc} \), \( I_{sc} \), and \( P_{sc} \)
   - The large impedance of the shunt branch (\( X_m \) and \( R_c \)) compared to \( X_s' \) and \( R_s' \) means that \( I_m \) is negligible, and this test determines the series branch parameters

   \[ Z_{series} = \frac{V_p}{I_p} \quad \quad \quad Z_{series} = Z_1 + Z_{21} \]
   \[ R_{series} = \frac{P_{sc}}{I_p^2} \quad \quad \quad R_{series} = R_1 + R_{21} \]
   \[ X_{series} = \sqrt{Z_{series}^2 - R_{series}^2} \quad \quad \quad X_{series} = X_1 + X_{21} \]

   - Note that this only give the combined \( Z_1 + Z_{21} \), and not the relative magnitudes of each. This be determine approximate ratio \( R_1 \) and \( R_2 \) by doing dc ohmmeter tests on the windings. A common rule of thumb is to say that \( Z_1 = Z_{21} \)
   - Note also that secondary quantities are referred to the primary in these calculations.

Alternate method with same results:

\[ \theta_{sc} = \cos^{-1} \left( \frac{P_{sc}}{V_{sc} \cdot I_{sc}} \right) \quad \quad \quad R_{series} = Z_{series} \cdot \cos(\theta_{sc}) \quad \quad \quad X_{eq} = Z_{series} \cdot \sin(\theta_{sc}) \]
2. Open circuit test:

- Open circuit the secondary.
- Apply rated voltage to the primary (nameplate voltage). It is important for this to be the nameplate voltage, since \( X_m \) is voltage dependent.
- Measure: \( V_{oc} \), \( I_{oc} \), and \( P_{oc} \)
- The large impedance of the shunt branch (\( X_m \) and \( R_c \)) compared to \( X_p \) and \( R_p \) means that the voltage across \( R_c \) and \( X_m \) is essentially the same as \( V_{oc} \).

\[
R_c = \frac{V_{oc}^2}{P_{oc}}
\]

\[
Z_{oc} = \frac{V_{oc}}{I_{oc}} = \frac{R_c \cdot (j \cdot X_m)}{R_c + j \cdot X_m} \quad \text{solve for } X_m
\]

Alternate Approach:

\[
Y_{oc} = \frac{I_{oc}}{V_{oc}} \quad \theta_{oc} = \acos\left( \frac{P_{oc}}{V_{oc} \cdot I_{oc}} \right)
\]

\[
R_c = \frac{1}{Y_{oc} \cdot \cos(\theta_{oc})}
\]

\[
X_m = \frac{1}{Y_{oc} \cdot \sin(\theta_{oc})}
\]